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COST- Towards Normal Equation Level Combination

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What is going on?

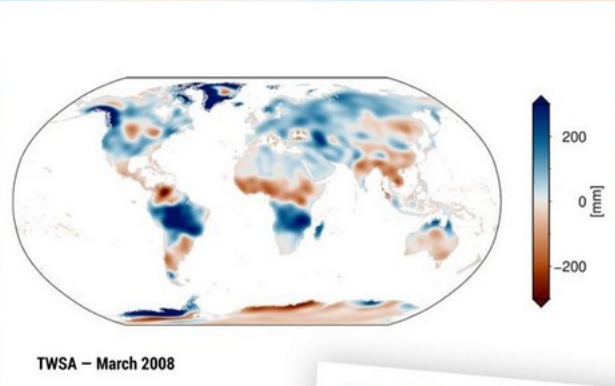
Gravity Field Solutions for dedicated Time Periods

FSM DOI	quarterly
Grace/Grace-FO RL02.1 DOI	monthly
Swarm DOI	monthly

C3S 





Now in the CDS:

The new Terrestrial Water Storage Anomalies (TWSA) dataset

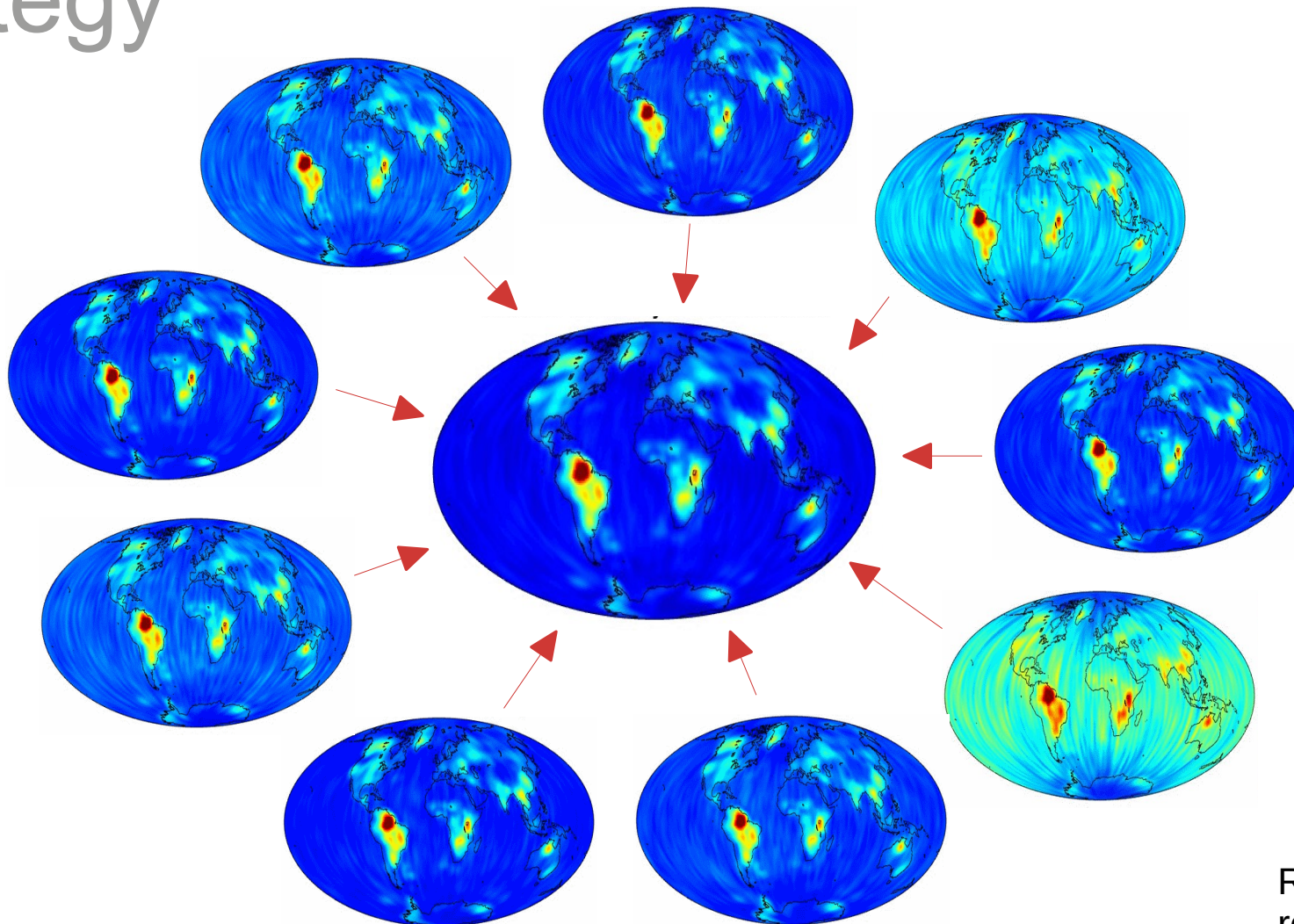


TWSA – March 2008

NOW AVAILABLE IN THE CLIMATE DATA STORE

 PROGRAMME OF THE EUROPEAN UNION   

Combination of Gravity Field Solutions Strategy



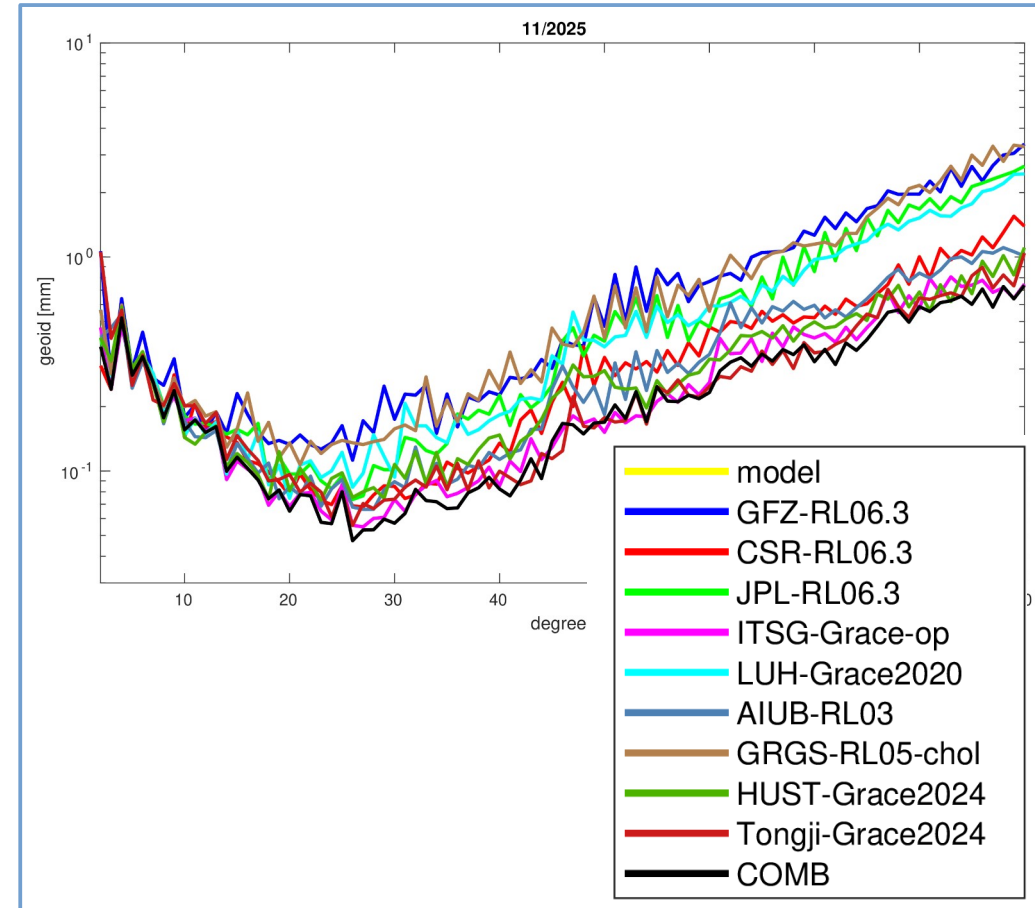
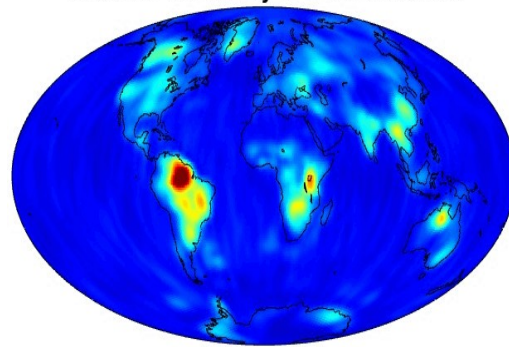
RMS of residual variation after
removing climatological signal

Combination of Gravity Field Solutions Strategy

Derive weights iteratively based on contribution to full solution

- start with average of all solutions
- no formal error information taken into account
- weight each spherical harmonic coefficient

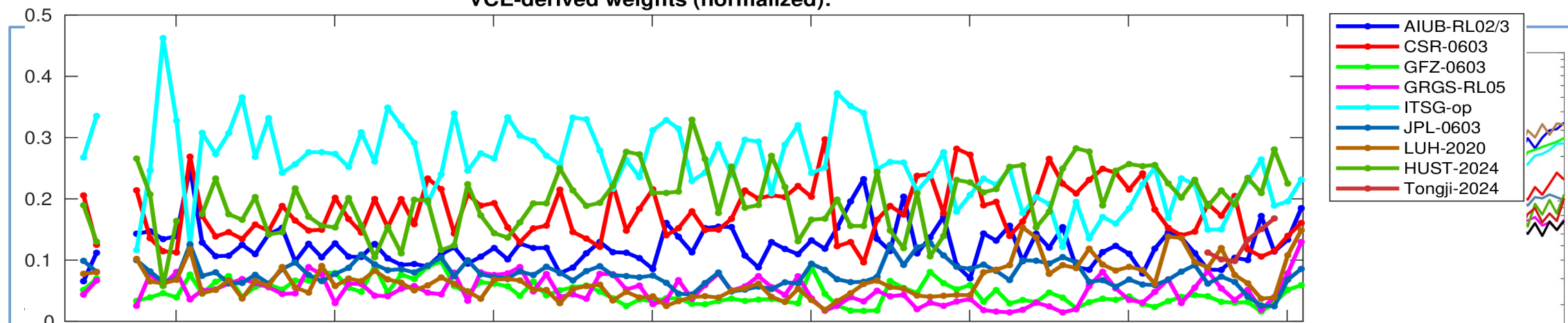
$$\bar{X}_{l,m}^t = \frac{\sum_{i=1}^{N_{\text{sol}}} w_{l,m}^{i,t} X_{l,m}^{i,t}}{\sum_{i=1}^{N_{\text{sol}}} w_{l,m}^{i,t}}$$



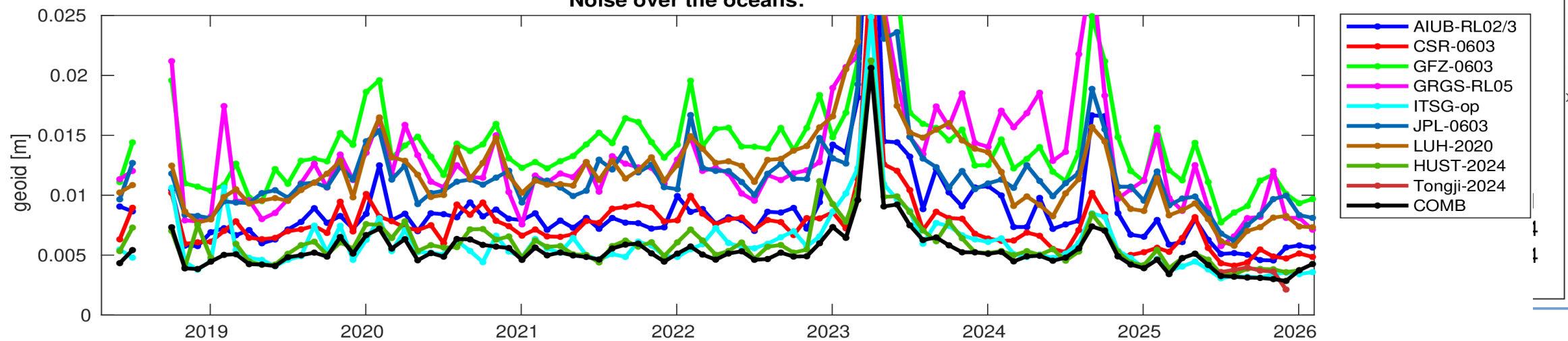
Combination on Solution Level Results

RL2.1 doi:10.5880/COST-G.ICGEM_02_L2

VCE-derived weights (normalized):



Noise over the oceans:

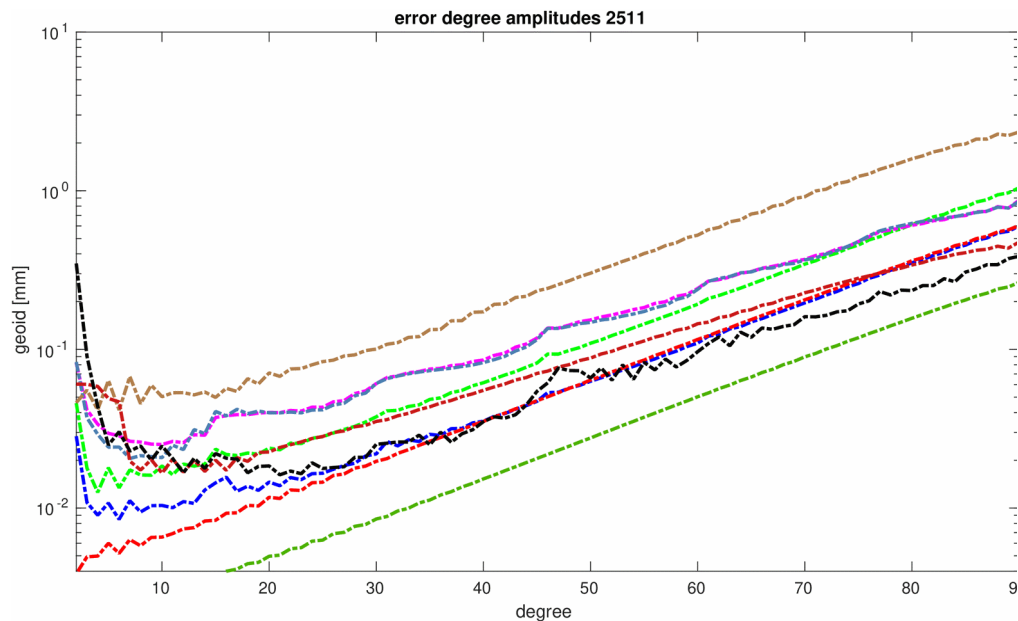


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Combination on Normal Equation Level

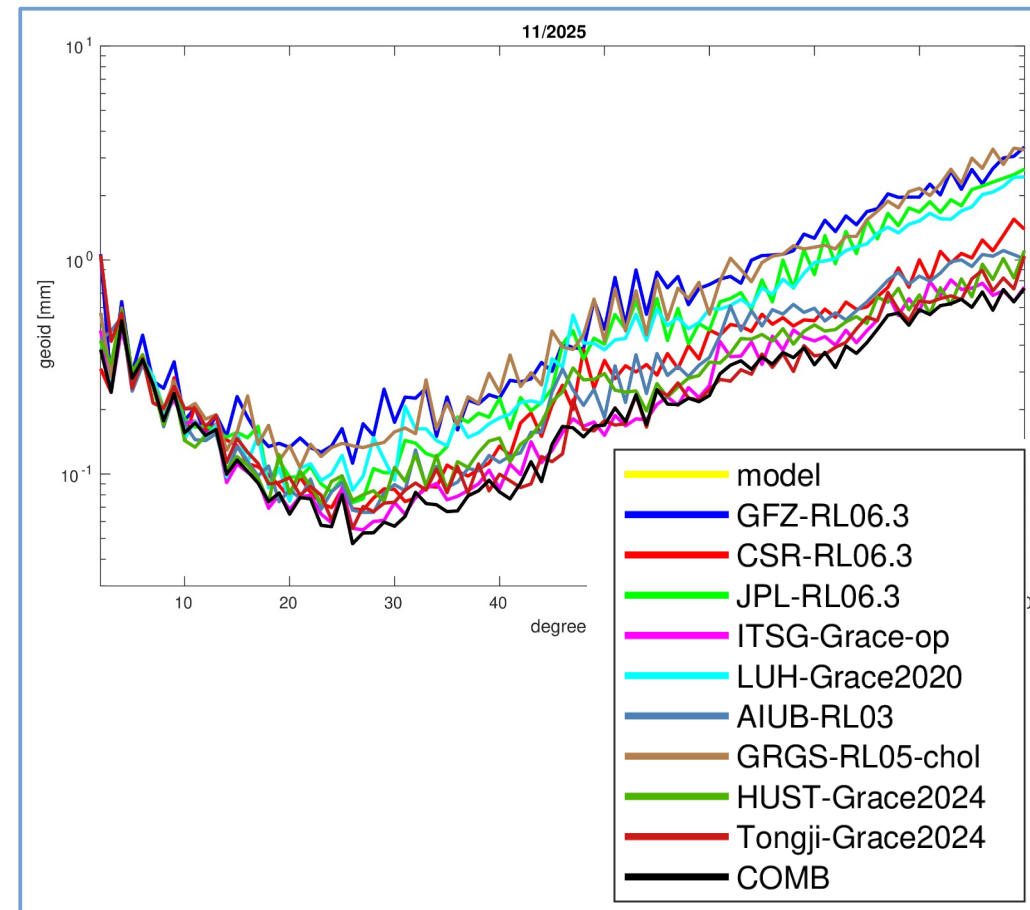
Origin of the problem



Formal error estimates (as given by each AC)
– and in NEQ system –
are not realistic due to different strategies for
noise compensation

See Meyer et al. [2019]
Combination of GRACE monthly gravity fields on the normal equation level

doi: [10.1007/s00190-019-01274-6](https://doi.org/10.1007/s00190-019-01274-6)



Combination on Normal Equation Level Strategy

$$\hat{\mathbf{x}}_1 = \mathbf{x}_{0,1} + \mathbf{N}_1^{-1} \mathbf{b}_1$$

$$\hat{\mathbf{x}}_2 = \mathbf{x}_{0,2} + \mathbf{N}_2^{-1} \mathbf{b}_2$$

$$\hat{\mathbf{x}}_3 = \mathbf{x}_{0,3} + \mathbf{N}_3^{-1} \mathbf{b}_3$$

$$\hat{\mathbf{x}}_4 = \mathbf{x}_{0,4} + \mathbf{N}_4^{-1} \mathbf{b}_4$$

ICGEM (gfc) as
exchange format



Combination on
solution level

$$\mathbf{N}_1 = (\mathbf{A}_1^T \mathbf{P}_1 \mathbf{A}_1) \quad \mathbf{b}_1 = \mathbf{A}_1^T \mathbf{P}_1 \mathbf{l} \quad \mathbf{l} \mathbf{P}_1^T \mathbf{l} \quad \mathbf{x}_{0,1}$$

$$\mathbf{N}_2 = (\mathbf{A}_2^T \mathbf{P}_2 \mathbf{A}_2) \quad \mathbf{b}_2 = \mathbf{A}_2^T \mathbf{P}_2 \mathbf{l} \quad \mathbf{l} \mathbf{P}_2^T \mathbf{l} \quad \mathbf{x}_{0,2}$$

$$\mathbf{N}_3 = (\mathbf{A}_3^T \mathbf{P}_3 \mathbf{A}_3) \quad \mathbf{b}_3 = \mathbf{A}_3^T \mathbf{P}_3 \mathbf{l} \quad \mathbf{l} \mathbf{P}_3^T \mathbf{l} \quad \mathbf{x}_{0,3}$$

$$\mathbf{N}_4 = (\mathbf{A}_4^T \mathbf{P}_4 \mathbf{A}_4) \quad \mathbf{b}_4 = \mathbf{A}_4^T \mathbf{P}_4 \mathbf{l} \quad \mathbf{l} \mathbf{P}_4^T \mathbf{l} \quad \mathbf{x}_{0,4}$$

SINEX as exchange format

\mathbf{A} design
matrix
 \mathbf{P} weight
matrix
 \mathbf{l} obser-
vations
 \mathbf{x}_0 a priori
params.



$$\mathbf{N} = \sum_i \mathbf{N}_i \quad \mathbf{b} = \sum_i \mathbf{b}_i$$

$$\hat{\mathbf{x}} = \mathbf{x}_0 + \mathbf{N}^{-1} \mathbf{b}$$

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Combination on Normal Equation Level

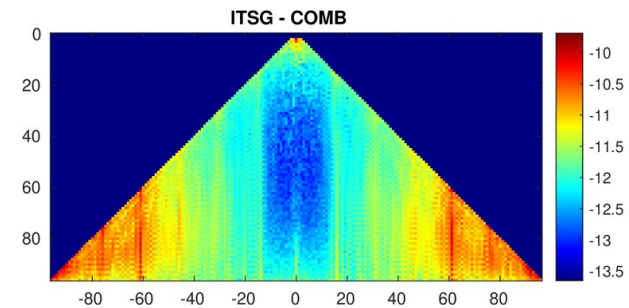
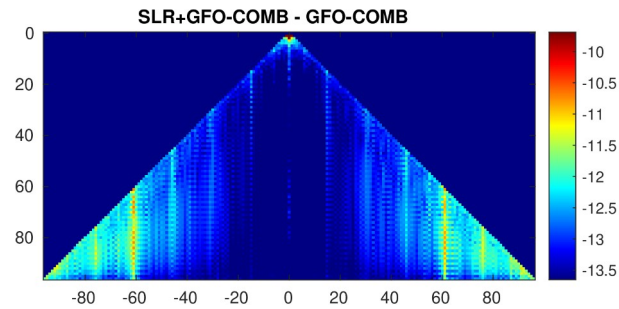
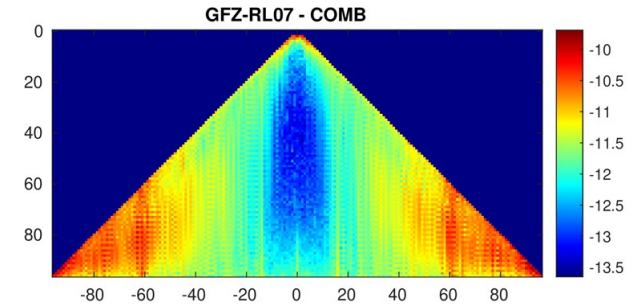
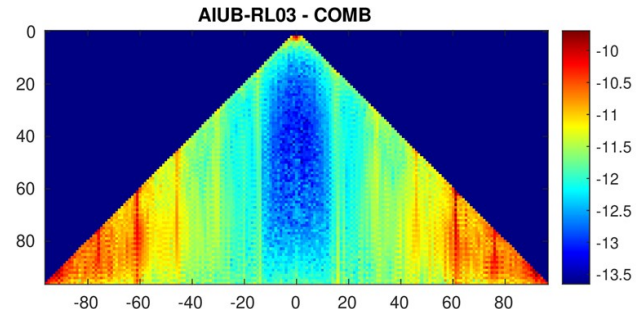
Data set

AIUB (RL03)

GFZ (RL07p)

TUG (ITSG-Grace_operational)

CNES (SLR)

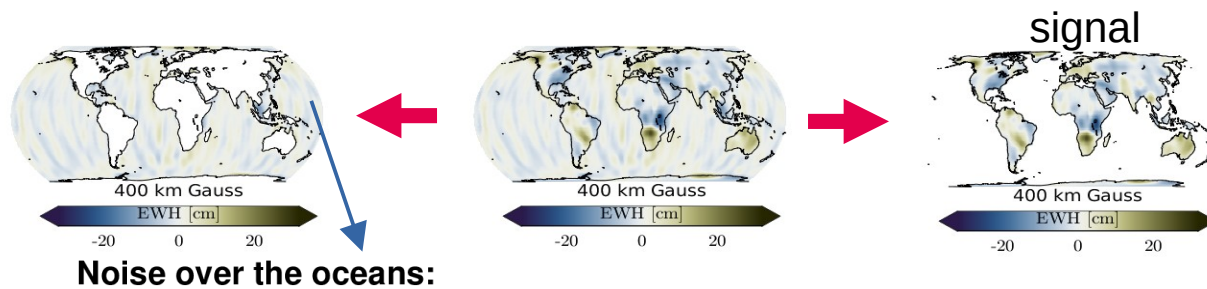


Exchange done via SINEX format
+ extension for GM, AE, permanent tide

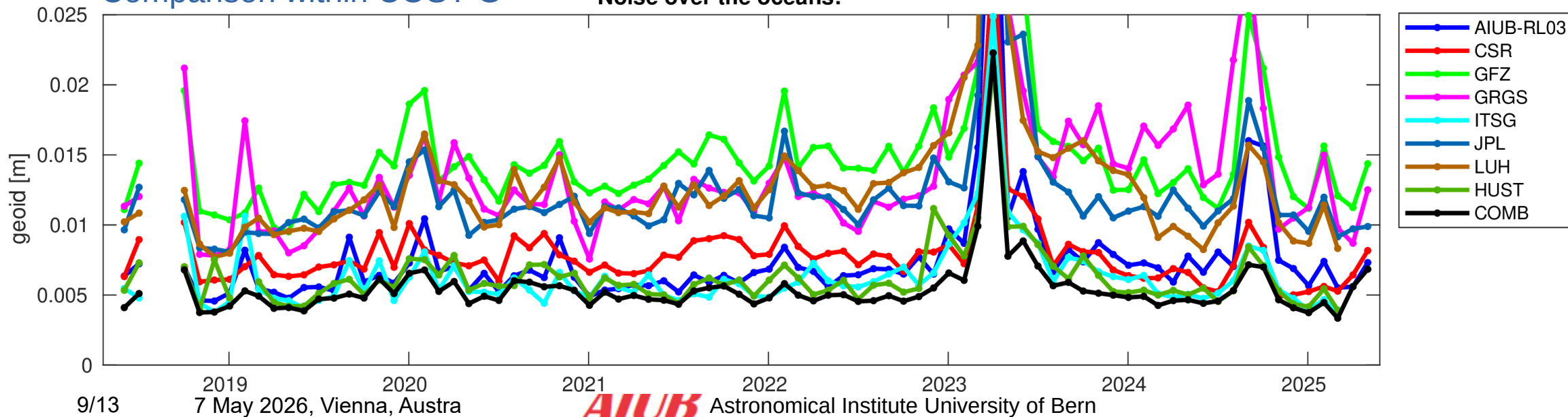
Combination on Normal Equation Level

AIUB

AIUB (RL03) solution based on MIXED2025, TIME22, AOD RL07 including AOe07 and JPL:ACH (ACX, ACX2) + empirical noise modelling

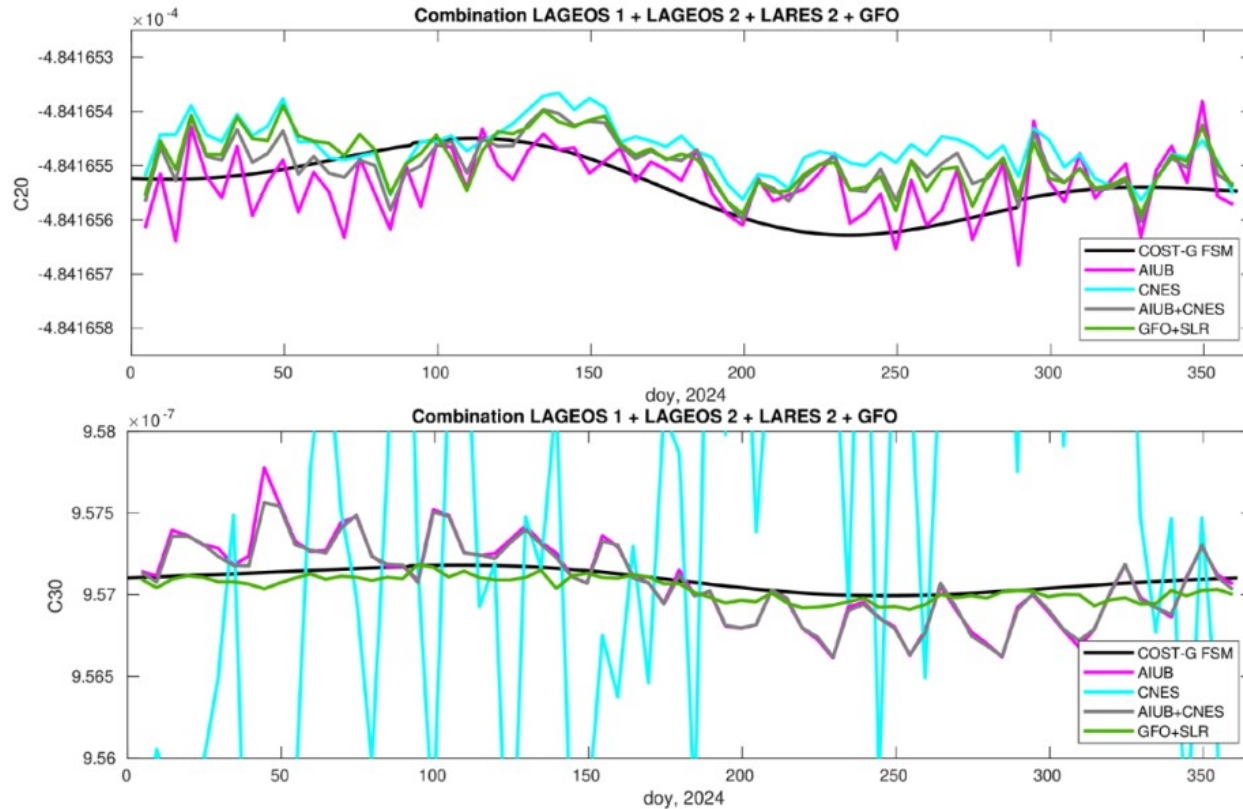


Comparison within COST-G



Combination on Normal Equation Level

Satellite laser ranging

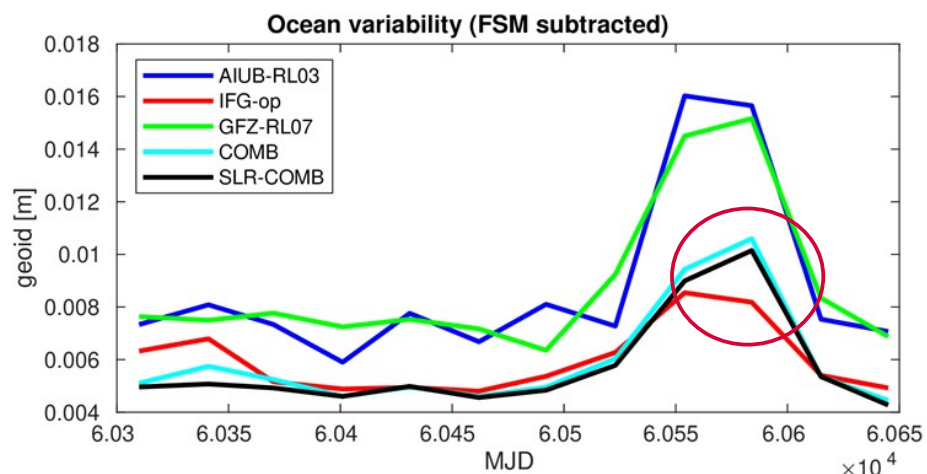
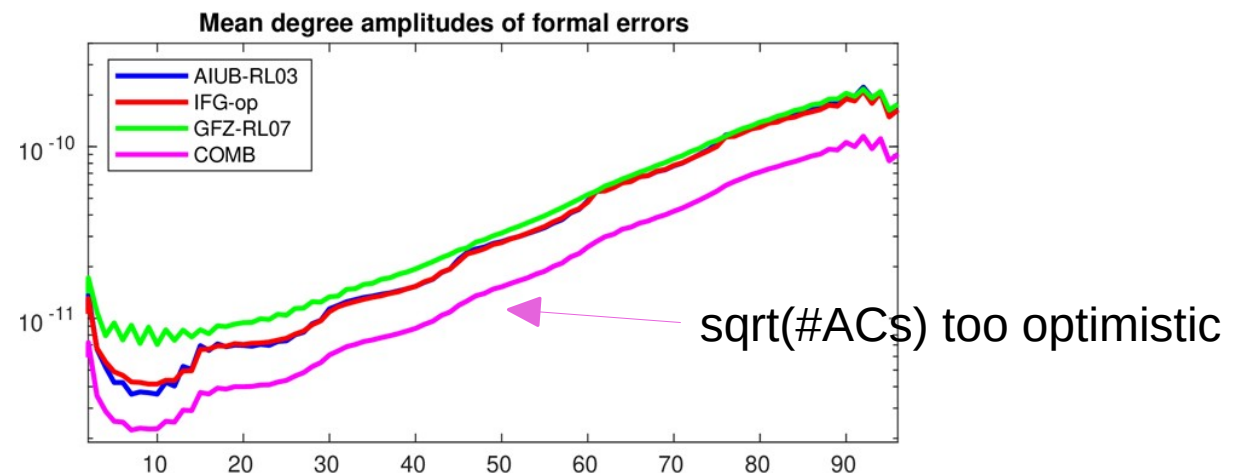
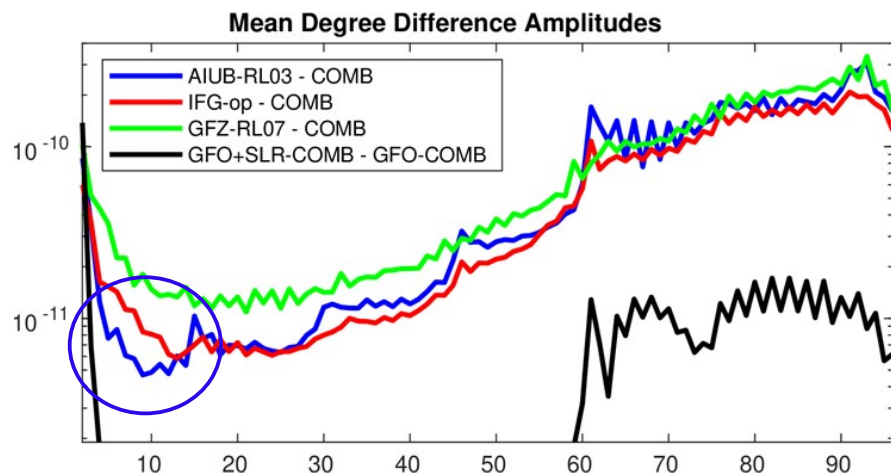


More variations for AIUB
 Compensated in combination with
 CNES

Large variations for CNES.
 Compensated in combination with
 AIUB.
 Smoothed by GRACE Follow-On
 contribution.

Combination on Normal Equation Level

Results



No noise modelling for AIUB for kinematic positions

Formal errors of combination not scaled

Slight degradation for Sep/Oct 2024

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Summary & Conclusions

- Combination on normal equation level feasible.
- Realistic / consistent noise modelling among ACs matters.
- Combining SLR on normal equation level works well.
- Solution level combination will remain method of choice.

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Thank you for your attention

Contact

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