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# Applications of Variance Component Estimation in GRACE-FO and SLR Spherical Geodetic Satellite Data Processing for Gravity Field Determination

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## Methodology

### Parameter Estimation Based on Least-Squares Adjustment (LSA)

Basics of LSA:

- Observation equation system: $\mathbf{l}' + \mathbf{v} = \mathbf{A} \cdot \Delta\hat{\mathbf{x}}$, $\mathbf{P}$: weight matrix (representing the noise model)
  - observed-computed
  - residuals
  - design matrix (containing derivatives wrt unknown parameters)
  - unknown parameters
- Normal Equation system: $\mathbf{N}\Delta\hat{\mathbf{x}} = \mathbf{b}$, with $\mathbf{A}^T\mathbf{P}\mathbf{A}$ and $\mathbf{A}^T\mathbf{P}\mathbf{l}'$

**Constraining parameters ($\hat{\mathbf{x}}_p \subset \hat{\mathbf{x}}$)** to their apriori values by introducing pseudo-observations: $(\mathbf{N} + \mathbf{W})\Delta\hat{\mathbf{x}} = \mathbf{b}$

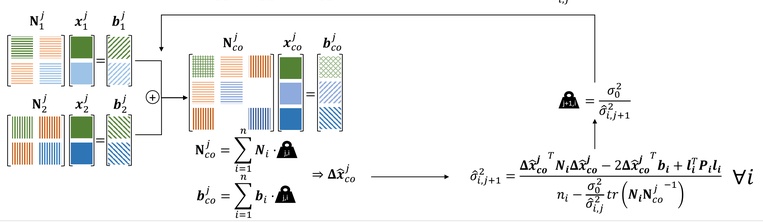
$\blacksquare = \frac{\sigma_0^2}{\sigma_{\hat{x}_p}^2}$

### Variance Component Estimation (VCE)

- $n$ individual NEQ systems: $\mathbf{N}_i\Delta\hat{\mathbf{x}}_i = \mathbf{b}_i, \forall\, i \in \{1, \dots, n\}$
- Combine NEQs iteratively by stacking common parameters:

$$\mathbf{N}_{co}^j\Delta\hat{\mathbf{x}}_{co}^j = \mathbf{b}_{co}^j, \quad \text{iteration: } j, \quad \blacksquare = \frac{\sigma_0^2}{\hat{\sigma}_{i,j}^2} \quad \forall i \in \{1, \dots, n\}$$

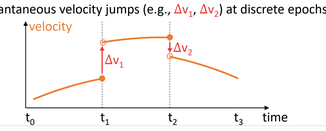
e.g.



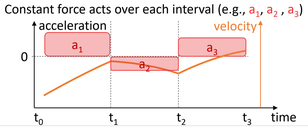

### Celestial Mechanics Approach (CMA)

CMA (Beutler et al., 2010), implemented in the Bernese GNSS Software (Dach et al., 2015), provides a unified and flexible framework for gravity field recovery by **jointly estimating spherical harmonic gravity field coefficients (SHCs)**, further geodetic parameters and (daily) **orbital parameters** in a least-squares adjustment. A defining **feature of the CMA** is the co-estimation of pseudo-stochastic parameters, i.e., regularly spaced **pseudo-stochastic pulses** (StPs) or **piecewise constant accelerations** (PCAs) in radial (R), along-track (S) and out-of-plane (W), to account for **force modelling deficiencies**.

Instantaneous velocity jumps (e.g., $\Delta v_1$, $\Delta v_2$) at discrete epochs




Constant force acts over each interval (e.g., $a_1$, $a_2$, $a_3$)




## Set Up (Observations, Parametrization, Background Models)

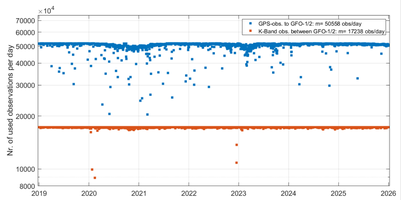
### K-Band, GPS

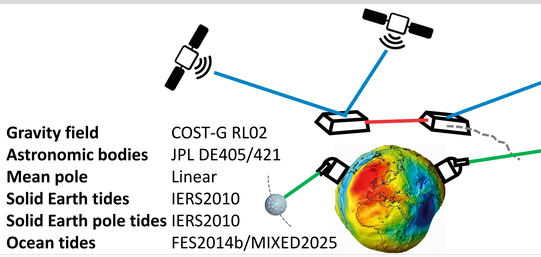
#### Parametrization

**Orbit parameters for GRACE-FO (GF1 and GF2):**

| | |
|---|---|
| Initial conditions: | 6/day |
| Constant accelerations: | 3/day |
| ACC scaling matrix | 9/day |
| PCAs in (R, S, W): | 288/day (15 min) |
| **AOerr** (sph. harm.) | up to d/o 96/day |
| **Gravity field parameters:** | up to d/o 96/month |

#### Observations



| | |
|---|---|
| **Gravity field** | COST-G RL02 |
| **Astronomic bodies** | JPL DE405/421 |
| **Mean pole** | Linear |
| **Solid Earth tides** | IERS2010 |
| **Solid Earth pole tides** | IERS2010 |
| **Ocean tides** | FES2014b/MIXED2025 |
| **Ocean pole tides** | DESAI2016 |
| **Atmospheric tides** | AOD RL06/7 |
| **De-aliasing** | AOD RL06/7 |
| **Relativistic effects** | IERS2010 |
| **Reference frame** | SLRF 2020 |
| **ERPs** | IERS-20-C04 |

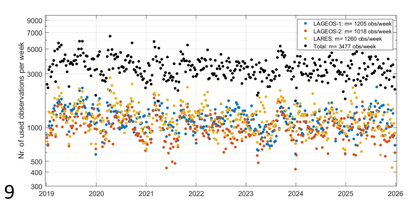
### Satellite Laser Ranging (SLR)

#### Parametrization

**Orbit parameters for LAGEOS-1/2 and LARES:**

| | |
|---|---|
| Initial conditions: | 6/week |
| Constant acc. in S: | 1/week |
| Once-per-revolution (sin/cos) acc. in S for LAGEOS-1/2: | 1/week |
| StPs in (R, S, W): | 2/day |
| **SLR range biases:** | for all stations |
| **Gravity field parameters:** | $C_{20}$ or up to d/o 9 |

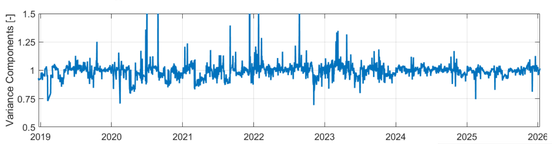
#### Observations

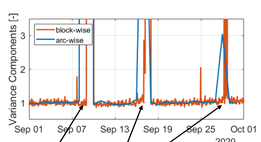



## K-Band, GPS

### Use VCE to determine:

- relative weights between arcs / blocks of observations:

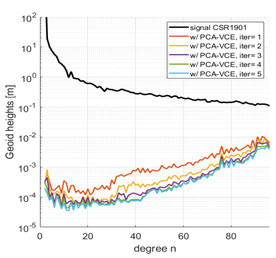


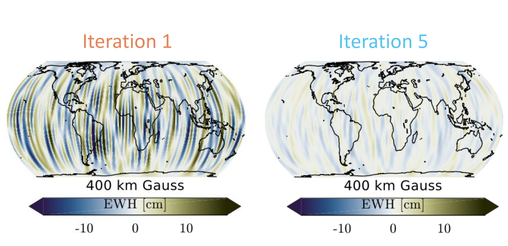




orbit manoeuvre

K-Band calibration manoeuvres

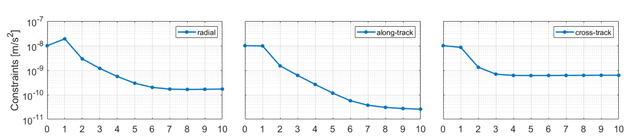
- constraints on estimated PCAs in an automated approach:



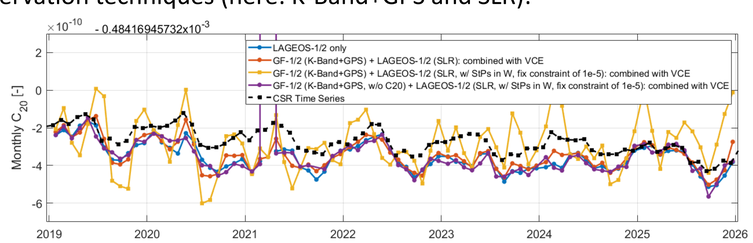










### Use VCE to determine relative weights between different space geodetic observation techniques (here: K-Band+GPS and SLR):



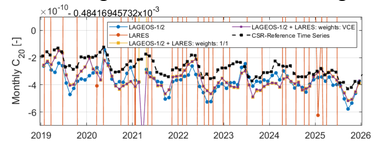

## Conclusions & Outlook

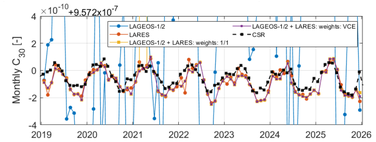
- **SLR:** The estimation of StPs degrades the stability of the $C_{20}$ time series due to signal absorption, but it improves the orbit determination and reduces the observation residuals. VCE provides a framework to balance these competing effects in an optimal manner.
- **K-Band + GPS:** VCE can be used as an automated outlier detection and with the estimation of the constraints on PCAs, it strengthens the resulting gravity field solution.
- **K-Band + GPS + SLR (to LAGEOS-1/2):** In combination, the additional StPs in the orbit parametrization of LAGEOS-1/2 seem to compensate inconsistencies and errors in both the background and stochastic models.
- **Next steps:**
  - increase consistency between single-technique NEQs
  - use VCE to determine relative weights between K-Band and GPS observations
  - use VCE to constrain estimated AO errors
  - add more spherical SLR satellites to the combination
  - investigate the multi-technique combination in more detail
  - extend the set of estimated parameters, e.g., Earth rotation parameters and station coordinates

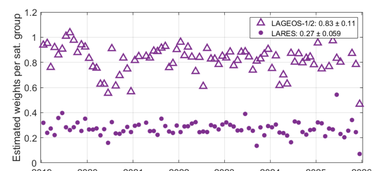
### SLR

#### Use VCE to determine:

- relative weights between different sat. groups (here: LAGEOS-1/2 and LARES):

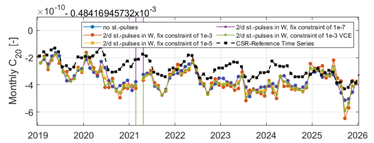



| | Standard Deviation of Differences wrt CSR Reference [1e-10] | | | |
|---|---|---|---|---|
| | LAGEOS-1/2 (A) | LARES (B) | A + B: 1/1 | A + B: VCE |
| $C_{20}/C_{30}$ | 0.45/10 | 57/0.48 | 0.47/0.45 | 0.52/0.46 |
| $C_{21}/S_{21}$ | 0.52/0.63 | 1.2/1.1 | 0.50/0.55 | 0.35/0.37 |
| $C_{22}/S_{22}$ | 0.37/0.38 | 0.46/0.46 | 0.34/0.39 | 0.28/0.29 |
| $C_{31}/S_{31}$ | 0.76/0.71 | 0.82/0.76 | 0.68/0.72 | 0.55/0.50 |
| $C_{32}/S_{32}$ | 1.5/1.3 | 1.3/1.2 | 1.3/1.1 | 0.90/0.82 |
| $C_{33}/S_{33}$ | 1.6/1.9 | 1.0/1.1 | 1.3/1.1 | 1.1/0.78 |
| **d > 3** | Same behaviour as seen for lower degrees | | | |

- constraints on estimated StPs in an automated approach (here LAGEOS-1/2):




| | Standard Deviation of Differences wrt CSR Reference [1e-10] | | | | |
|---|---|---|---|---|---|
| | no StPs | 2/d StPs in W with constraint of | | | |
| | | 1e-3 | 1e-5 | 1e-7 | VCE |
| $C_{20}$ | 0.44 | 0.71 | 0.53 | 0.44 | 0.58 |