GRACE Follow-On Orbit and Gravity Field Determination from GPS Carrier Phase Observations

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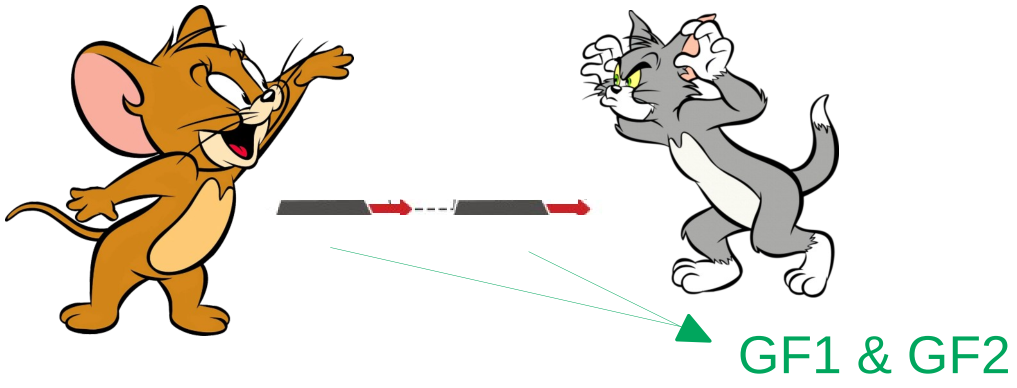
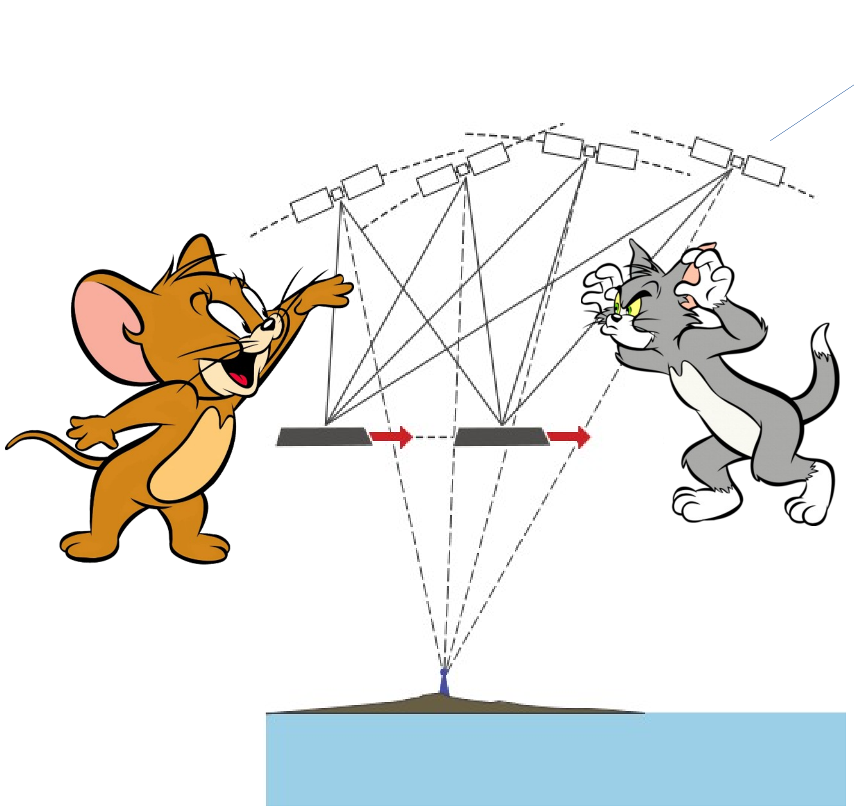
GRACE Follow-On

Observation concept

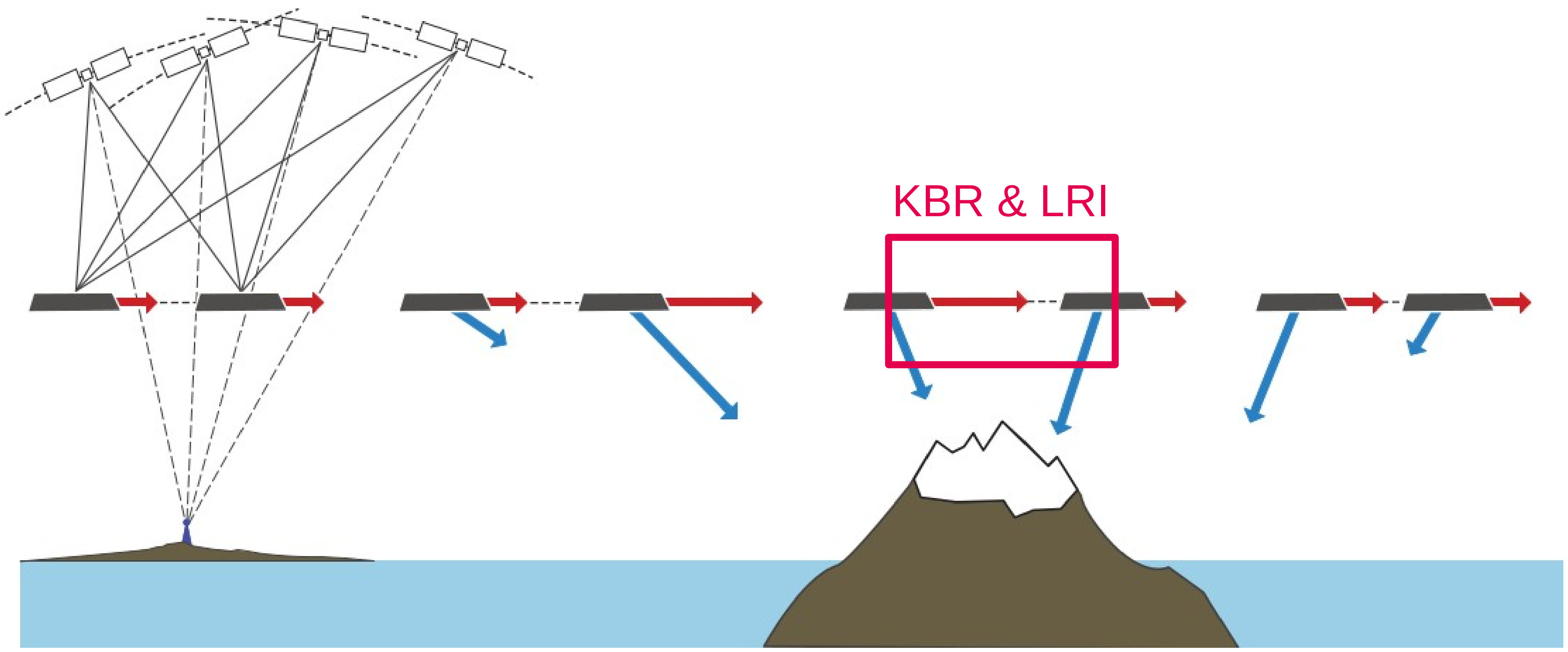


GRACE Follow-On

Observation concept

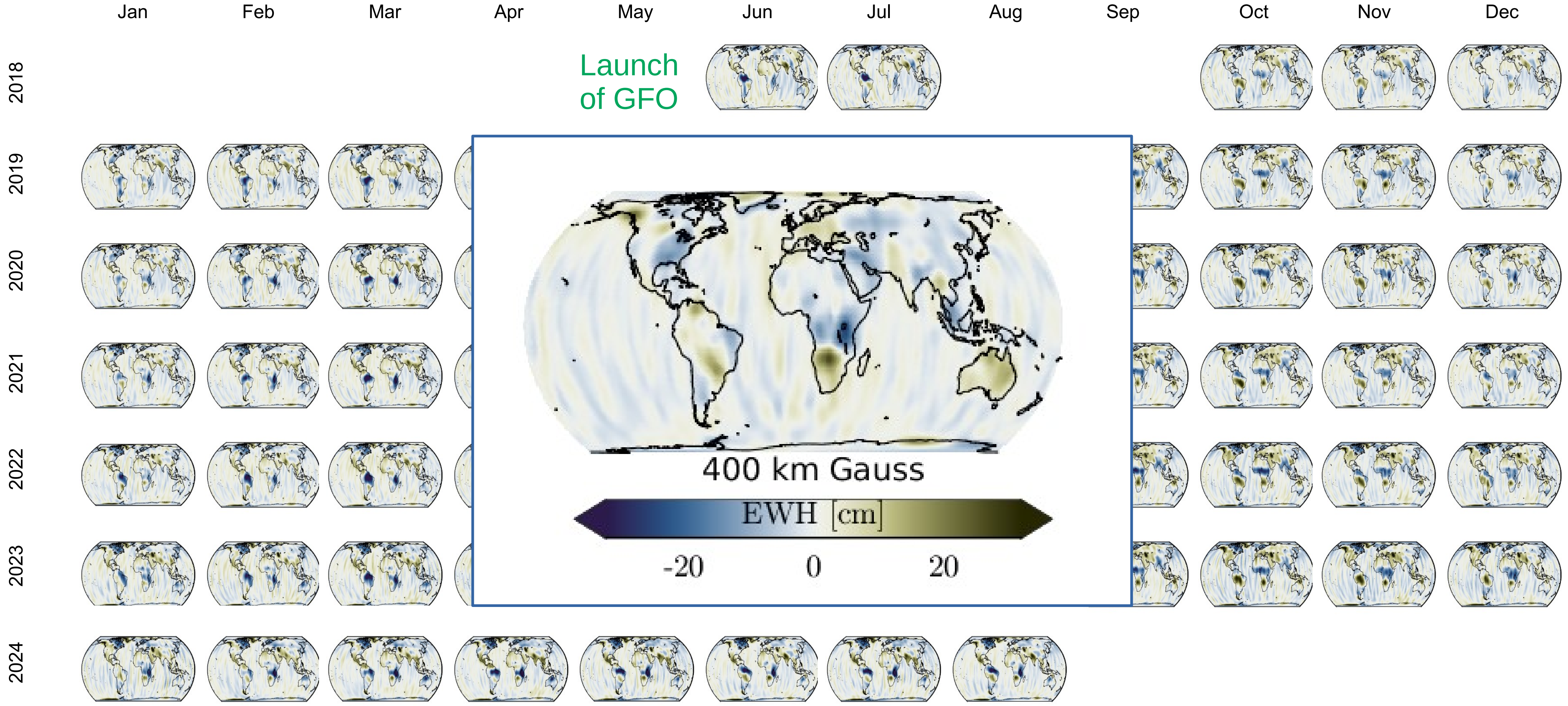


GRACE Follow-On
Observation concept

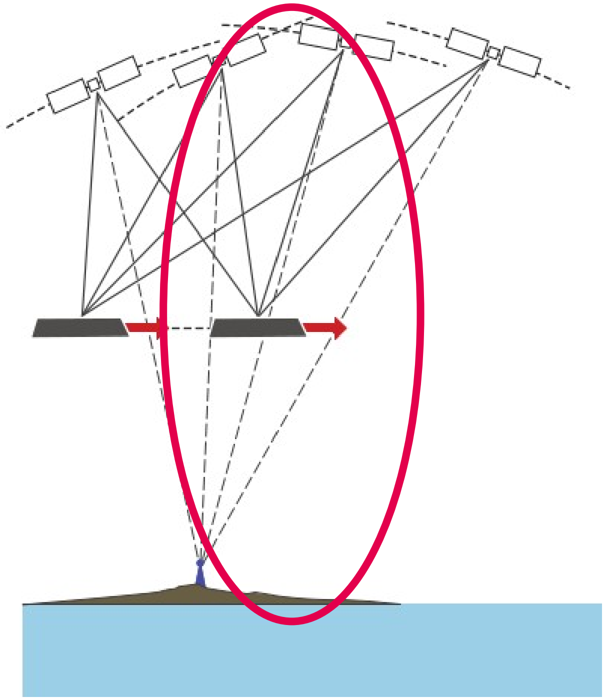


GRACE Follow-On

Results

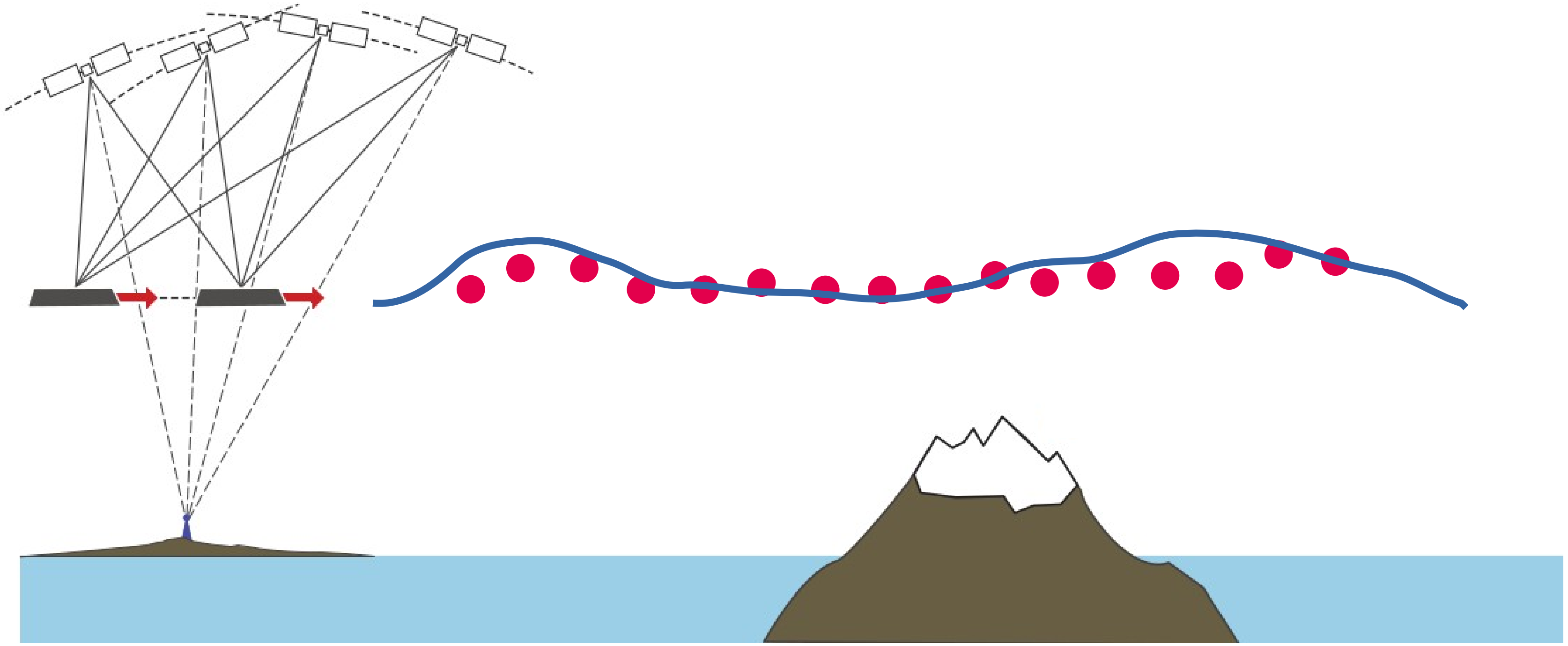


GRACE Follow-On
Orbit representation



GRACE Follow-On

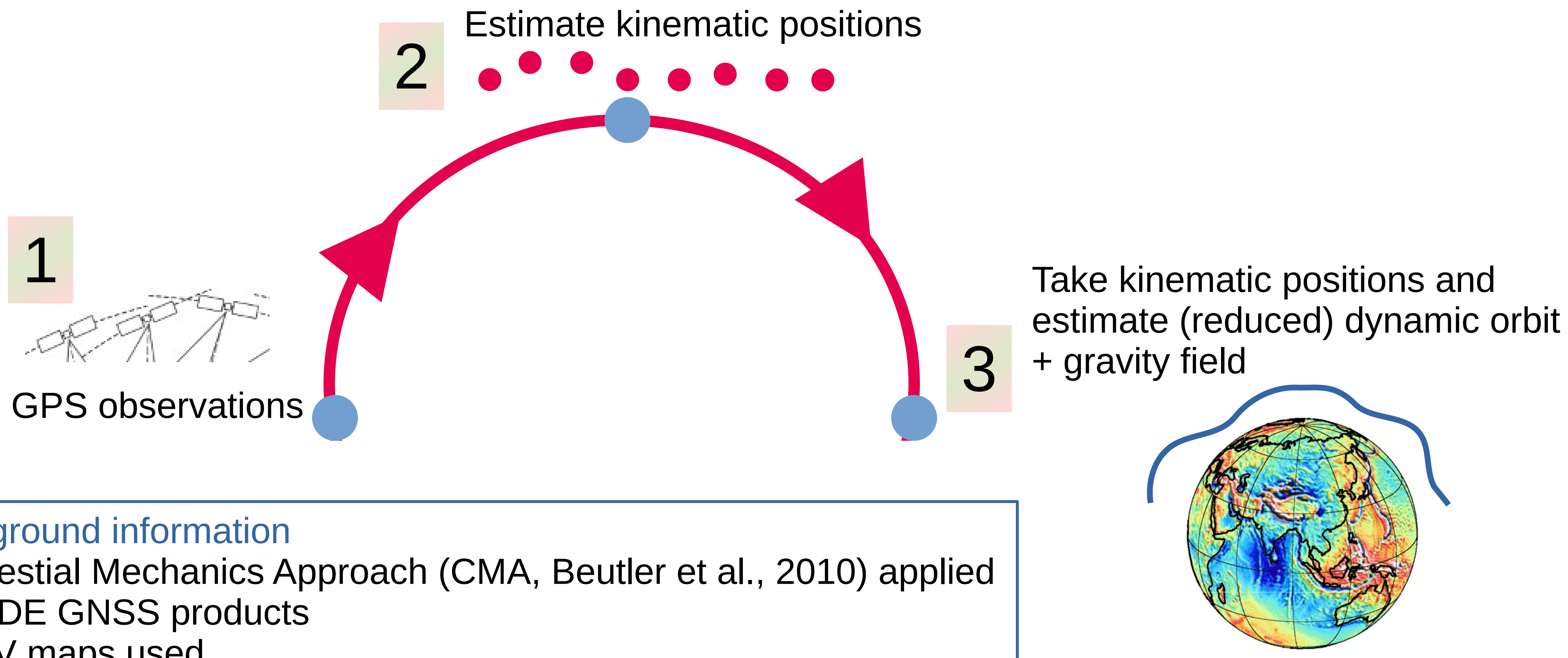
Orbit representation



Kinematic ‹orbit›

(Reduced)
dynamic orbit

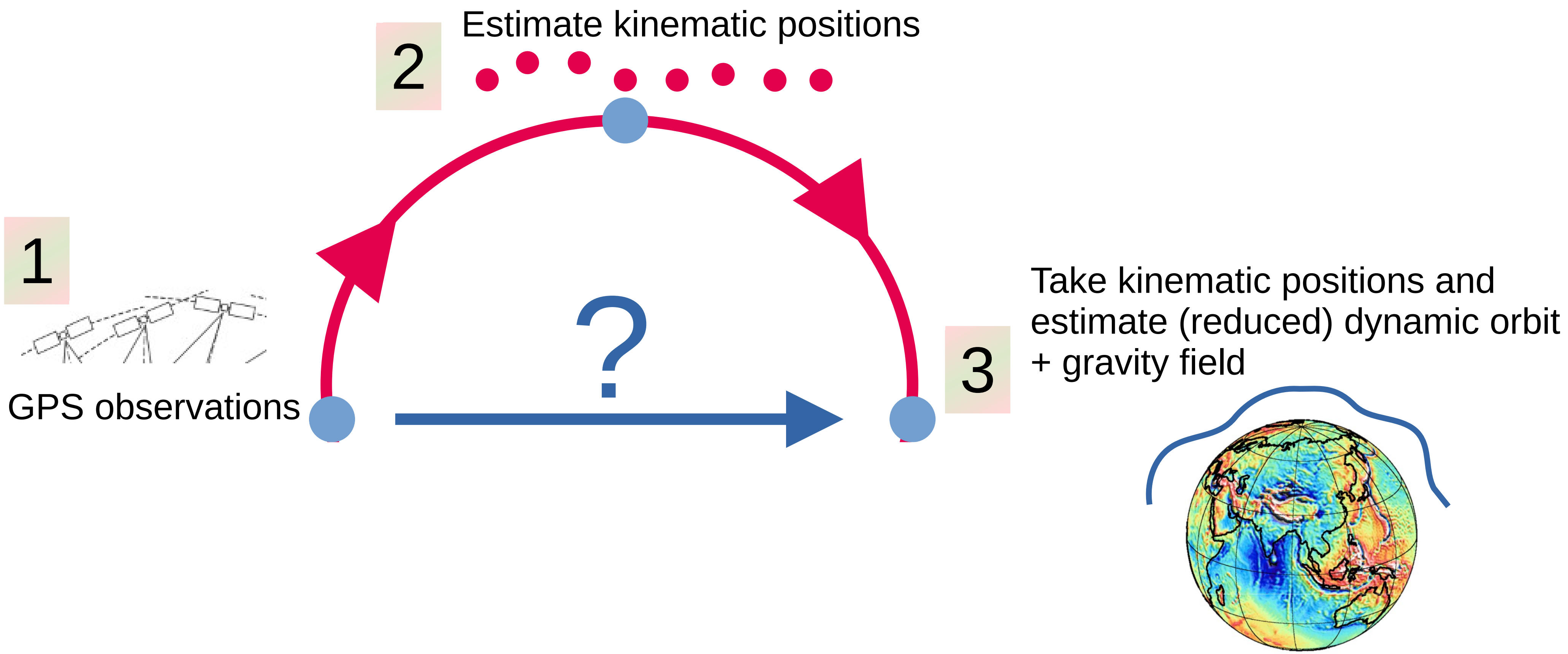
Gravity Field Recovery
Detour



Background information

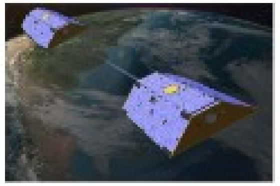
- Celestial Mechanics Approach (CMA, Beutler et al., 2010) applied
- CODE GNSS products
- PCV maps used
- Ambiguities float or integer-fixed

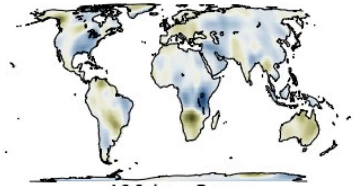
Gravity Field Recovery
Shortcut



Parametrisation

For estimating gravity fields





Basic parametrisation
- Initial conditions
- Bias in radial, along-track, cross-track
- Clock per epoch
- Ambiguities

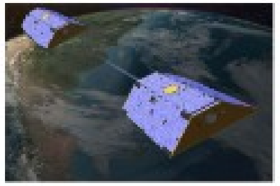
Additional parameters
- 15 min PCA per satellite in
  - → radial
  - → along-track
  - → cross-track

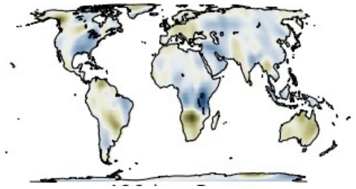
in daily arcs

+ gravity field d/o=2..70

Parametrisation

For estimating gravity fields





Basic parametrisation

- Initial conditions
- Bias in radial, along-track, cross-track
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- Ambiguities

Additional parameters

- 15 min PCA per satellite in
  - → radial
  - → along-track
  - → cross-track

+ gravity field

Force models	
Gravity field	GOCO06s
Astronomic bodies	JPL DE421 (all planets)
Mean pole	Linear
Solid Earth tides	IERS2010
Solid Earth pole tides	IERS2010
Ocean tides	FES2014b (+ admittances from TUG)
Ocean pole tides	Desai
Atmospheric tides	AOD RL06
Atmospheric & oeanic dealiasing	AOD RL06
Relativistic effects	IERS2010

The Detour in a Nutshell

A bit of math ...

Least-squares to estimate kinematic positions

$$\hat{\boldsymbol{x}}_{\text{KIN}} = (\mathbf{A}^{\text{T}}\mathbf{P}_{\text{ph}}\mathbf{A})^{-1}\mathbf{A}^{\text{T}}\mathbf{P}_{\text{ph}}\boldsymbol{\ell}_{\text{ph}} \qquad \text{with} \qquad \mathbf{A} := \frac{\partial \boldsymbol{\ell}_{\text{ph}}}{\partial \boldsymbol{x}_{\text{KIN}}}$$

$$= \mathbf{N}_{\text{ph}}^{-1}\mathbf{A}^{\text{T}}\mathbf{P}_{\text{ph}}\boldsymbol{\ell}_{\text{ph}} \quad .$$

Least-squares to estimate orbit and gravity field parameters based on kinematic positions

$$\hat{\boldsymbol{p}} = (\mathbf{B}^{\text{T}}\mathbf{Q}_{\text{KIN}}^{-1}\mathbf{B})^{-1}\mathbf{B}^{\text{T}}\mathbf{Q}_{\text{KIN}}^{-1}\boldsymbol{\ell}_{\text{KIN}} \qquad \text{with} \qquad \mathbf{B} := \frac{\partial \boldsymbol{\ell}_{\text{KIN}}}{\partial \boldsymbol{p}} \qquad \mathbf{Q}_{\text{KIN}} := \mathbf{N}_{\text{ph}}^{-1}$$

$$= \mathbf{N}_{\text{p}}^{-1}\mathbf{B}^{\text{T}}\mathbf{Q}_{\text{KIN}}^{-1}\hat{\boldsymbol{x}}_{\text{KIN}} \quad .$$

The Detour in a Nutshell

A bit of math ...

Or summarise…

$$\mathbf{M} := \mathbf{A}\mathbf{B}$$

$$\hat{\boldsymbol{p}} = (\mathbf{M}^{\mathrm{T}}\mathbf{P}_{\mathrm{ph}}\mathbf{M})^{-1}\mathbf{M}^{\mathrm{T}}\mathbf{P}_{\mathrm{ph}}\boldsymbol{\ell}_{\mathrm{ph}}$$

Stochastic Properties of Kinematic Positions

And how to consider them

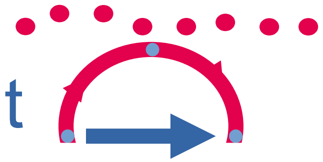
Recall kinematic positions as observations

$$\hat{\boldsymbol{p}} = (\mathbf{B}^{\mathrm{T}} \mathbf{Q}_{\mathrm{KIN}}^{-1} \mathbf{B})^{-1} \mathbf{B}^{\mathrm{T}} \mathbf{Q}_{\mathrm{KIN}}^{-1} \boldsymbol{\ell}_{\mathrm{KIN}} \qquad \text{with} \qquad \mathbf{B} := \frac{\partial \boldsymbol{\ell}_{\mathrm{KIN}}}{\partial \boldsymbol{p}}$$

$$= \mathbf{N}_{\mathrm{p}}^{-1} \mathbf{B}^{\mathrm{T}} \mathbf{Q}_{\mathrm{KIN}}^{-1} \hat{\boldsymbol{x}}_{\mathrm{KIN}} \ .$$

$$\mathbf{Q}_{\mathrm{KIN}} := \mathbf{N}_{\mathrm{ph}}^{-1}$$

Remark for the shortcut



When directly starting with phase observations all correlations are implicitly considered.

Stochastic Properties Kinematic Positions

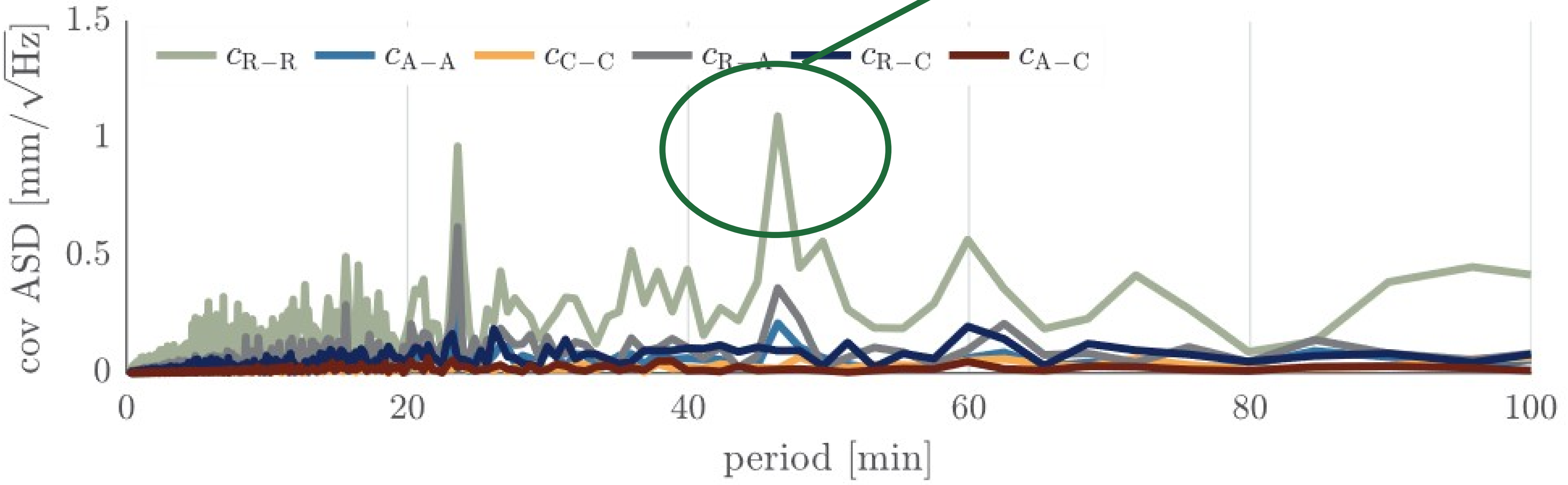
Power-spectrum – epoch-wise

$$\mathbf{Q}_{\mathrm{KIN}} := \mathbf{N}_{\mathrm{ph}}^{-1}$$

$$\mathbf{C}_{\mathrm{epo}}^{\mathrm{KIN}} = \begin{bmatrix} c_{xx} & c_{xy} & c_{xz} \\ c_{xy} & c_{yy} & c_{yz} \\ c_{xz} & c_{yz} & c_{zz} \end{bmatrix}$$

→ radial
along-track
cross-track

2/rev peak due to polar gap of GPS constellation



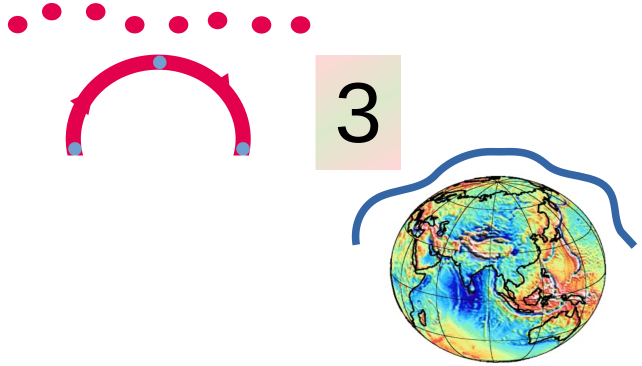
Results Reduced Dynamic Orbit

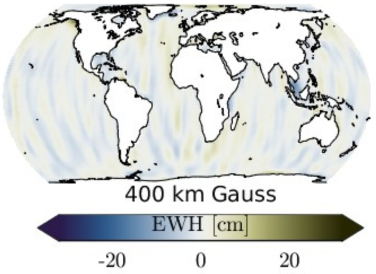
A priori orbit (pre-fit)

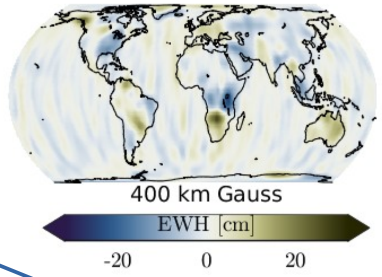
	float			fixed		
MAD carrier phase residuals [cm]	0.21			0.29		
MAD w.r.t. kinematic positions [cm]	radial	along	cross	radial	along	cross
	1.05 –	0.84 –	0.69	0.63 –	0.26 –	0.19
MAD of K-band validation [cm]	0.17			0.14		

Results Gravity Field

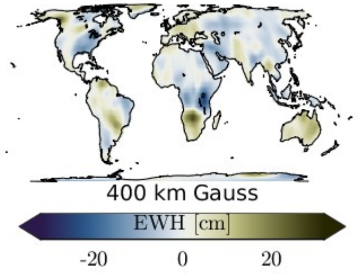
From kinematic positions



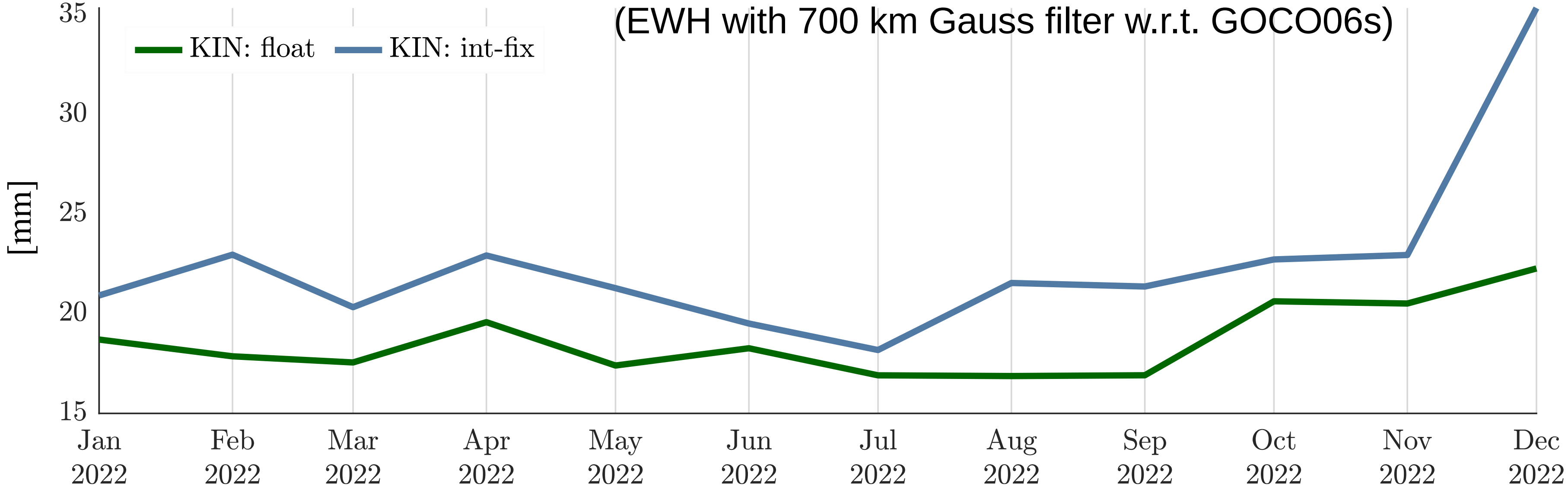




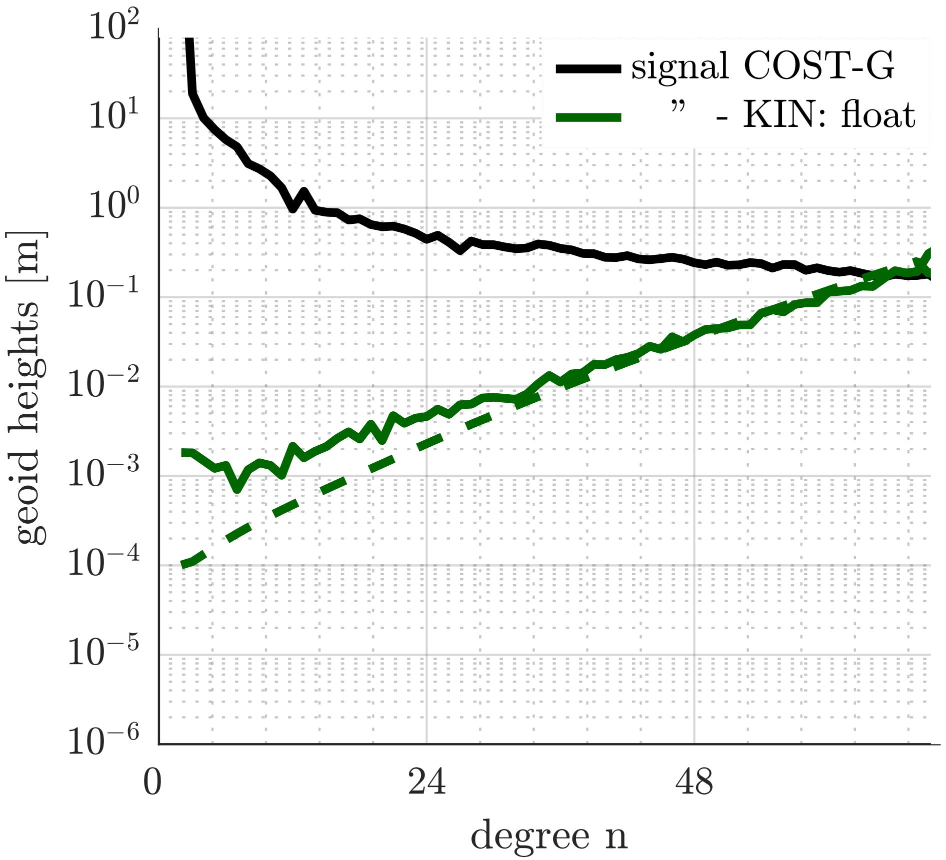
signal



RMS over the oceans
(EWH with 700 km Gauss filter w.r.t. GOCO06s)



Results Gravity Field
Directly from phase observations

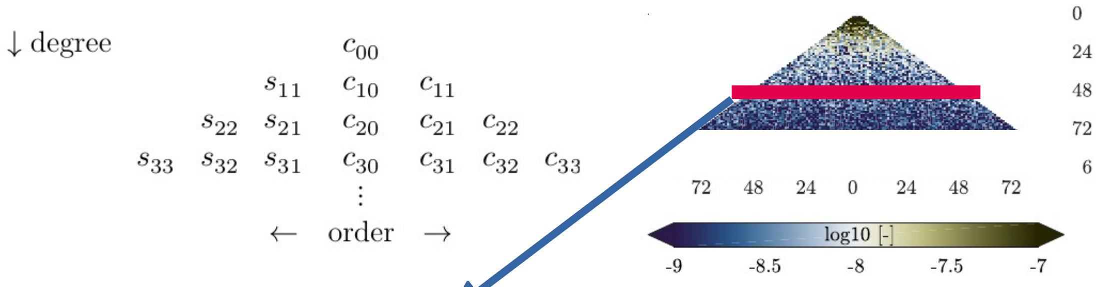


Gravity field representation in spherical harmonics

unknown coefficients

$$V=\frac{GM}{R}\sum_{n=0}^{N}\left(\frac{a_e}{r}\right)^{n}\sum_{m=0}^{n}P_{nm}(\sin(\varphi))[c_{nm}\cos(m\lambda)+s_{nm}\sin(m\lambda)]$$

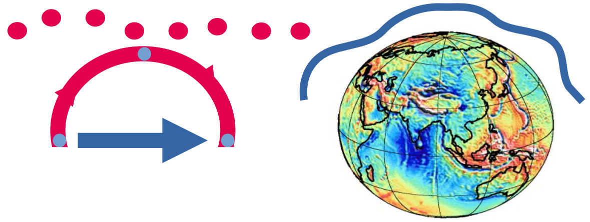
degree N point of evaluation (r,φ,λ)

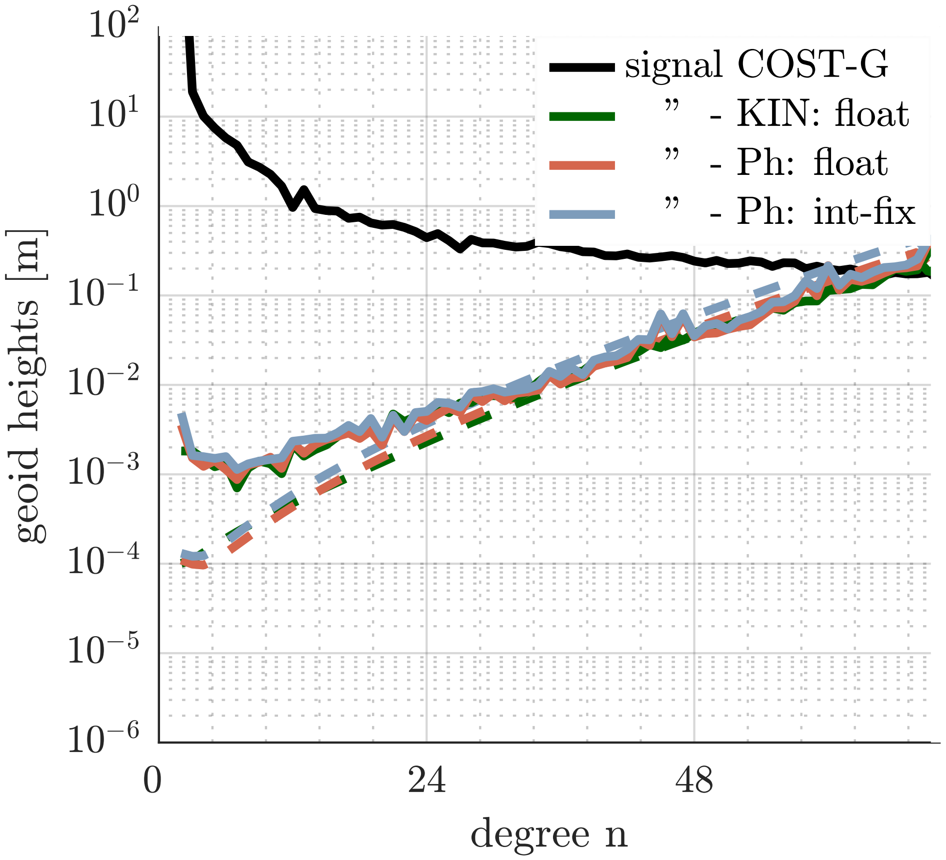


$$\sigma_n=\sqrt{\sum_{m=0}^{n}\left(c_{nm}^2+s_{nm}^2\right)}$$

Results Gravity Field

Directly from phase observations

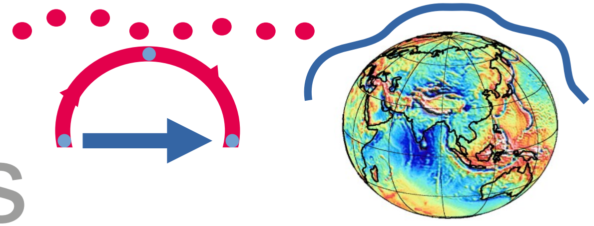


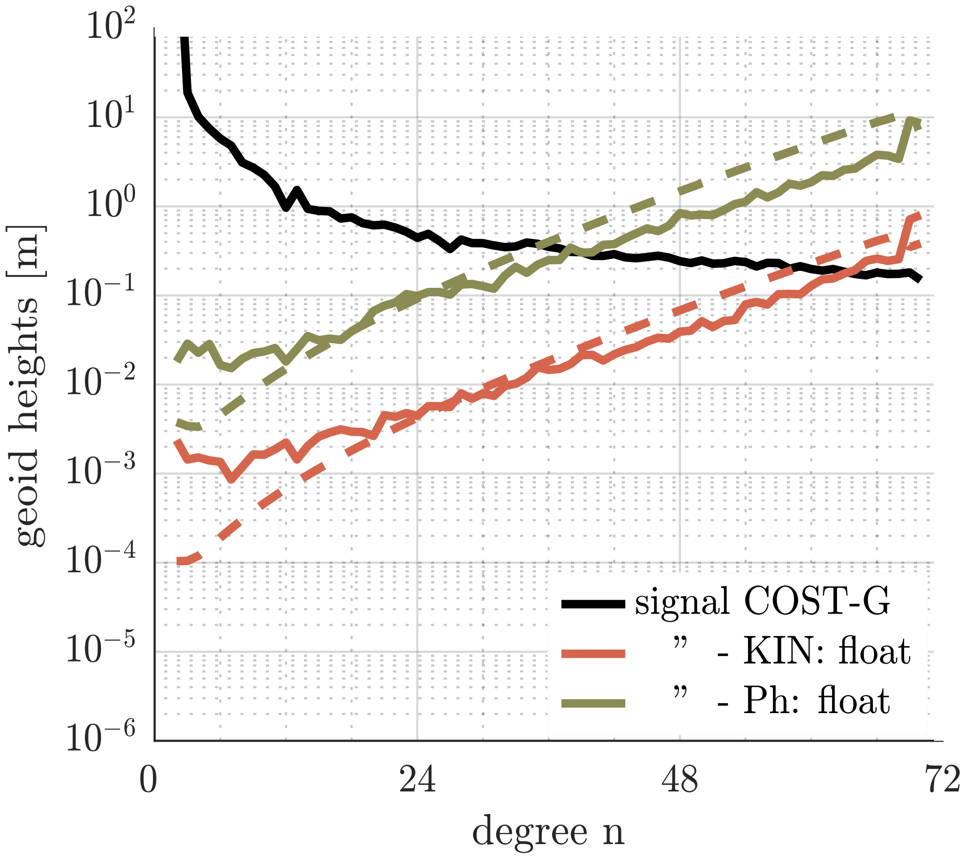


- Ambiguity-float solution almost the same for KIN or phase observations
- Ambiguity-integer-fixed solution slightly deteriorated
- Formal errors over-estimate low-degree coefficient quality

Results Gravity Field

Robustness of kinematic positions





- Imperfect PCV model used (aka 10° elevation cut-off ignored)
- Solution directly from phase observations heavily affected
- Kinematic positions do not really suffer

Results Reduced Dynamic Orbit

Jointly estimated with the gravity field (pre-fit & post-fit)

	float	fixed
MAD carrier phase residuals [cm]	0.21	0.29
MAD w.r.t. kinematic positions [cm]	radial – along – cross 1.05 – 0.84 – 0.69 1.08 – 0.86 – 0.70	radial – along – cross 0.63 – 0.26 – 0.19 0.64 – 0.26 – 0.20
MAD of K-band validation [cm]	0.17 0.17	0.14 0.13

Summary
& Conclusions

- Gravity field determination for carrier phase observations introduced
- Challenges for ambiguity resolution strategy remain in all cases
- Computational efficiency needs to be improved
- For GRACE Follow-On:
    - Extend with inter-satellite link
    - Combine carrier phase data and K-band/LRI data

Thank you for your attention

Contact

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