# Joint GRACE Follow-On Orbit and Gravity Field Determination from GPS Carrier Phase Observations

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AIUB Astronomical Institute University of Bern

# GRACE Follow-On

## Observation concept

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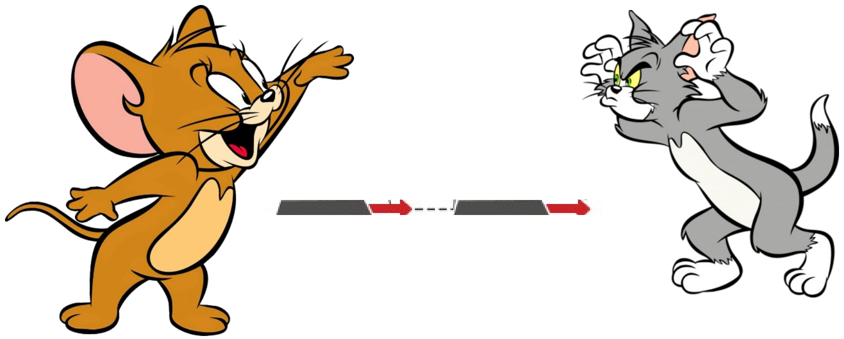

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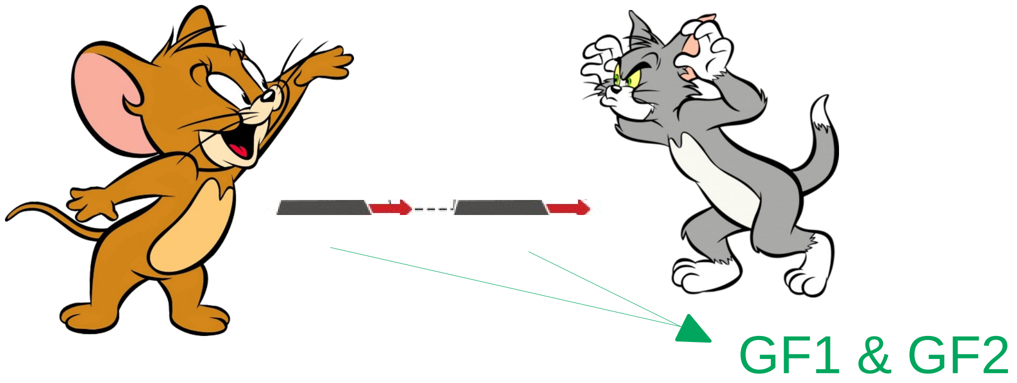

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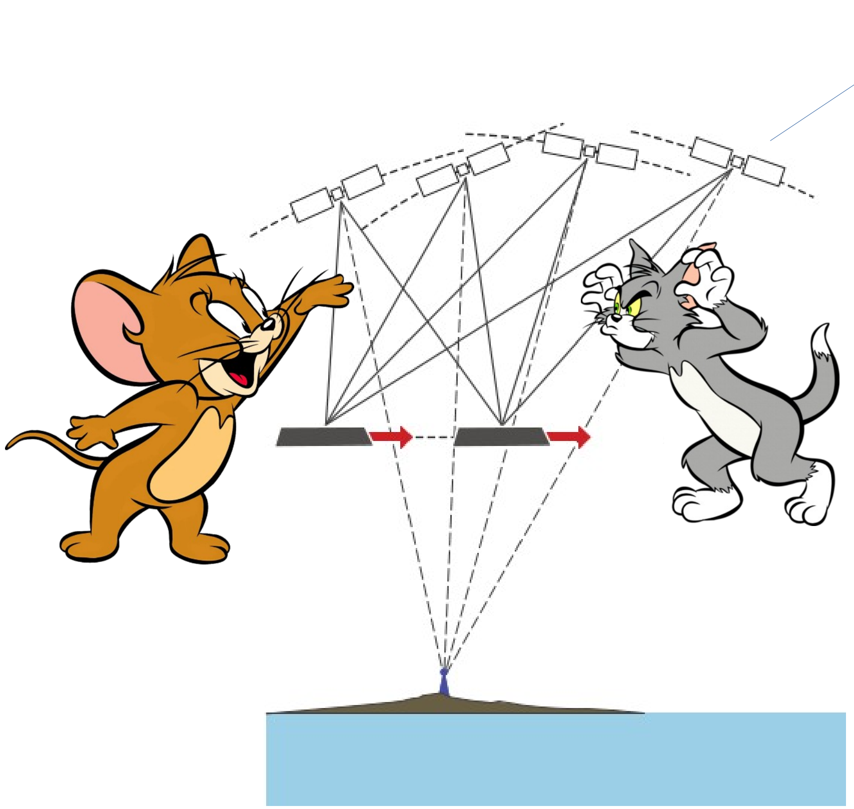

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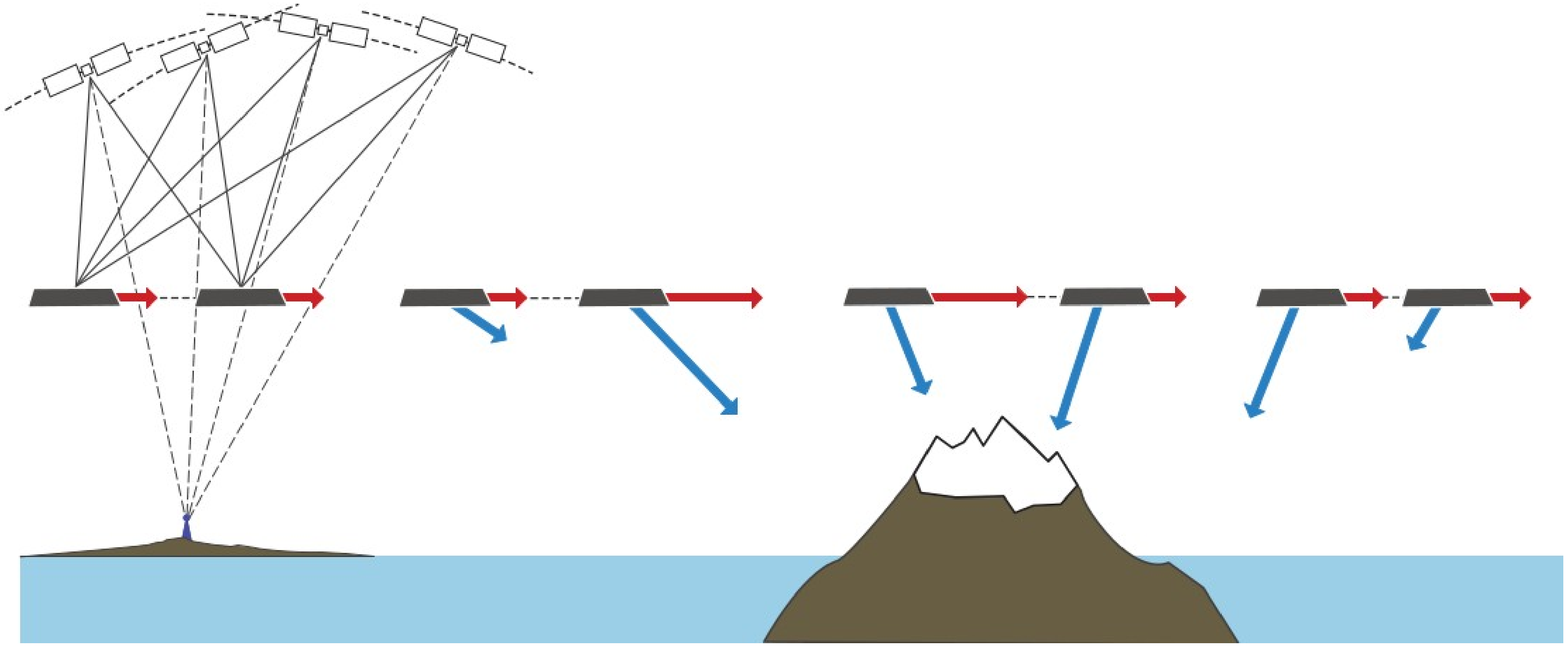

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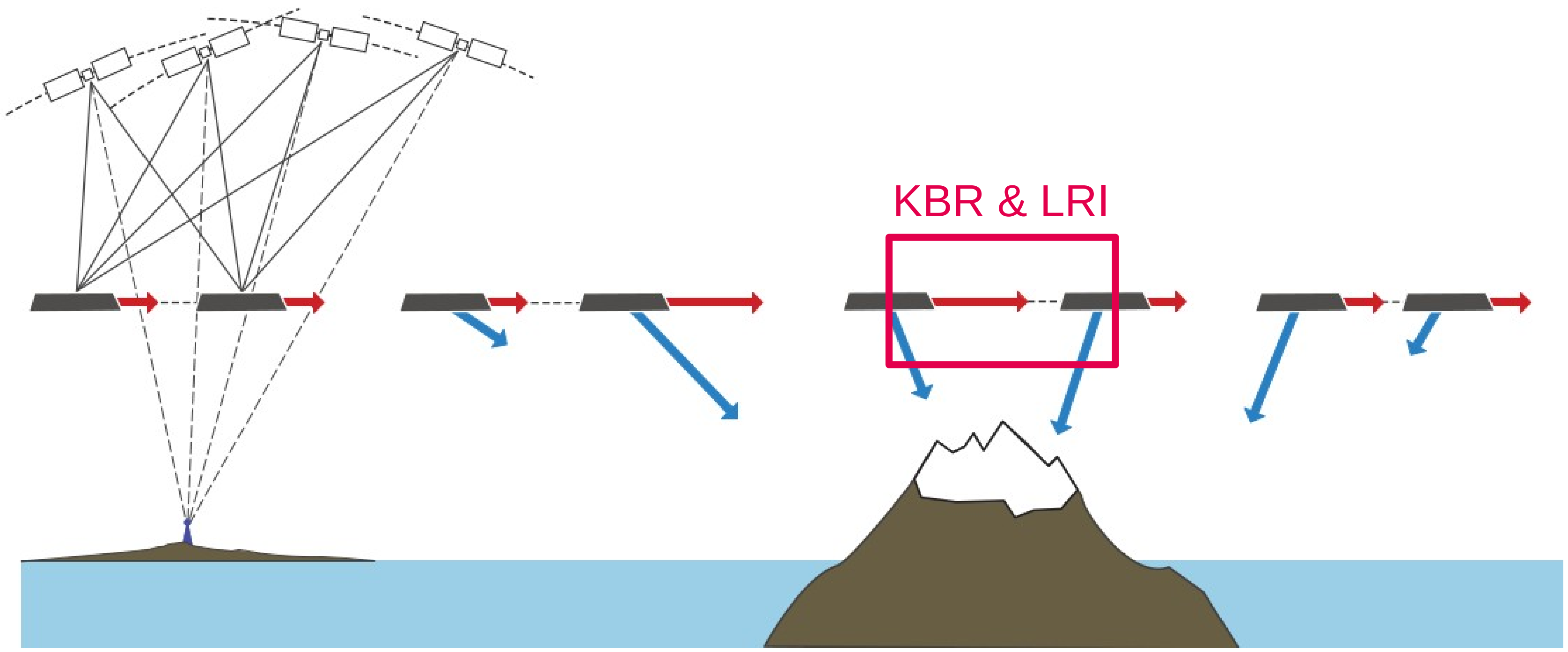

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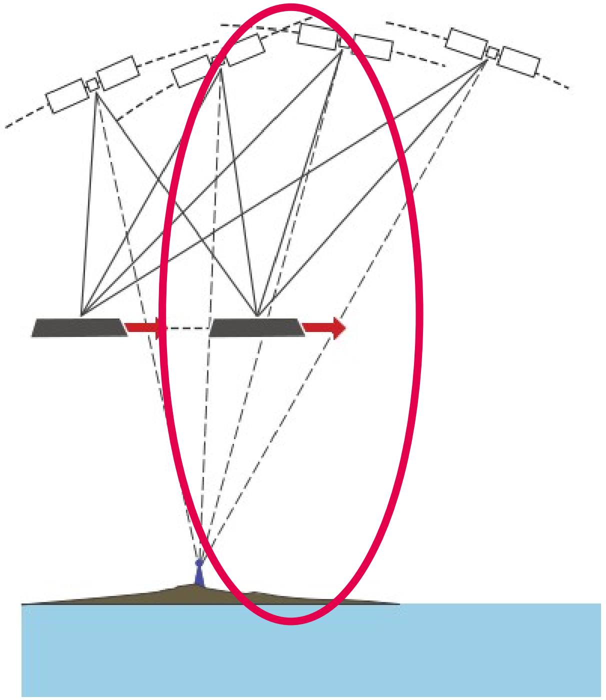
## Orbit representation



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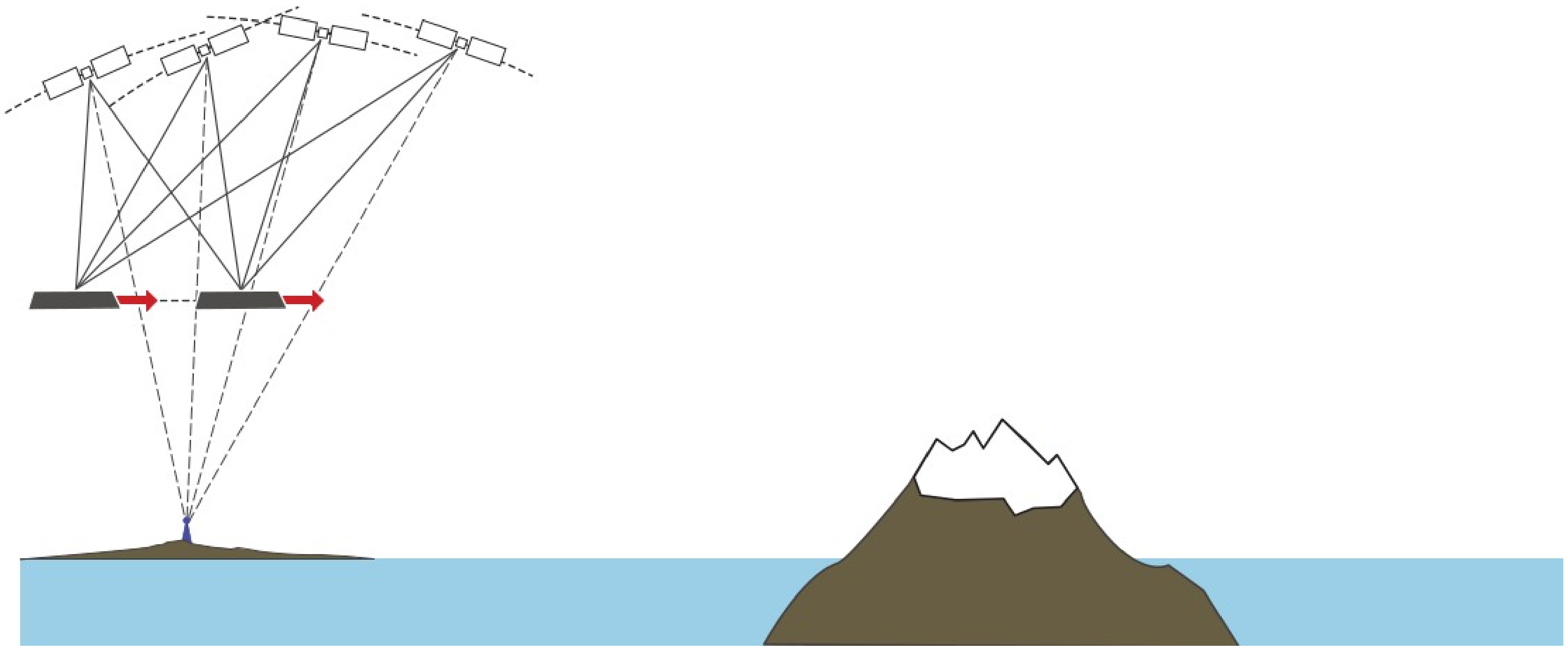

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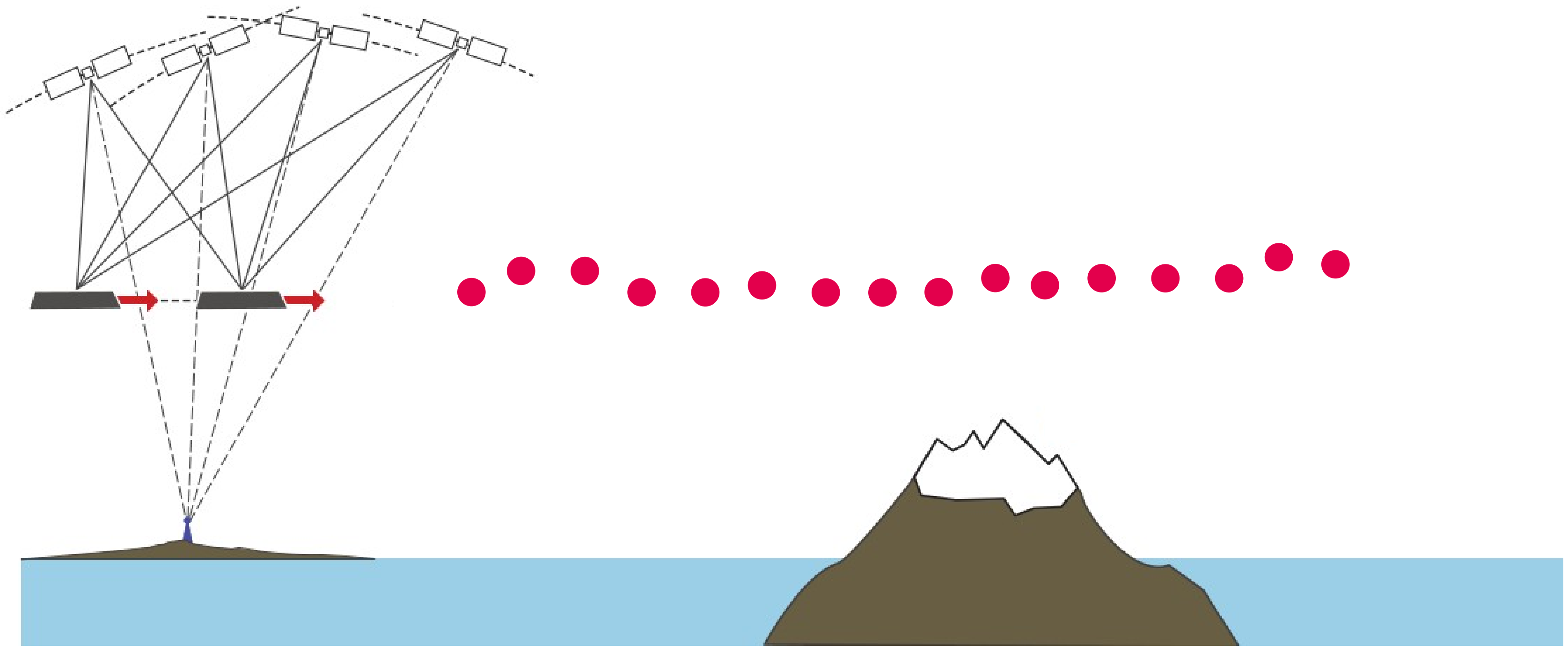

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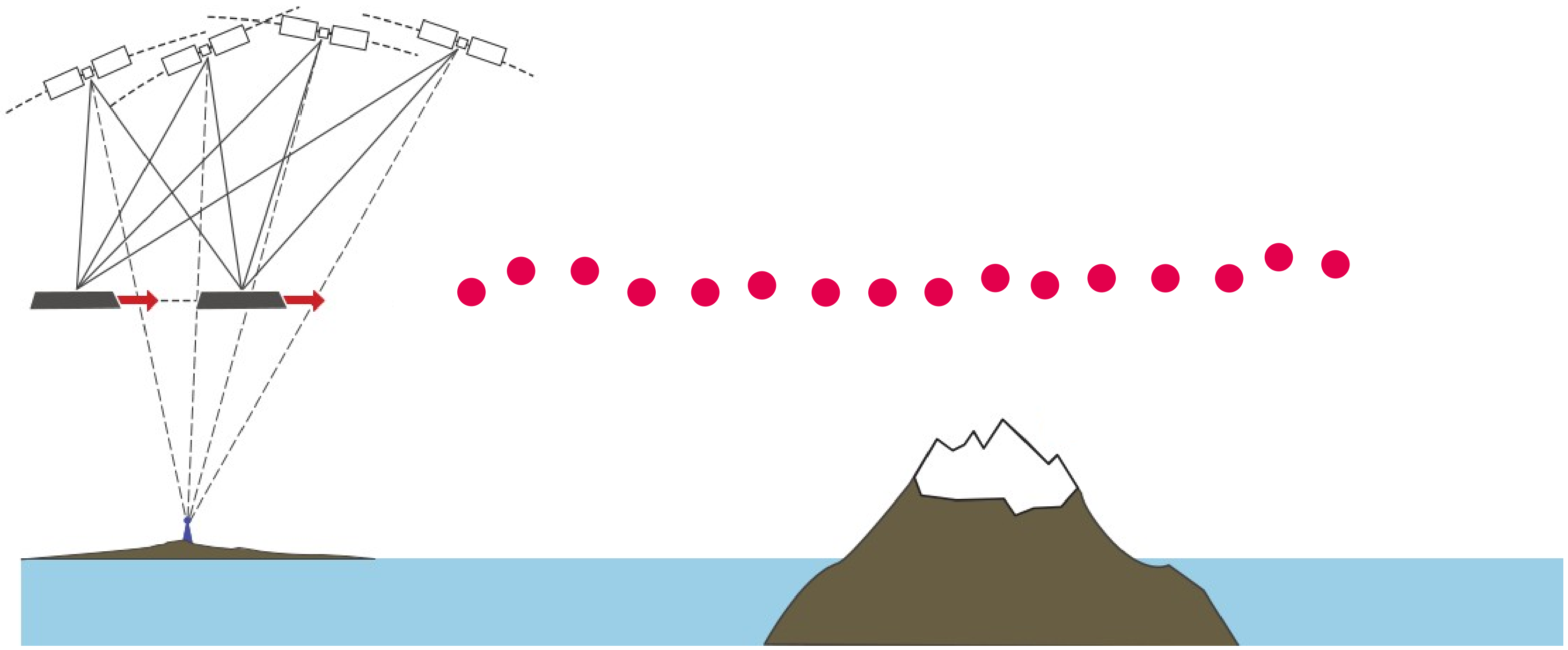

Kinematic ‹orbit›

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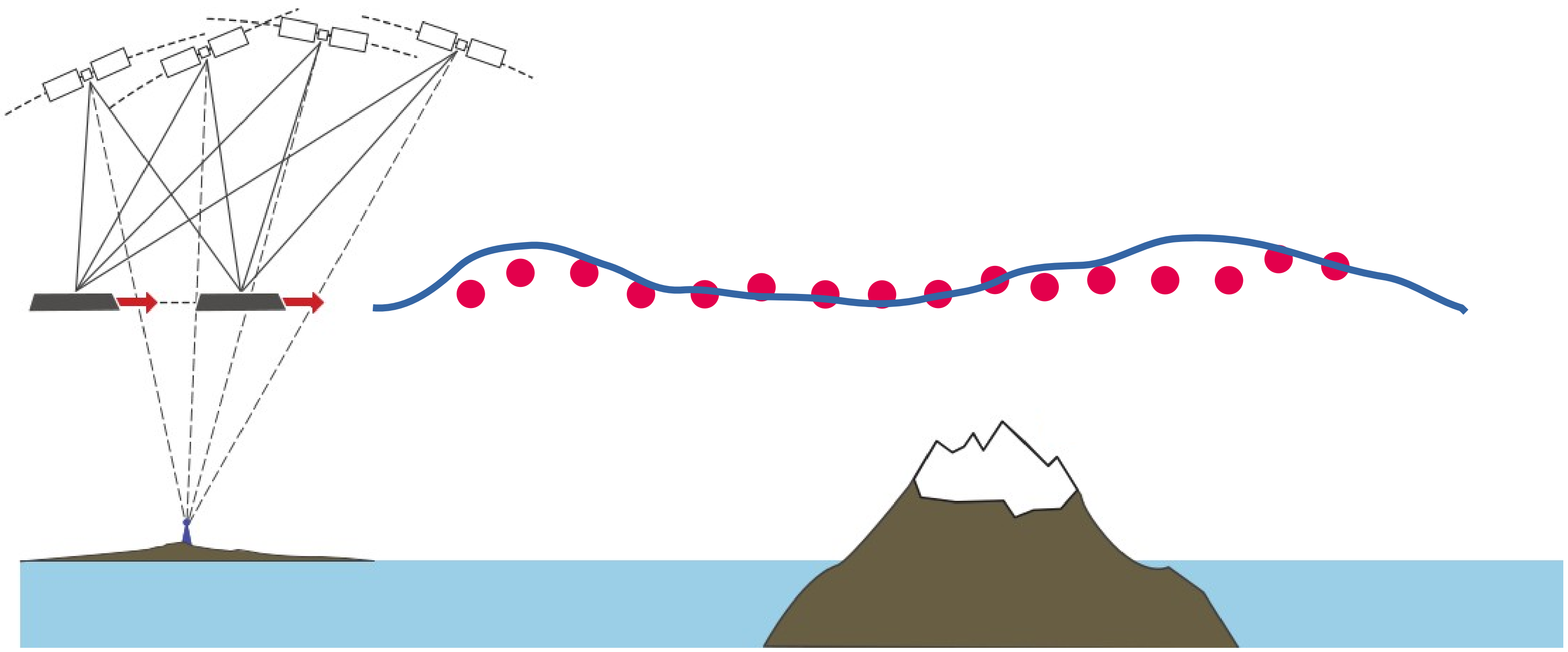
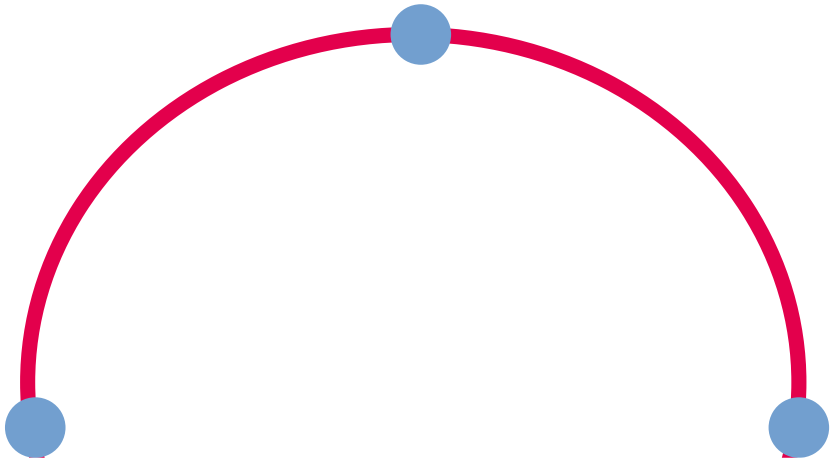


Kinematic ‹orbit›

(Reduced)
dynamic orbit

# Gravity Field Recovery

## Detour



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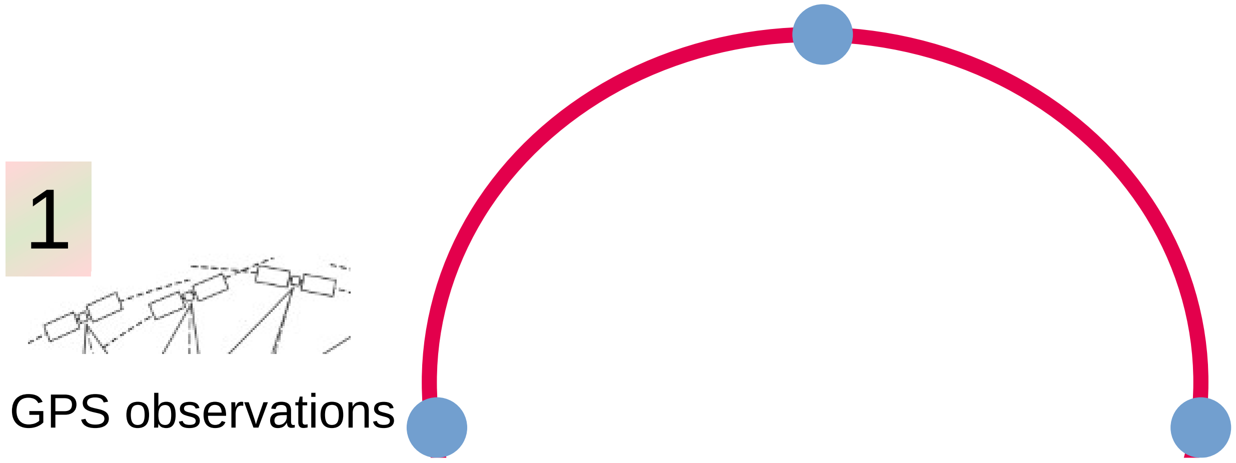

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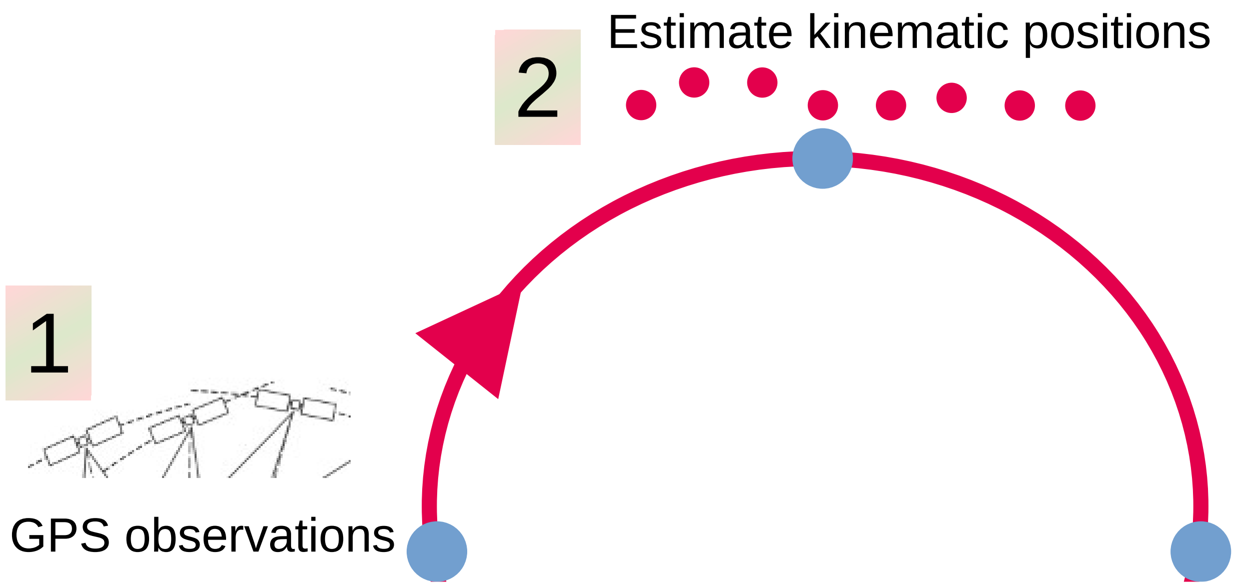

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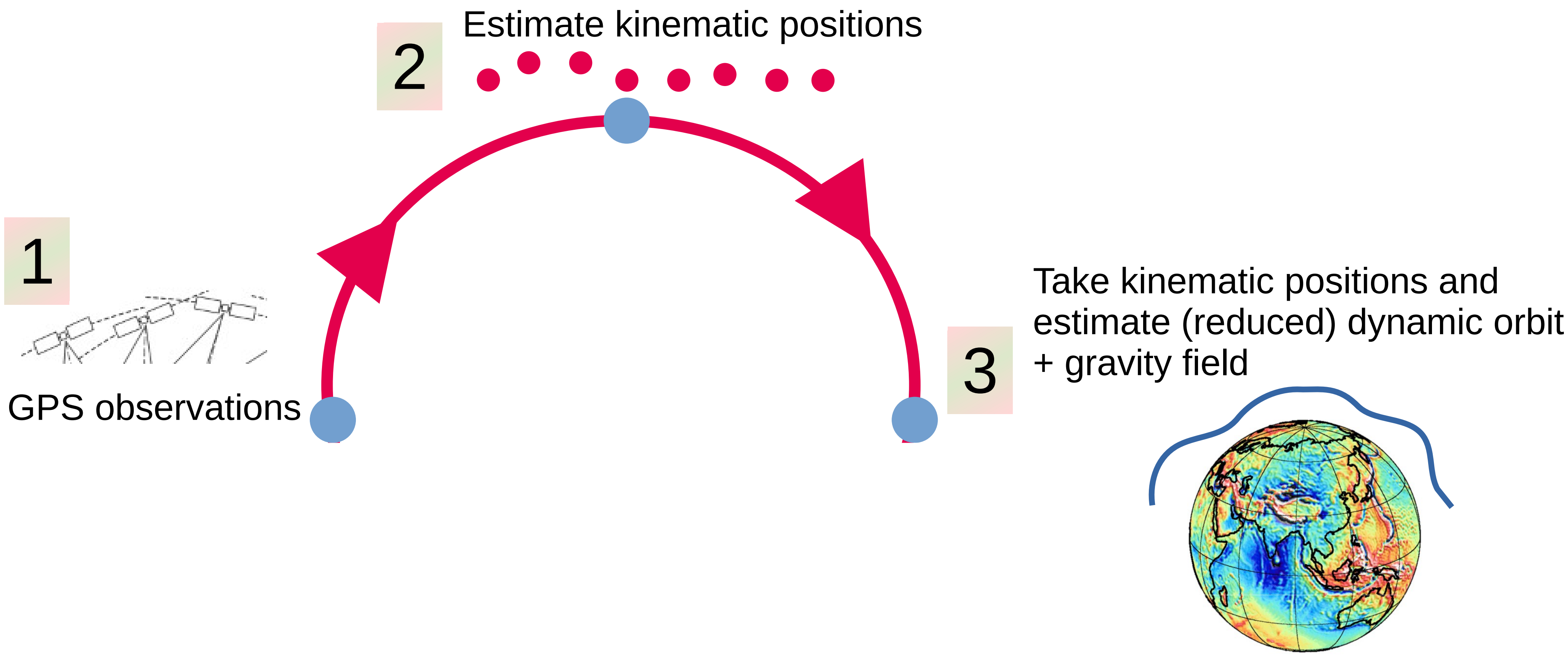
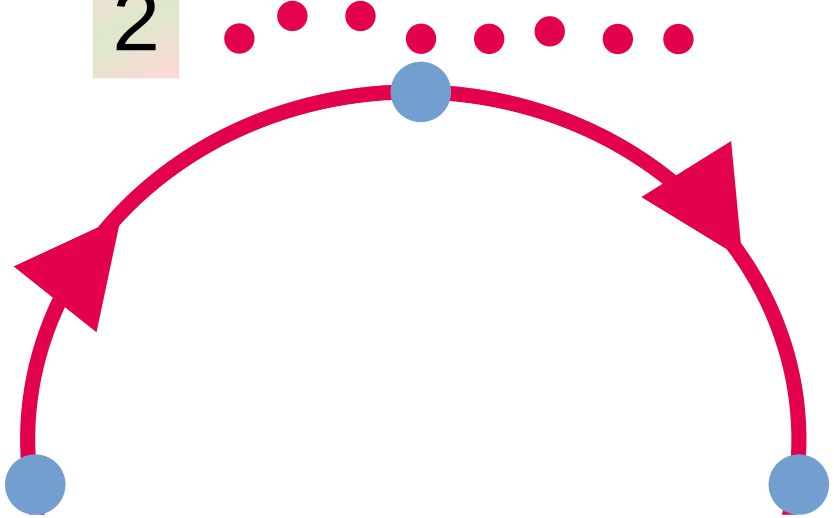


# Gravity Field Recovery

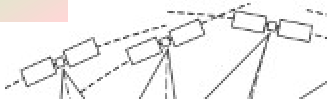
## Detour

2

Estimate kinematic positions



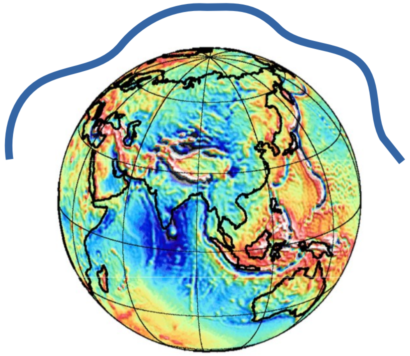
1



GPS observations

3

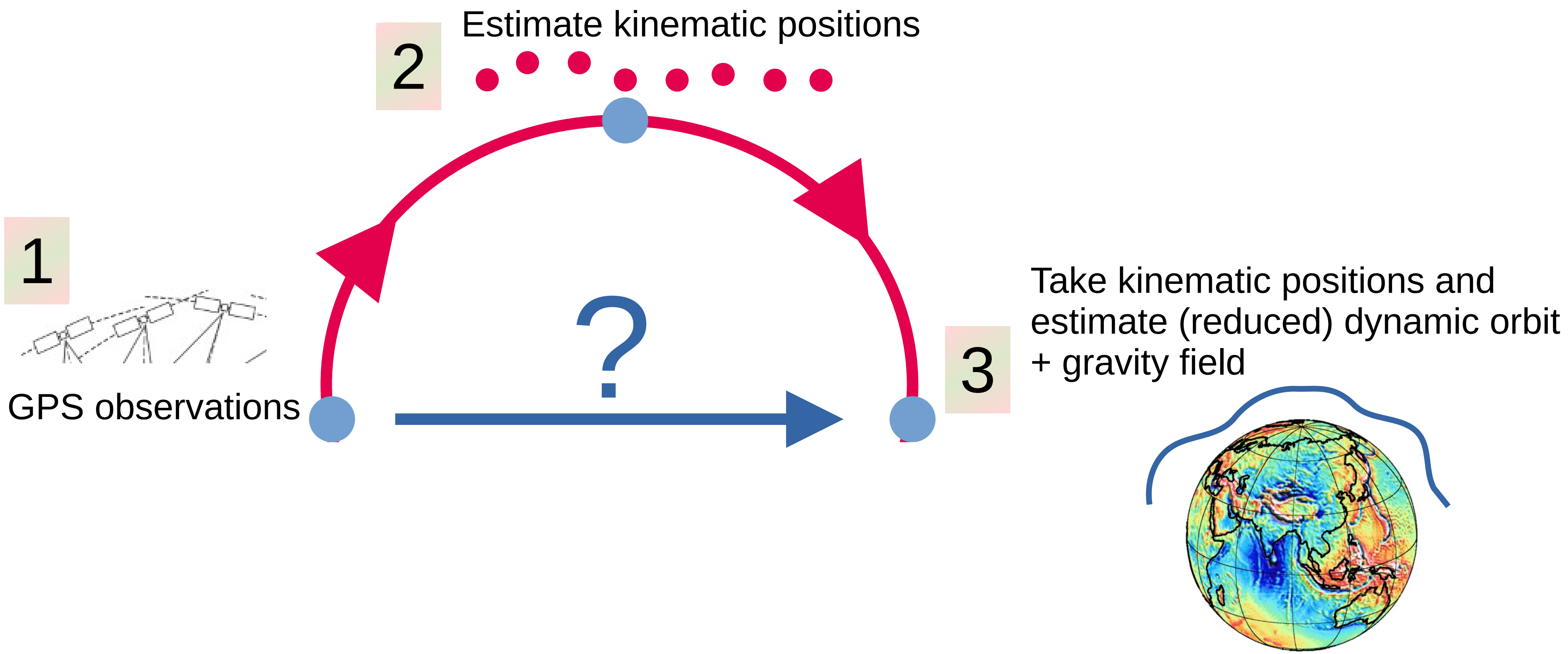
Take kinematic positions and estimate (reduced) dynamic orbit + gravity field



**Background information**

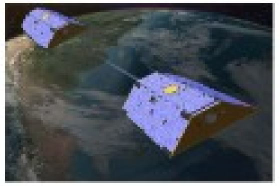
- Celestial Mechanics Approach (CMA, Beutler et al., 2010) applied
- CODE GNSS products
- PCV maps used
- Ambiguities float or integer-fixed

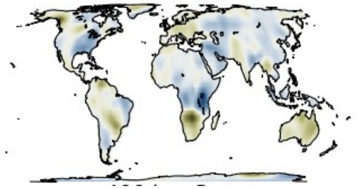
# Gravity Field Recovery
## Shortcut



# Parametrisation

## For estimating gravity fields





**Basic parametrisation**
- Initial conditions
- Bias in radial, along-track, cross-track
- Clock per epoch
- Ambiguities

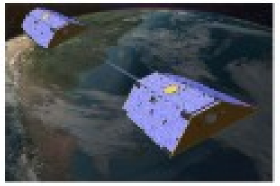
**Additional parameters**
- 15 min PCA per satellite in
  - → radial
  - → along-track
  - → cross-track

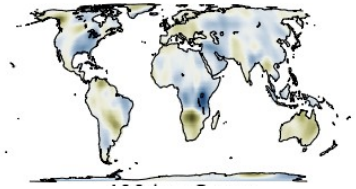
in daily arcs

+ gravity field d/o=2..70

# Parametrisation

## For estimating gravity fields





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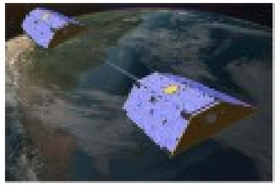

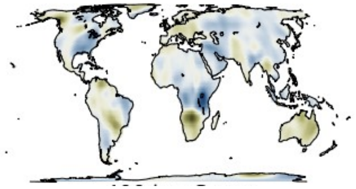

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- Initial conditions
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### Additional parameters
- 15 min PCA per satellite in
  - → radial
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  - → cross-track

+ gravity field

| Force models | |
|---|---|
| Gravity field | GOCO06s |
| Astronomic bodies | JPL DE421 (all planets) |
| Mean pole | Linear |
| Solid Earth tides | IERS2010 |
| Solid Earth pole tides | IERS2010 |
| Ocean tides | FES2014b<br>(+ admittances from TUG) |
| Ocean pole tides | Desai |
| Atmospheric tides | AOD RL06 |
| Atmospheric & oeanic dealiasing | AOD RL06 |
| Relativistic effects | IERS2010 |

# The Detour in a Nutshell

## A bit of math ...

### Least-squares to estimate kinematic positions

$$\hat{\boldsymbol{x}}_{\text{KIN}} = (\mathbf{A}^{\text{T}}\mathbf{P}_{\text{ph}}\mathbf{A})^{-1}\mathbf{A}^{\text{T}}\mathbf{P}_{\text{ph}}\boldsymbol{\ell}_{\text{ph}} \qquad \text{with} \qquad \mathbf{A} := \frac{\partial \boldsymbol{\ell}_{\text{ph}}}{\partial \boldsymbol{x}_{\text{KIN}}}$$

$$= \mathbf{N}_{\text{ph}}^{-1}\mathbf{A}^{\text{T}}\mathbf{P}_{\text{ph}}\boldsymbol{\ell}_{\text{ph}} \quad .$$

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$$= \mathbf{N}_{\mathrm{ph}}^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}_{\mathrm{ph}}\boldsymbol{\ell}_{\mathrm{ph}} \quad .$$

### Least-squares to estimate orbit and gravity field based on kinematic positions

$$\hat{\boldsymbol{p}} = (\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\boldsymbol{\ell}_{\mathrm{KIN}} \qquad \text{with} \qquad \mathbf{B} := \frac{\partial \boldsymbol{\ell}_{\mathrm{KIN}}}{\partial \boldsymbol{p}} \qquad \mathbf{Q}_{\mathrm{KIN}} := \mathbf{N}_{\mathrm{ph}}^{-1}$$

$$= \mathbf{N}_{\mathrm{p}}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\hat{\boldsymbol{x}}_{\mathrm{KIN}} \quad .$$

# The Detour in a Nutshell

## A bit of math ...

### Or summarise…

$$\mathbf{M} := \mathbf{AB}$$

$$\hat{\boldsymbol{p}} = (\mathbf{M}^{\mathrm{T}}\mathbf{P}_{\mathrm{ph}}\mathbf{M})^{-1}\mathbf{M}^{\mathrm{T}}\mathbf{P}_{\mathrm{ph}}\boldsymbol{\ell}_{\mathrm{ph}}$$

# The Detour in a Nutshell

## A bit of math ...

Or summarise…

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$$\hat{\boldsymbol{p}} = (\mathbf{M}^{\mathrm{T}}\mathbf{P}_{\mathrm{ph}}\mathbf{M})^{-1}\mathbf{M}^{\mathrm{T}}\mathbf{P}_{\mathrm{ph}}\boldsymbol{\ell}_{\mathrm{ph}}$$

# Stochastic Properties of Kinematic Positions

## And how to consider them

### Recall kinematic positions as observations

$$\hat{\boldsymbol{p}} = (\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\boldsymbol{\ell}_{\mathrm{KIN}} \qquad \text{with} \qquad \mathbf{B} := \frac{\partial \boldsymbol{\ell}_{\mathrm{KIN}}}{\partial \boldsymbol{p}}$$

$$= \mathbf{N}_{\mathrm{p}}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\hat{\boldsymbol{x}}_{\mathrm{KIN}} \quad .$$

# Stochastic Properties of Kinematic Positions

## And how to consider them

### Recall kinematic positions as observations

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$$= \mathbf{N}_{\mathrm{p}}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\hat{\boldsymbol{x}}_{\mathrm{KIN}} \; .$$

$$\mathbf{Q}_{\mathrm{KIN}} := \mathbf{N}_{\mathrm{ph}}^{-1}$$

# Stochastic Properties of Kinematic Positions

## And how to consider them

### Recall kinematic positions as observations

$$\hat{\boldsymbol{p}} = (\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\boldsymbol{\ell}_{\mathrm{KIN}} \qquad \text{with} \qquad \mathbf{B} := \frac{\partial \boldsymbol{\ell}_{\mathrm{KIN}}}{\partial \boldsymbol{p}}$$

$$= \mathbf{N}_{\mathrm{p}}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{KIN}}^{-1}\hat{\boldsymbol{x}}_{\mathrm{KIN}} \; .$$

$$\mathbf{Q}_{\mathrm{KIN}} := \mathbf{N}_{\mathrm{ph}}^{-1}$$

### Remark for the shortcut

When directly starting with phase observations all correlations are implicitly considered.

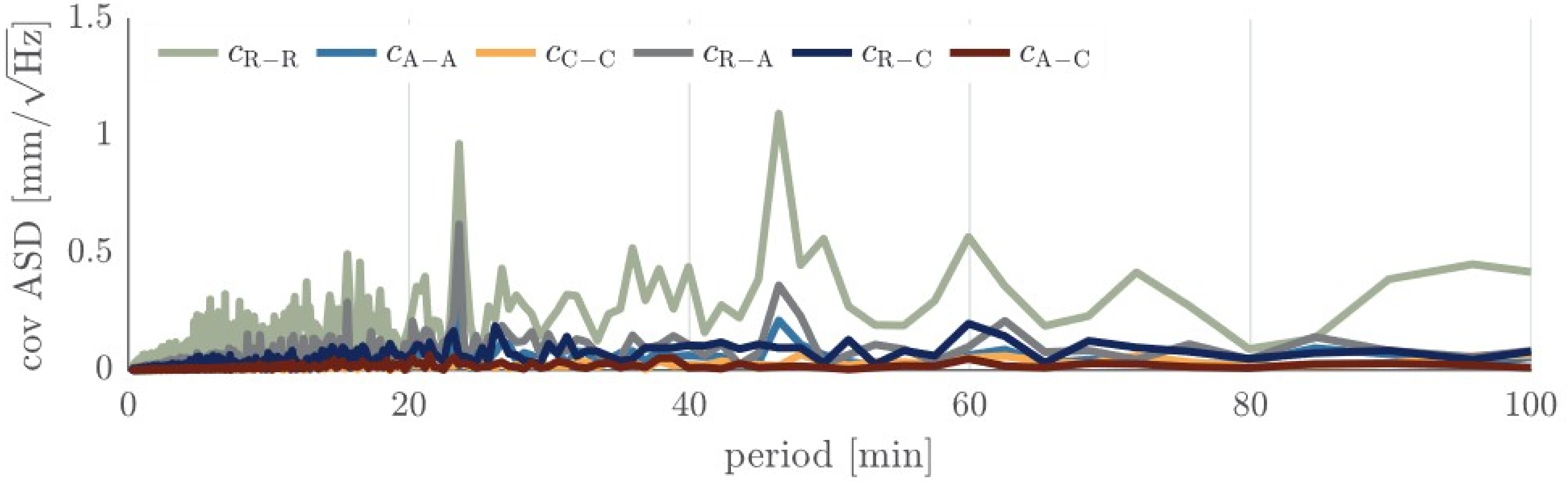
# Stochastic Properties Kinematic Positions

## Power-spectrum – epoch-wise

$$\mathbf{Q}_{\mathrm{KIN}} := \mathbf{N}_{\mathrm{ph}}^{-1}$$

$$\mathbf{C}_{\mathrm{epo}}^{\mathrm{KIN}} = \begin{bmatrix} c_{xx} & c_{xy} & c_{xz} \\ c_{xy} & c_{yy} & c_{yz} \\ c_{xz} & c_{yz} & c_{zz} \end{bmatrix} \rightarrow$$

radial
along-track
cross-track



# Stochastic Properties Kinematic Positions

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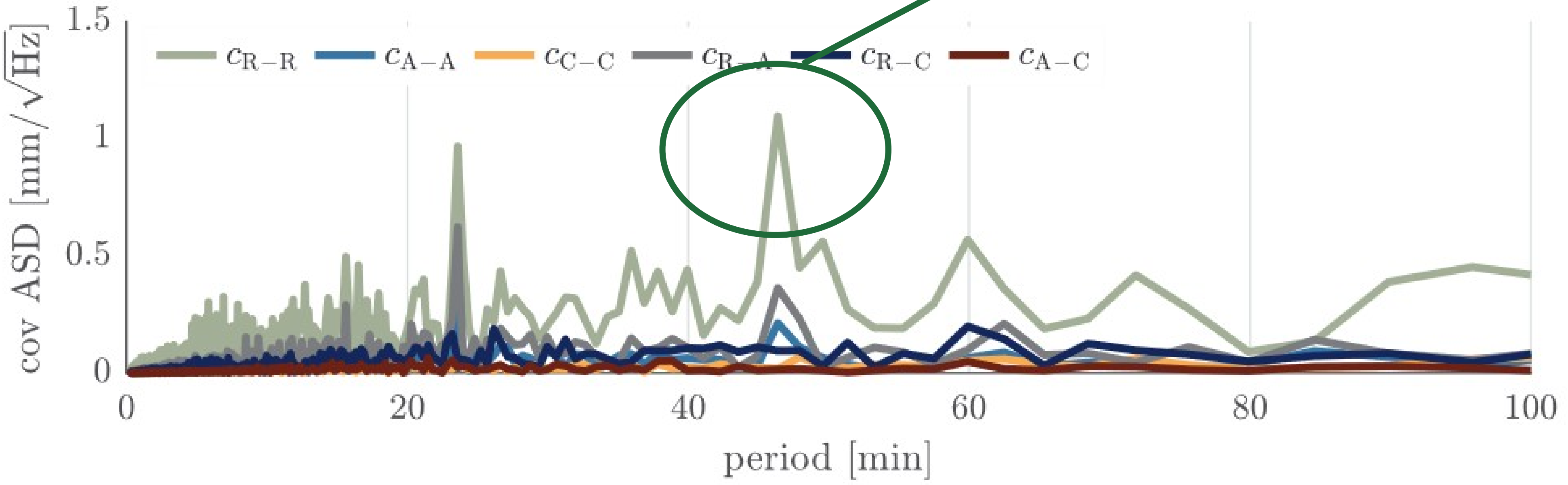
→ radial, along-track, cross-track

2/rev peak due to polar gap of GPS constellation



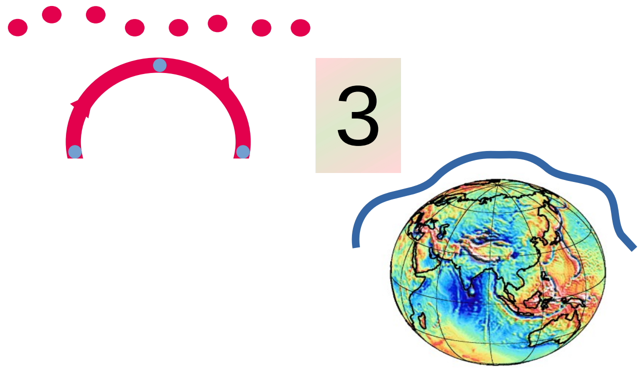
# Results Reduced Dynamic Orbit

## A priori orbit (pre-fit)

| | float | fixed |
|---|---|---|
| MAD carrier phase residuals [cm] | 0.21 | 0.29 |
| MAD w.r.t. kinematic positions [cm] | radial along cross<br>1.05 – 0.84 – 0.69 | radial along cross<br>0.63 – 0.26 – 0.19 |
| MAD of K-band validation [cm] | 0.17 | 0.14 |

# Results Gravity Field

## From kinematic positions



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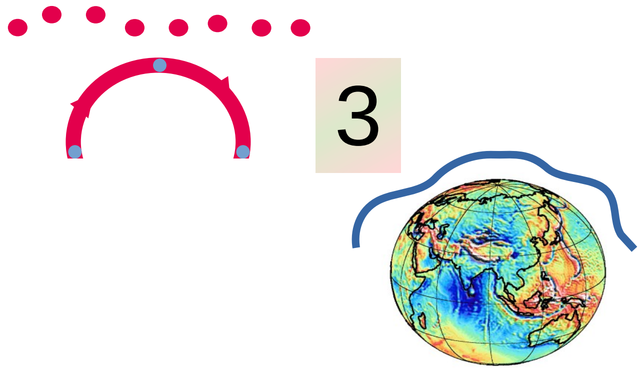
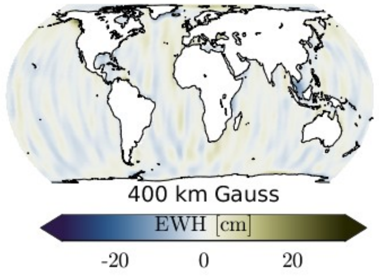
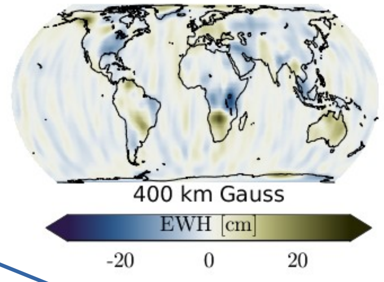
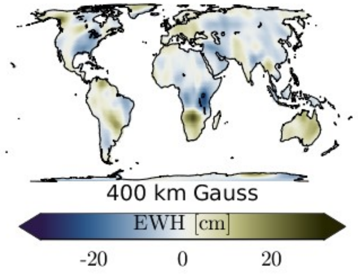



signal




RMS over the oceans
(EWH with 700 km Gauss filer w.r.t. GOCO06s)

# Results Gravity Field

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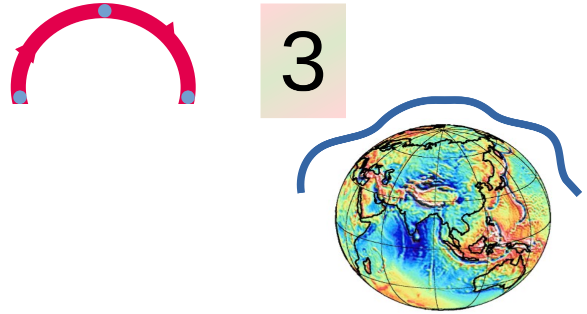


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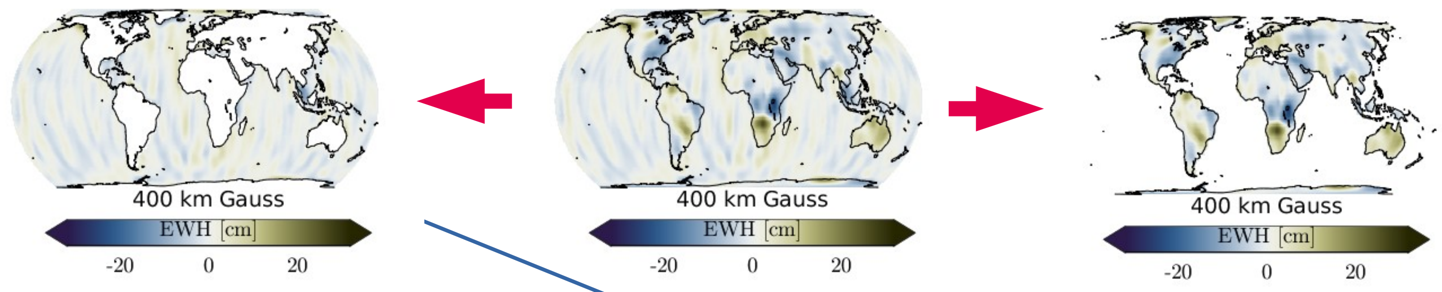


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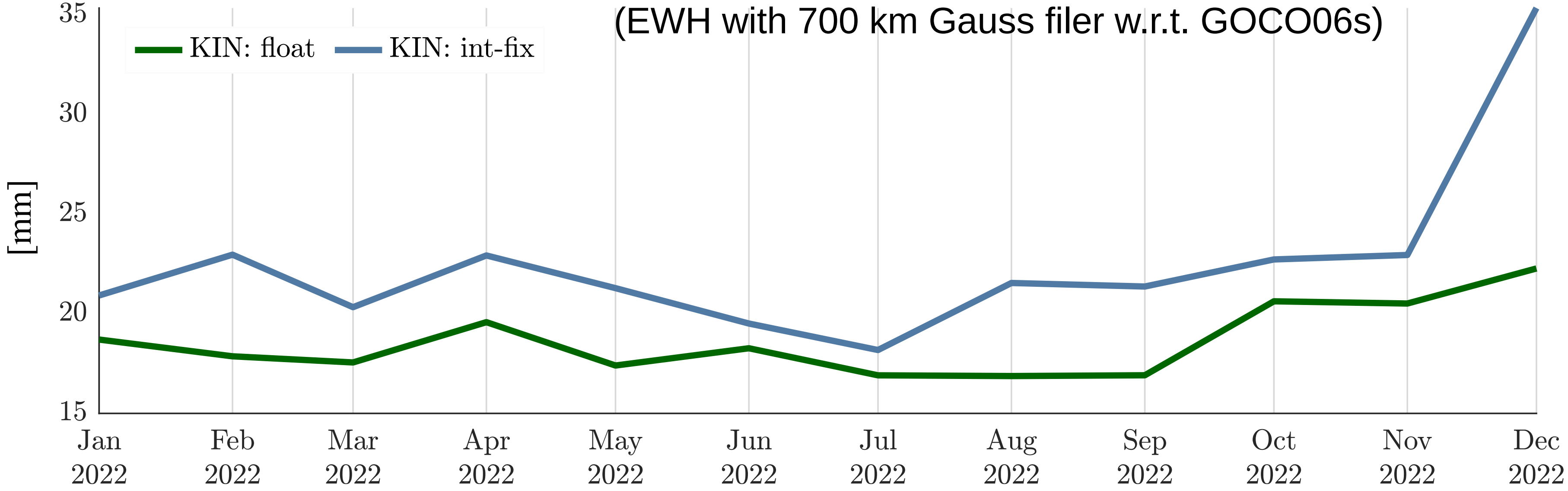
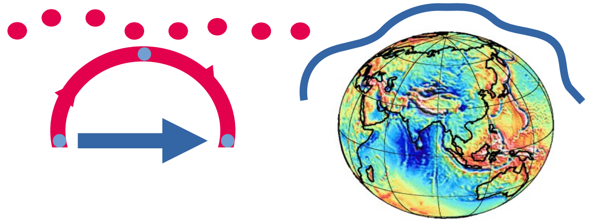
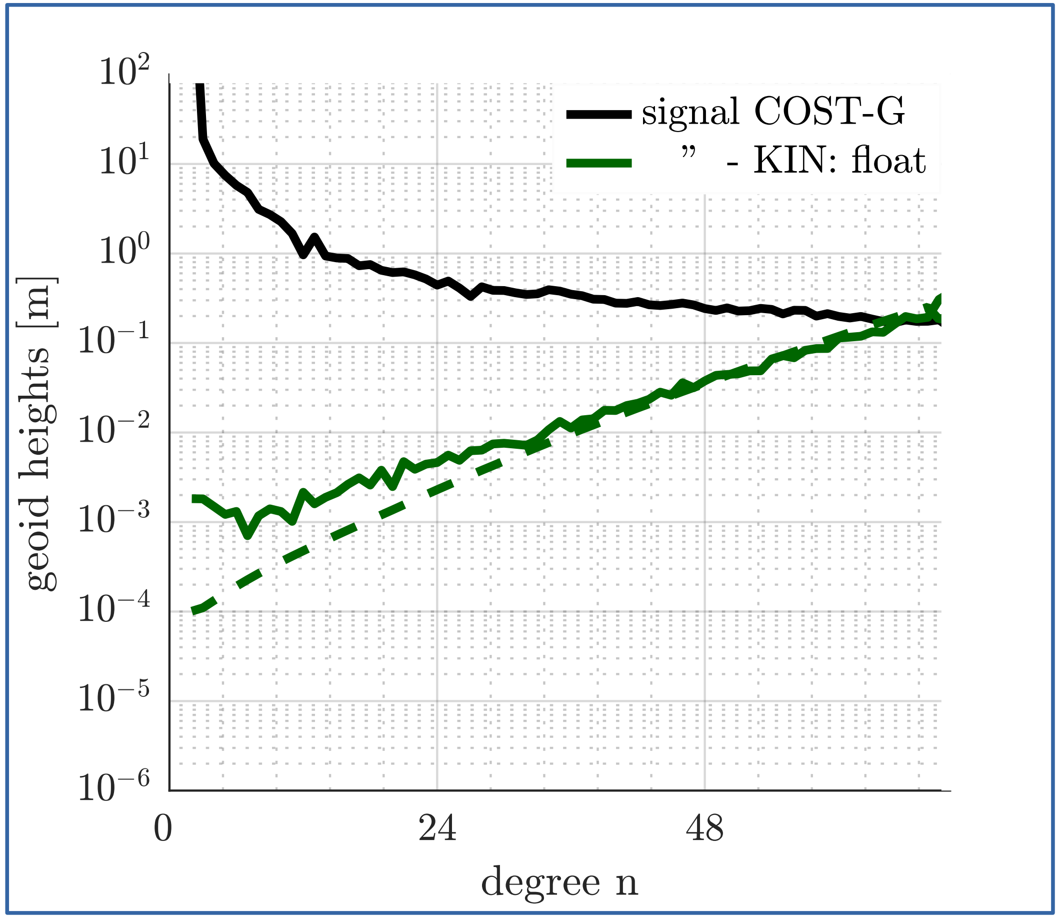


# Results Gravity Field
## Directly from phase observations





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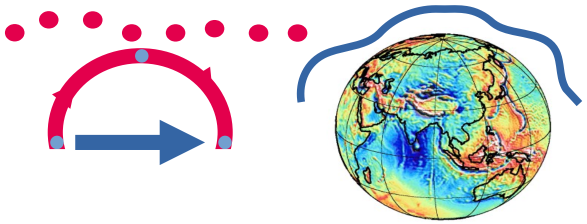
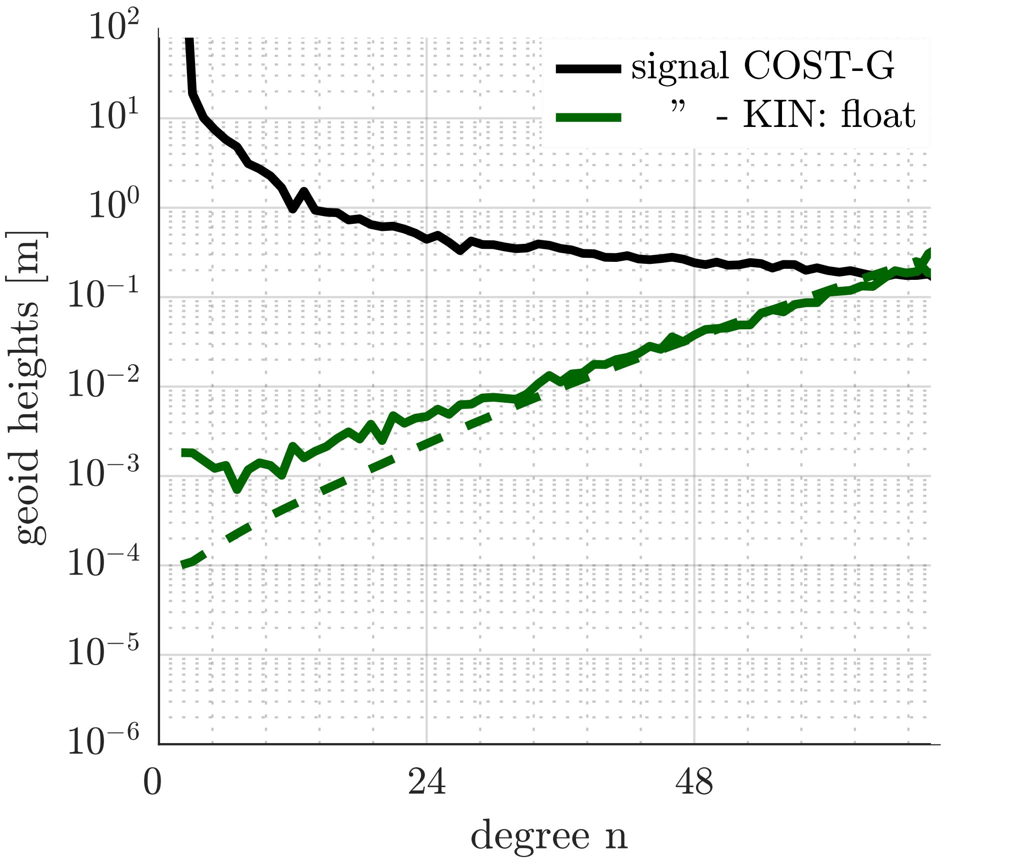





### Gravity field representation in spherical harmonics

unknown coefficients

$$V=\frac{GM}{R}\sum_{n=0}^{N}\left(\frac{a_e}{r}\right)^n\sum_{m=0}^{n}P_{nm}(\sin(\varphi))[c_{nm}\cos(m\lambda)+s_{nm}\sin(m\lambda)]$$

degree $N$ point of evaluation $(r,\varphi,\lambda)$

↓ degree

$c_{00}$

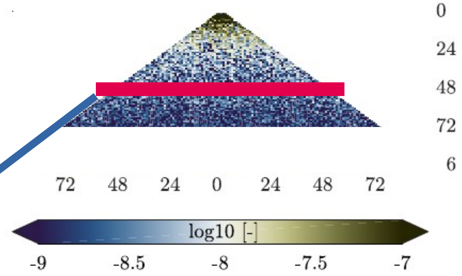
$s_{11}$ $c_{10}$ $c_{11}$

$s_{22}$ $s_{21}$ $c_{20}$ $c_{21}$ $c_{22}$

$s_{33}$ $s_{32}$ $s_{31}$ $c_{30}$ $c_{31}$ $c_{32}$ $c_{33}$

⋮

← order →




$$\sigma_n=\sqrt{\sum_{m=0}^{n}\left(c_{nm}^2+s_{nm}^2\right)}$$

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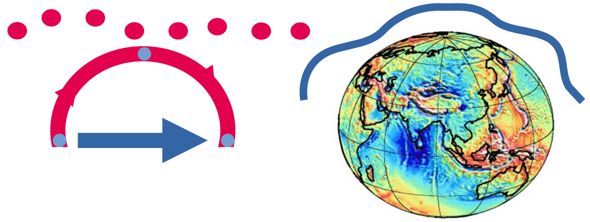

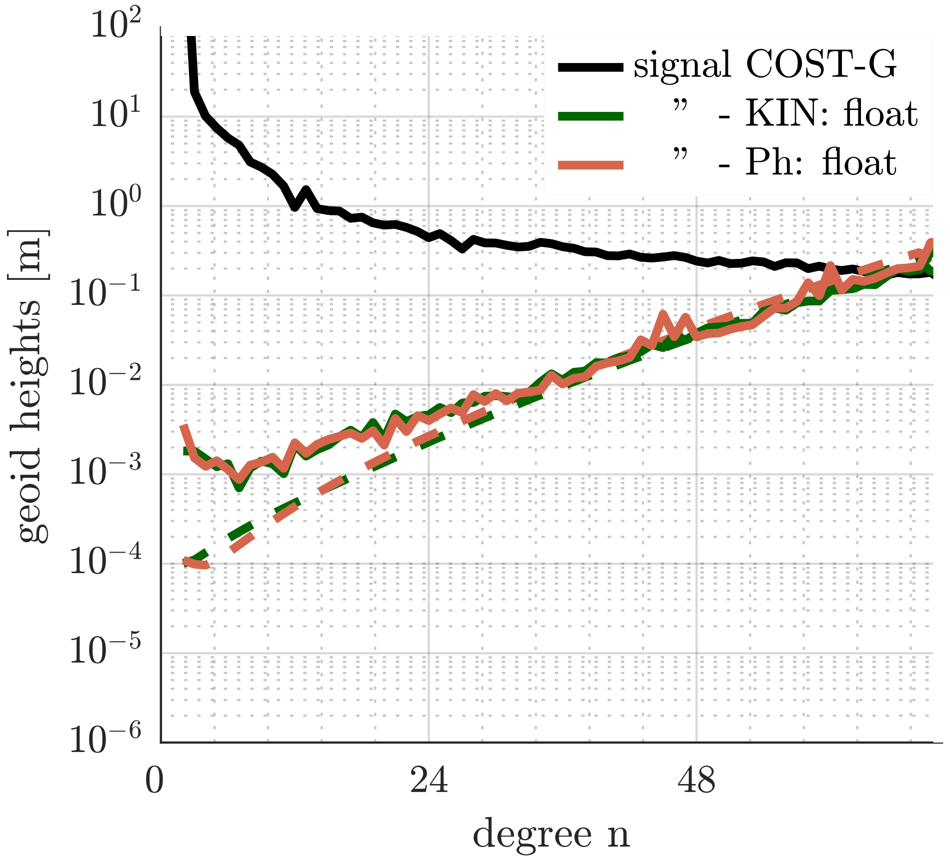

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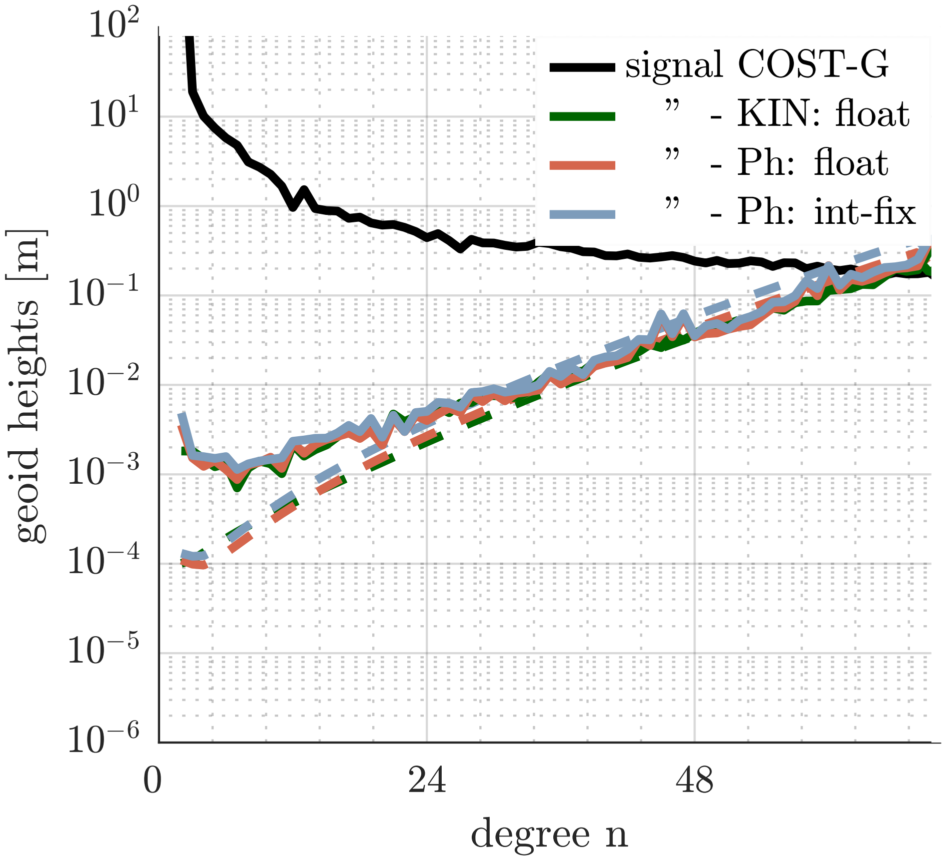


- Ambiguity-float solution almost the same for KIN or phase observations
- Ambiguity-integer-fixed solution slightly deteriorated
- Formal errors over-estimate low-degree coefficient quality

# Results Reduced Dynamic Orbit

## Jointly estimated with the gravity field (pre-fit & post-fit)

| | float | fixed |
|---|---|---|
| MAD carrier phase residuals [cm] | 0.21 | 0.29 |
| MAD w.r.t. kinematic positions [cm] | radial – along – cross<br>1.05 – 0.84 – 0.69<br>1.08 – 0.86 – 0.70 | radial – along – cross<br>0.63 – 0.26 – 0.19<br>0.64 – 0.26 – 0.20 |
| MAD of K-band validation [cm] | 0.17<br>0.17 | 0.14<br>0.13 |

# Summary

## & Conclusions

- Gravity field determination for carrier phase observations introduced
- Challenges for ambiguity resolution strategy remain in all cases
- Computational efficiency needs to be improved
- For GRACE Follow-On:
  - Extend with inter-satellite link
  - Combine carrier phase data and K-band/LRI data

# Thank you for your attention

**Contact**

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# References A

Beutler, G., Jäggi, A., Mervart, L. and Meyer, U. [2010]: The celestial mechanics approach: theoretical foundations. Journal of Geodesy, vol. 84(10), pp. 605-624. https://doi.org/10.1007/s00190-010-0401-7

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