

Introduction

In collaboration with the Bundesamt für Kartographie und Geodäsie Frankfurt (BKG), which acts as an analysis center of the International Laser Ranging Service (ILRS), the Bernese GNSS software, which is developed and maintained at the Astronomical Institute of the University of Bern (AIUB), has been transformed into a multi-technique software, capable of processing Satellite Laser Ranging (SLR) data.

The classical operational SLR-routine, as performed for the ILRS, is based on data of the two LAGEOS and the two high-orbiting ETALON satellites. Seven day orbits are determined, together with loosely constrained station coordinates, range biases for selected stations, and Earth Orientation Parameters (EOPs), i.e. x and y coordinates of the rotation pole and the length of day (LOD). For each 7d arc five empirical acceleration parameters, i.e. an along-track bias and periodic variations with 1/revolution frequency in along-track and cross-track, are estimated to absorb deficiencies in the force model.

With the accessibility of observations to the LARES-2 satellite, launched in July 2022 into an orbit altitude of 5900 km, comparable to the LAGEOS satellites, the new satellite was added to the operational weekly orbit and network solutions. Currently, a full set of range biases is estimated for LARES-2, until consolidated apriori values are available.

Solution setup for Low Earth Orbiters

The first LARES satellite was launched in February 2012 into a much lower orbit at 1450km altitude. Due to the more complex force modeling at this altitude the satellite has not yet been included into the routine SLR analysis. We present test solutions where LARES orbits have either been parameterized as 7d arcs, with one set of empirical accelerations per arc, or as long-arcs (Beutler et al., 1996), where continuous 7d arcs are accumulated from seven 1d arcs, each with a set of five empirical parameters. The solutions are compared in terms of the EOP results (Figs. 1-3), station coordinates (Figs. 4-5), and normal point residuals (Fig. 7).

Due to the increased sensitivity of the low-orbiting LARES to the Earth's gravity field we also generate solutions, where gravity field coefficients of spherical harmonic degrees 2 - 4 are co-estimated. Due to strong correlations between the C_{20} coefficient and the periodic 1/rev cross-track, or C_{30} and the 1/rev along-track accelerations, we constrain these empirical parameters to zero and study the impact on the satellite orbits (Fig. 7) and the estimated gravity field parameters (Fig. 6).

Tab. 1: A priori observation uncertainties, relative weights, mean number of normal point observations per 7d solution and satellite orbit characteristics.

	σ_o	weight	num. obs.	altitude	rev. period
LAGEOS 1/2	1.0 cm	1.00	1784	5860/5620 km	225/223 min
ETALON 1/2	3.0 cm	0.11	216	19120 km	675 min
LARES	1.5 cm	0.44	1045	1450 km	115 min
LARES 2	1.0 cm	1.00	739	5900 km	225 min

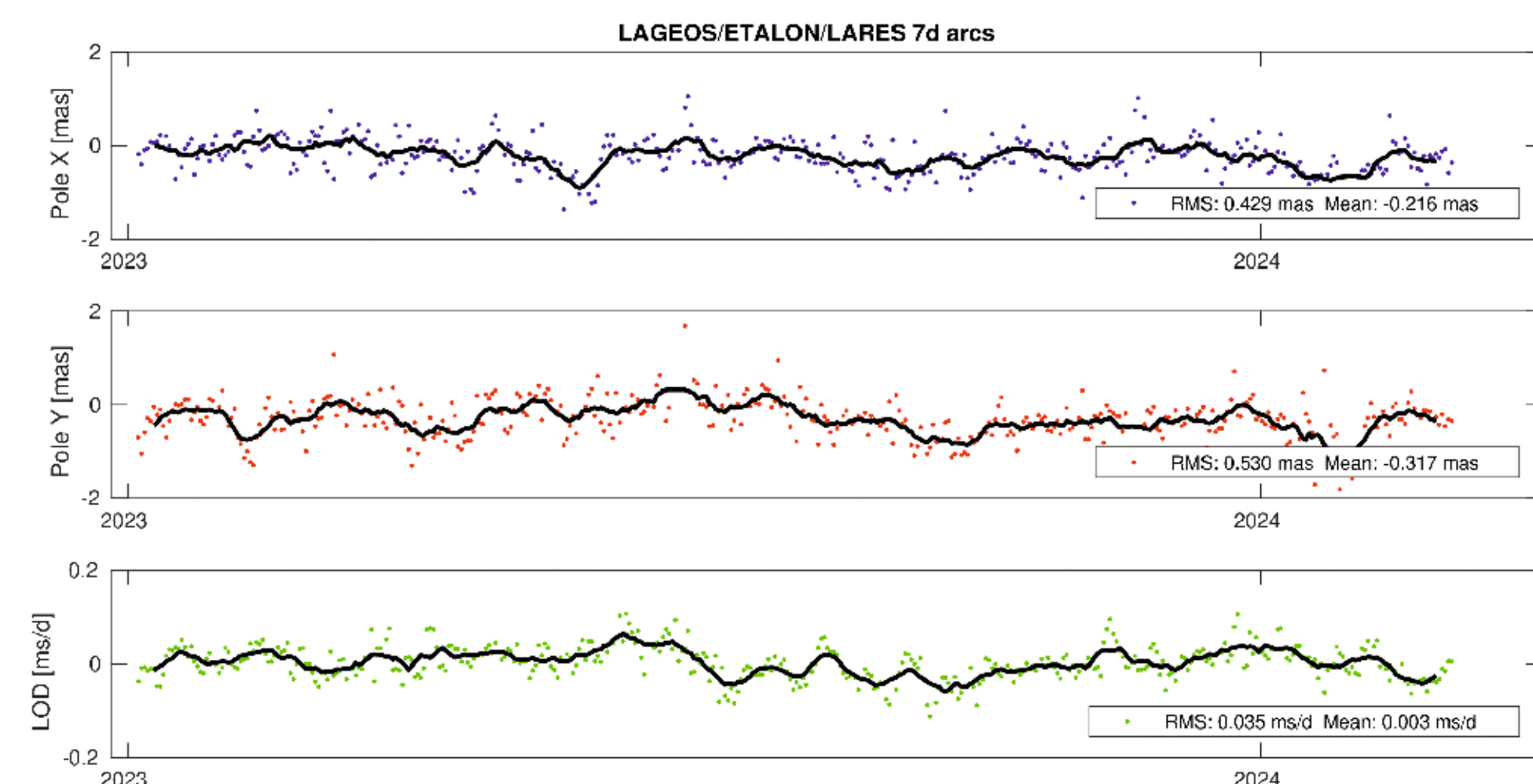


Fig. 1: Earth Orientation Parameters (EOPs) determined by a classical 7 day solution including 7d arcs of the low-orbiting LARES satellite.

Orbit parameterization aspects in global solutions of spherical Laser Ranging Satellites

Earth orientation parameters

EOPs are commonly determined in SLR-analysis. Due to the strong correlation of the time argument dUT with the precession of the ascending node only its rate, i.e. length-of-day (LOD) is estimated, while dUT is fixed to the daily apriori values of the IERS C04-series. Pole coordinates are estimated as piecewise constant, as requested by the ILRS, and daily rates are taken over from the C04 apriori time-series. Here, we focus on the impact of LARES on the scatter of the solutions, represented by the RMS of the EOP, where C04 serves as reference.

Tab. 2: Quality of EOP-estimates in terms of scatter and bias with respect to the IERS C04 time-series.

	RMS [mas]	Bias [mas]	RMS [mas]	Bias [mas]	RMS [ms/d]	Bias [ms/d]
true 7d arcs						
Pole X			Pole Y		LOD	
LAG/ET	0.257	-0.005	0.322	-0.052	0.032	0.001
LAG/ET/LAR2	0.248	-0.038	0.268	-0.067	0.030	0.001
LAG/ET/LAR	0.429	-0.216	0.530	-0.317	0.035	0.003
LARES: long-arc						
LAG/ET/LAR/LAR2	0.371	-0.191	0.405	-0.200	0.022	-0.004
grav. 2-4 est.	0.254	0.083	0.285	0.031	0.020	-0.003
no cross-track 1/rev	0.371	0.059	0.455	-0.073	0.026	-0.008
no 1/rev	0.609	0.049	0.747	-0.026	0.036	-0.003

Compared to the LAGEOS/ETALON combination, the addition of LARES 2 allows to decrease the scatter (Tab. 2), while an increase in the bias of the pole coordinates has been attributed to the imbalance of pro-/retro-grade orbital motions introduced by the 5th satellite (Geisser et al., 2022). Considering LARES instead of LARES-2 negatively impacts scatter and bias, as long as 7d arcs are used (Fig. 1).

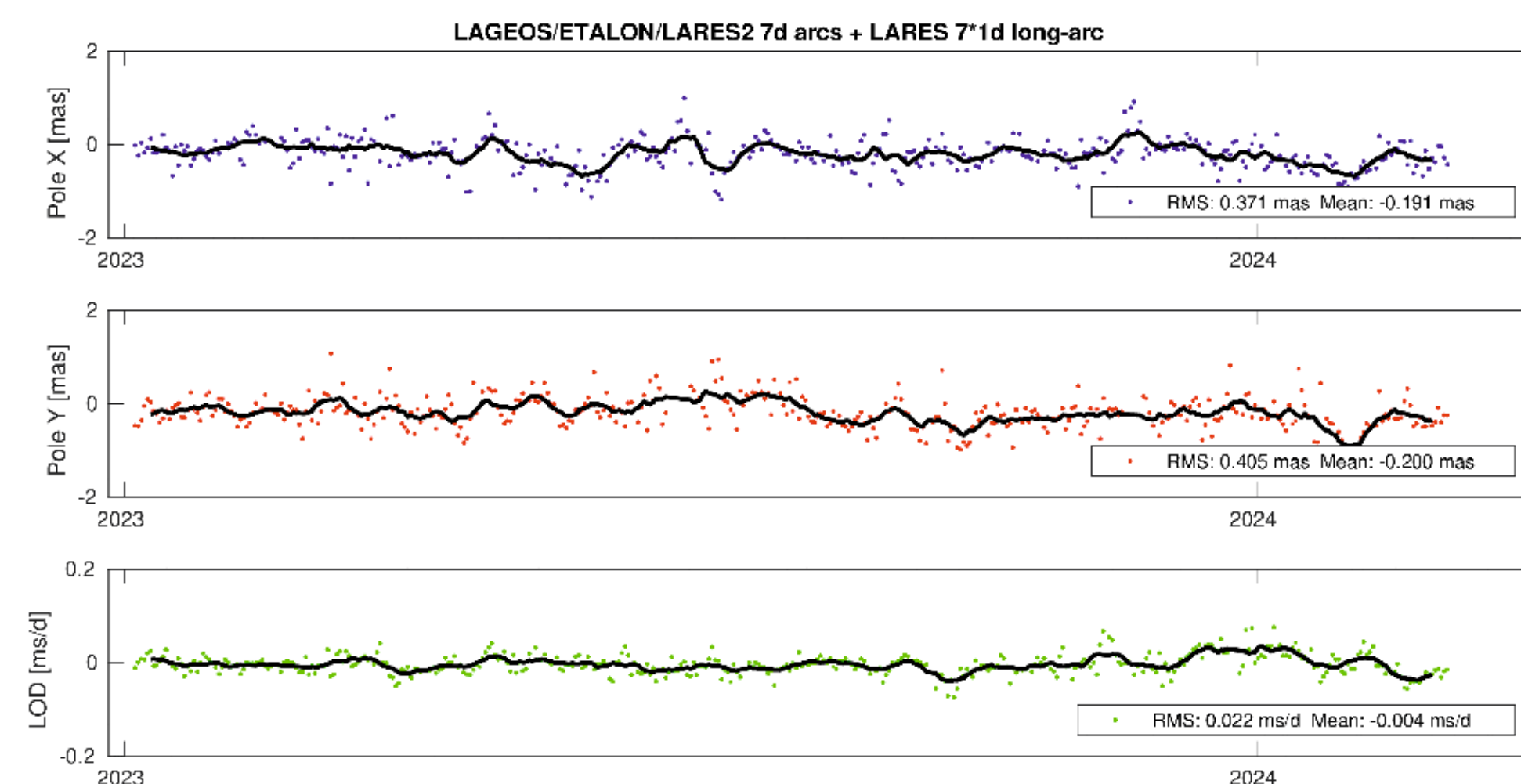


Fig. 2: EOPs derived from 7 day solutions, where LARES was added as long-arc, composed of seven 1d arcs.

Parameterizing LARES with long-arcs helps to decrease the scatter of the ERPs (Fig. 2), but the quality of the LAGEOS/ETALON/(LARES 2) solutions can only be reached, if low-degree gravity field coefficients are co-estimated (Fig. 3). This quality gain is lost again, if the periodic 1/rev accelerations are fixed to 0 by strong constraints in order to improve the quality of the gravity estimates (Tab. 2).

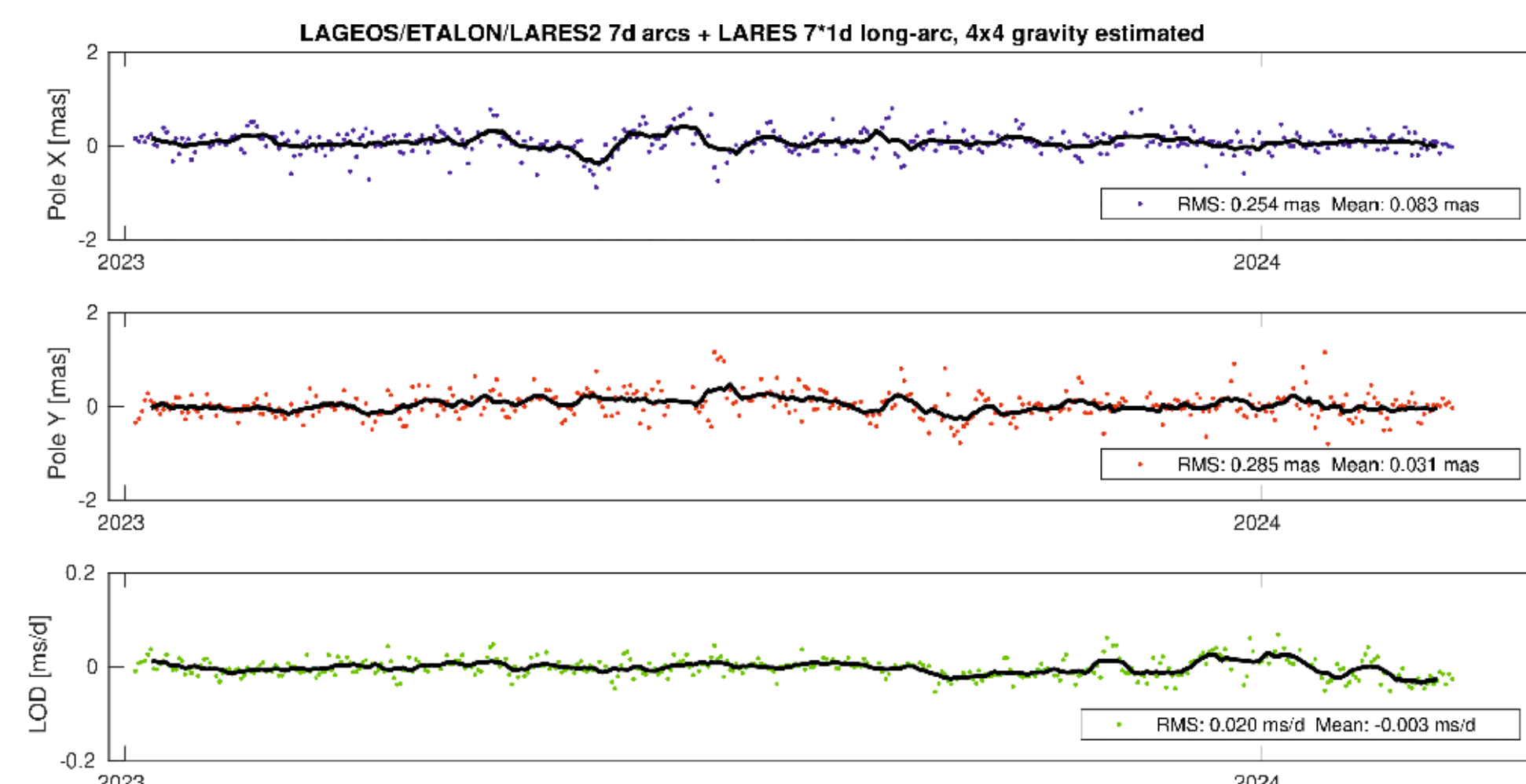


Fig. 3: Same as in Fig. 2, but this time with co-estimated gravity field coefficients of degrees 2-4.

Contact address

Astronomical Institute, University of Bern, Sidlerstrasse 5, 3012 Bern (Switzerland)
ulrich.meyer@unibe.ch

Station Coordinates and Scale

Station coordinates are routinely determined together with the satellite orbits and geophysical parameters. By request of the ILRS the datum is only loosely defined by 1m constraints, where SLRF2020 is introduced as apriori. For quality monitoring the network solutions are compared to the a priori reference frame by applying a seven parameter (translation, rotation, and scale) Helmert transformation, based on the reference stations as defined by the ILRS (considering only reference stations with more than 29 observations per weekly solution). The best consistency, at the level of 10 mm RMS, is achieved for the LAGEOS/ETALON combined solutions (Fig. 4). The addition of further satellites (LARES, LARES 2) has little impact, neither on the station coordinates, nor on the scale, as long as the orbits are parameterized as 7d arcs.

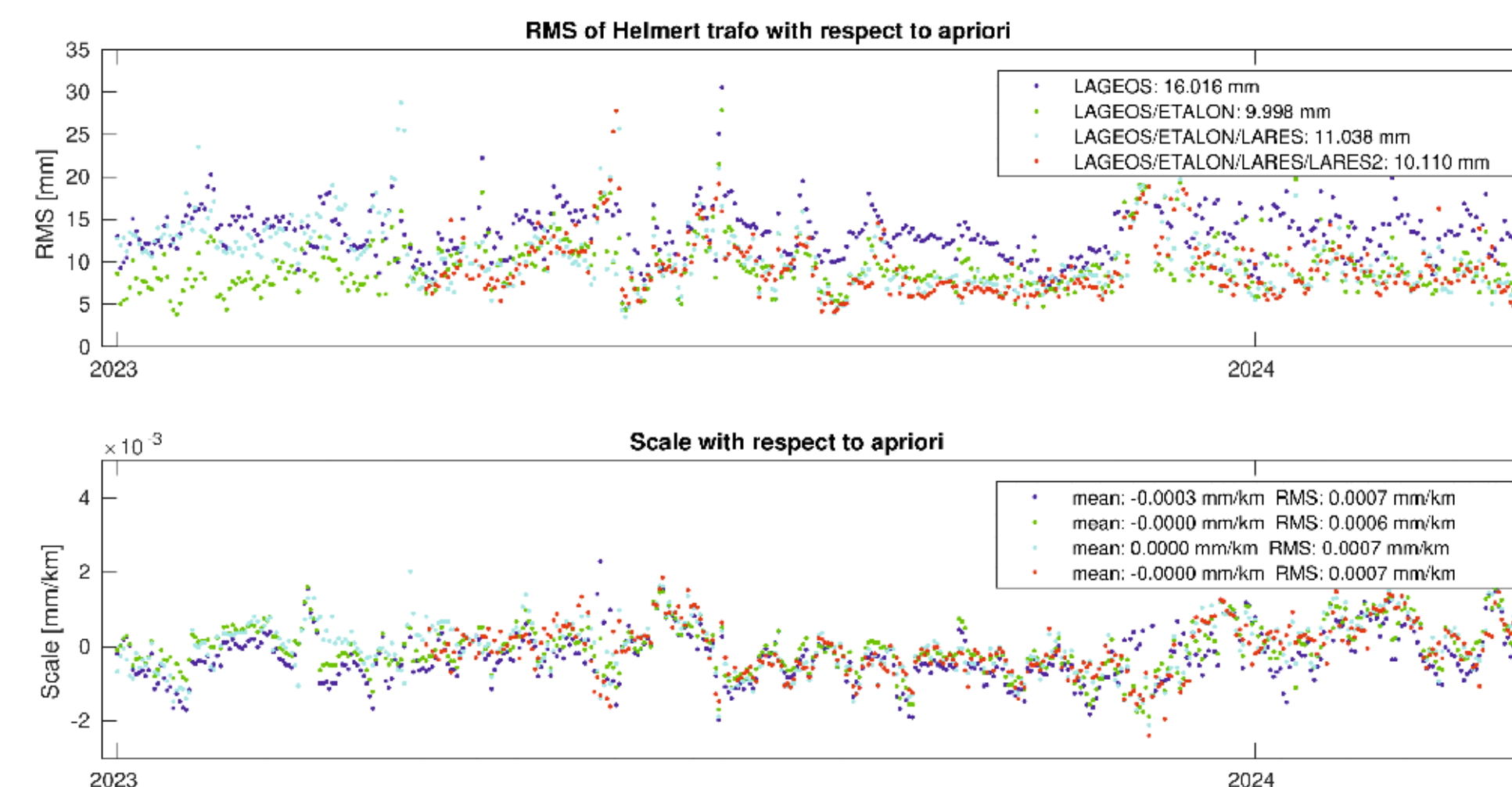


Fig. 4: Comparison of network solutions in terms of scatter (top) and scale (bottom) of the well-observing ILRS reference stations with respect to SLRF2020.

If LARES is added, parameterized as long-arc, we observe an increase of the scatter of the station coordinates from 10.1 mm to 14.6 mm, that may be slightly reduced if gravity field coefficients are co-estimated (Fig. 4). Constraining the empirical periodic 1/rev accelerations leads to drastically enlarged scatter, while the scale is less affected.

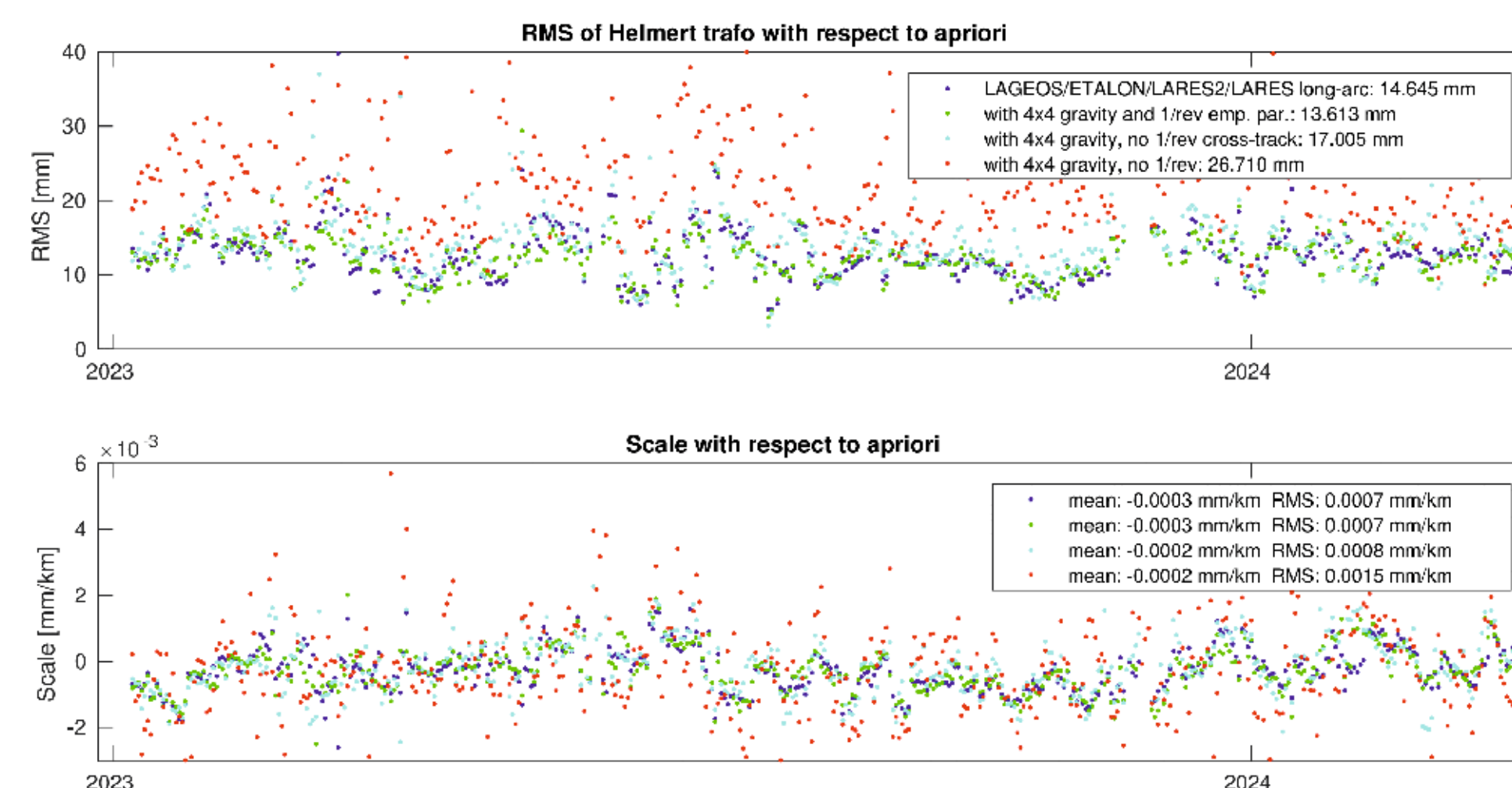


Fig. 5: Same as in Fig. 4, with LARES parameterized as long-arc.

Co-estimation of gravity field parameters

Due to its lower orbit altitude, LARES is more sensitive to temporal variations of the gravity field. We therefore co-estimate weekly gravity field coefficients of degrees 2 - 4 (Geisser et al., 2023). The gravity field estimation has a positive impact on the determination of the EOPs (Fig. 3) and the station coordinates (Fig. 5), but a closer look at the estimates of C_{20} and C_{30} reveals that this is caused by over-parameterization. Compared to the COST-G FSM apriori gravity field model that already includes secular and seasonal variations (Peter et al., 2023), the estimates of C_{20} and C_{30} show unreasonable scatter due to the correlations with the empirical periodic 1/rev accelerations. A strict constraining of these empirical parameters leads to reasonable estimates of the gravity field coefficients, but negatively affects the orbit quality (Fig. 7) and station coordinate estimates (Fig. 5).

U. Meyer¹, L. Geisser¹, R. Dach¹, D. König², D. Thaller², and A. Jäggi¹

¹Astronomical Institute, University of Bern, Switzerland

²Bundesamt für Kartographie und Geodäsie, Frankfurt a.M., Germany

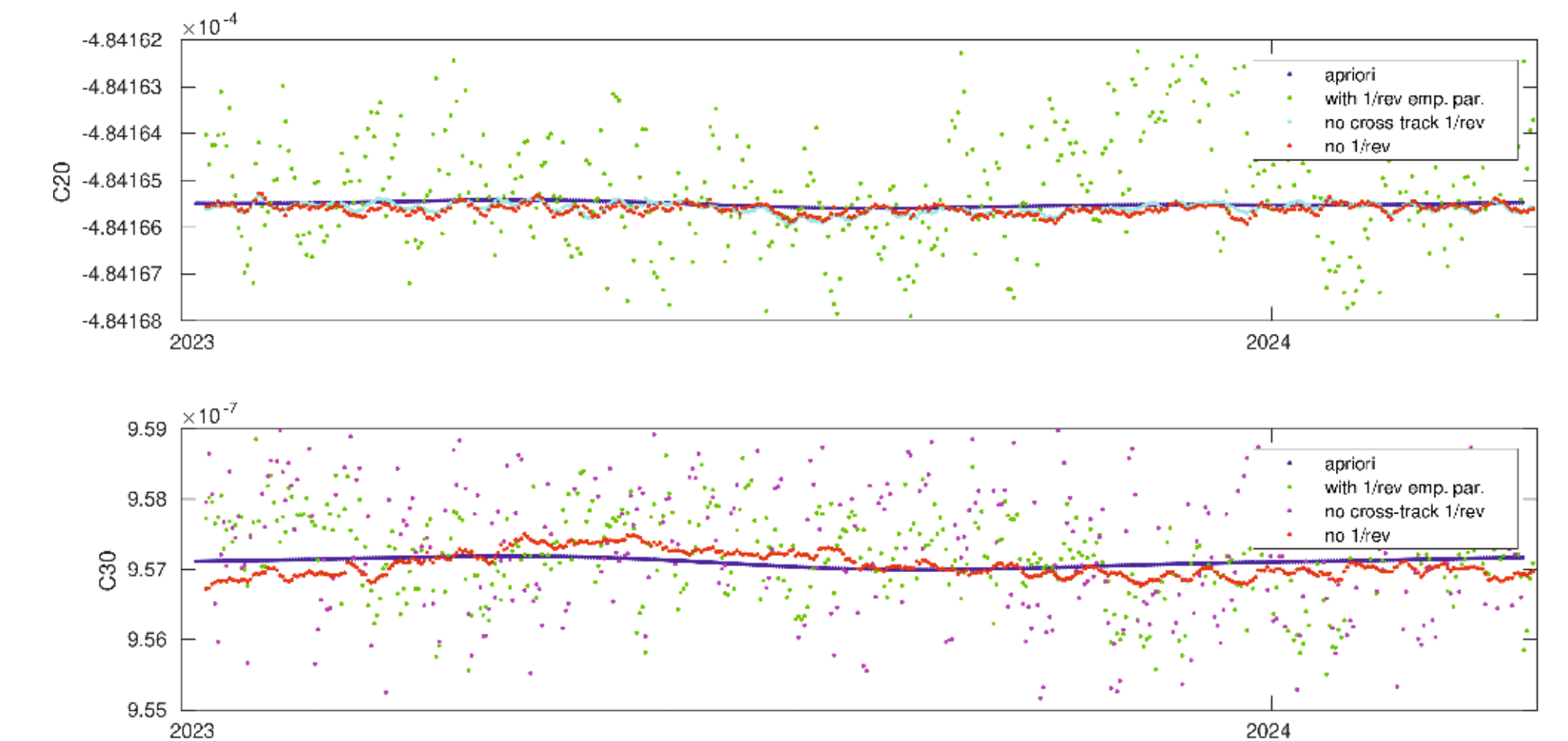


Fig. 6: Apriori and estimated gravity field coefficients from 7 day LAGEOS/ETALON/LARES/LARES-2 combinations.

Orbit quality

The LARES orbit quality is assessed by the scatter of the post fit normal point residuals (Fig. 7). In case of a seven day LARES-only solution with 7d arcs a mean residual RMS of 11.6mm is achieved. In combination with LAGEOS and ETALON the RMS of the LARES residuals increases to 18.7mm, indicating tensions that are caused by the improper LARES orbit parameterization.

If the LARES orbit is modeled as long-arc with daily sets of empirical parameters, the residual RMS of the LARES-only orbits is reduced to more realistic 8.4mm, but in the combination the orbit fit again gets worse with a mean RMS of 12.4 mm. The tension in the combination can only be released by the co-estimation of gravity field coefficients. Co-estimating coefficients of degrees 2-4 results in a mean post-fit RMS of the LARES normal point residuals of 7.6 mm, if the full set of empirical parameters is estimated. When interested in realistic C_{20} and C_{30} gravity field estimates, the periodic 1/rev accelerations have to be fixed to 0 (Fig. 6), and the orbit fit is significantly degraded (Fig. 7).

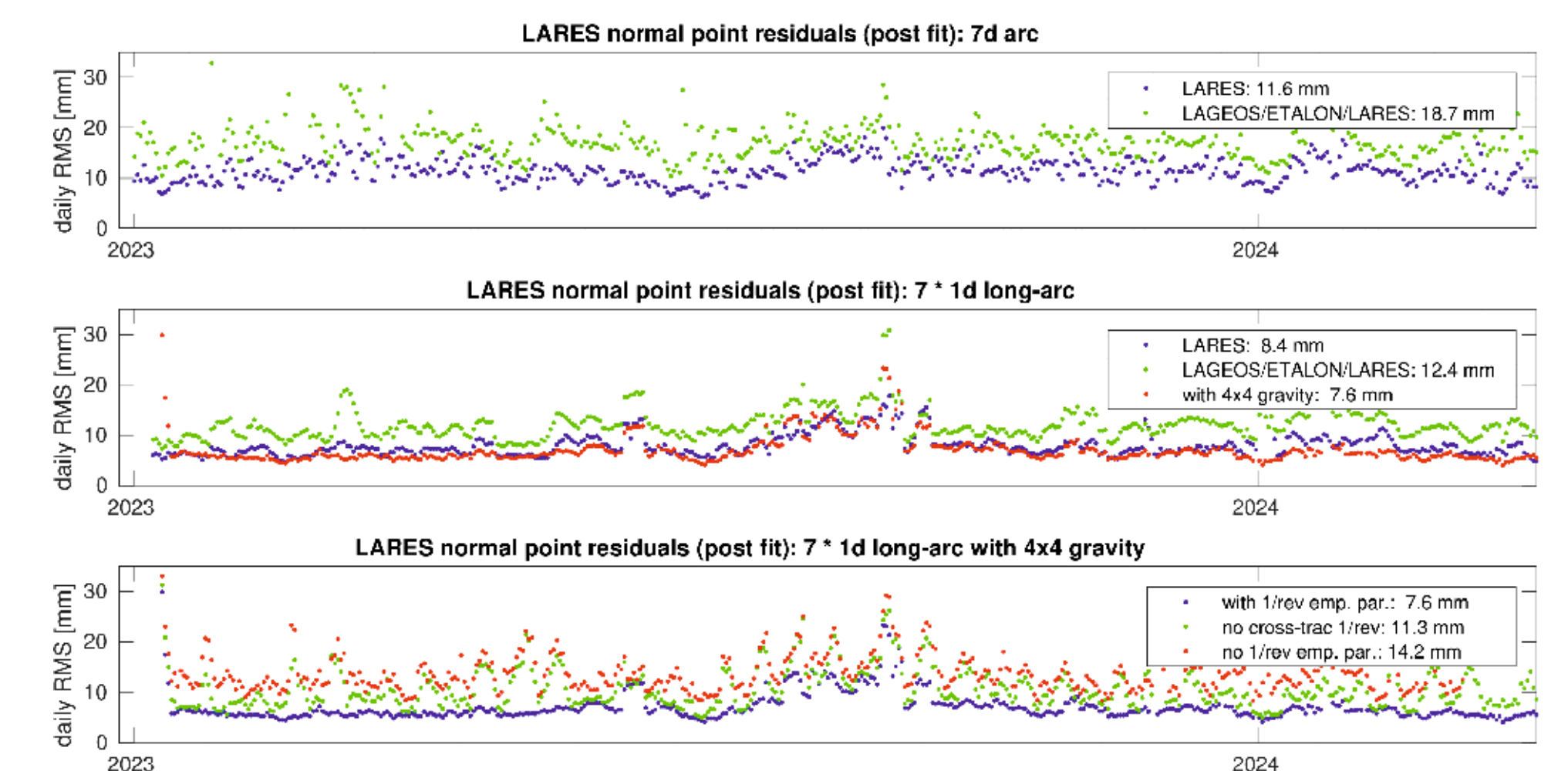


Fig. 7: Post-fit LARES normal point residuals of 7d arcs (top), long-arcs (middle), and when co-estimating gravity field coefficients (bottom).

Future plans

To achieve a proper orbit fit for Low Earth Orbiters (LEOs) and realistic gravity field solutions, a careful fine-tuning of constraints on the empirical periodic accelerations or alternative orbit parameterizations, e.g. using stochastic pulses, have to be considered. With the extension of the combination to further LEOs, e.g. AJISAI, Stella. Starlette and LARETS, satellite specific constraints will have to be determined due to the diverse orbit and satellite characteristics.

References:

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