Time-variable gravity field determination from GRACE Follow-On data using empirical noise modelling

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AIUB Astronomical Institute University of Bern

Satellite gravimetry with GRACE Follow-On

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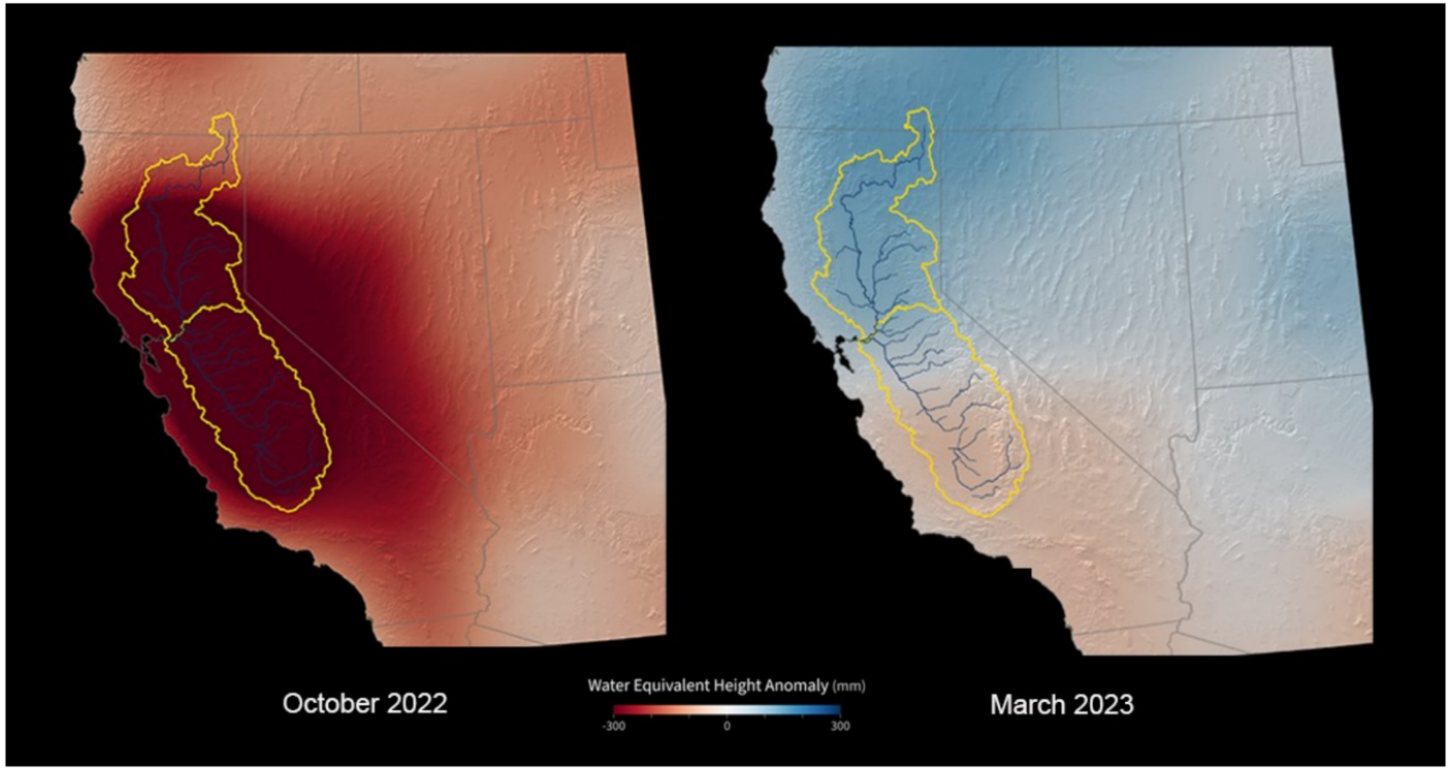

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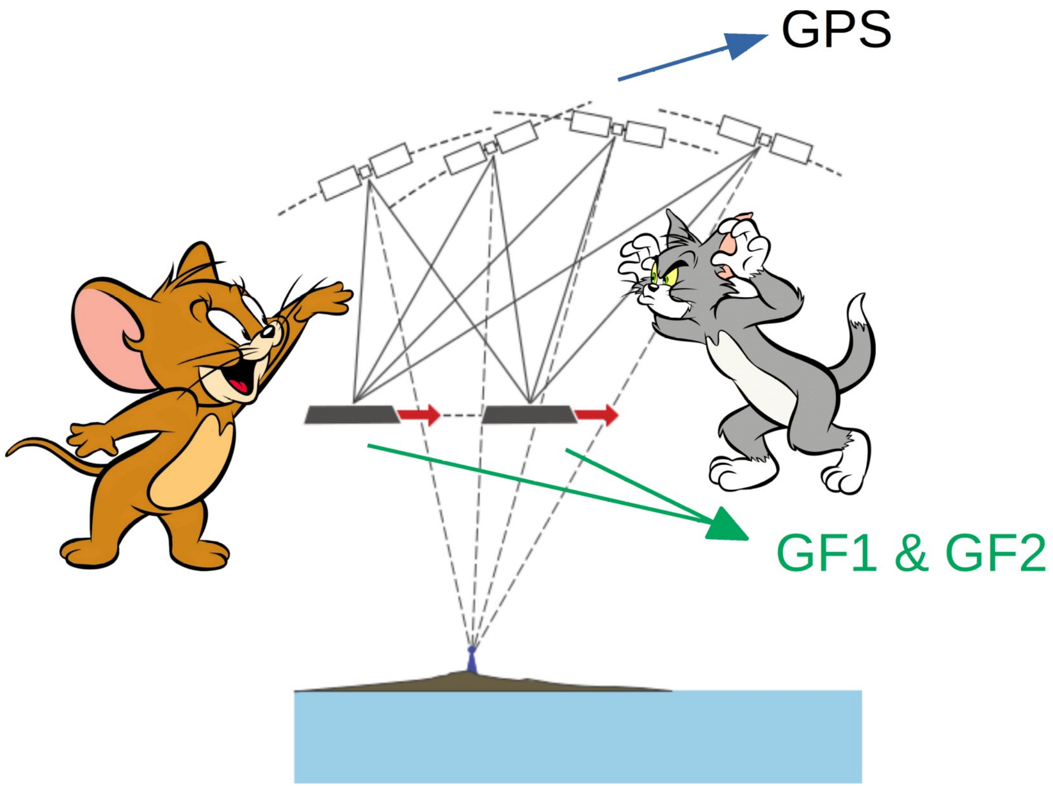
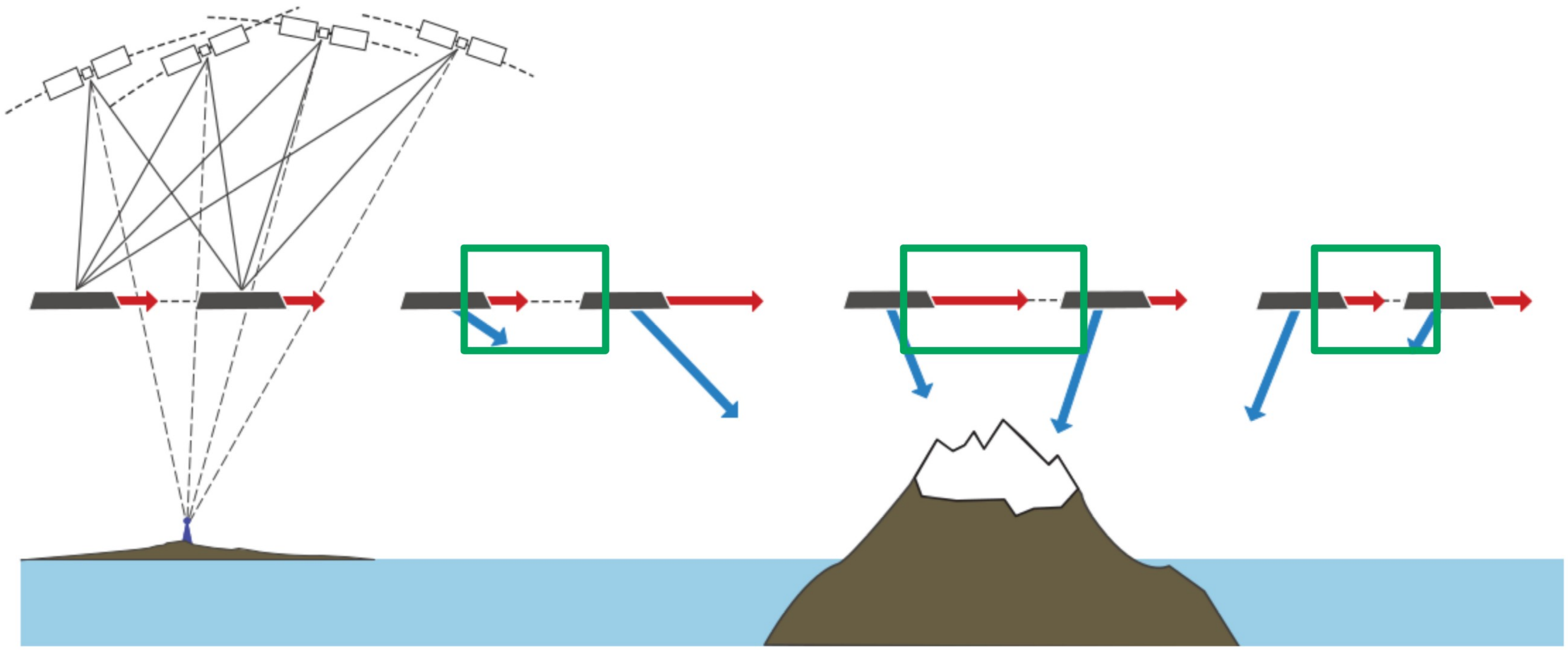
GRACE Follow-On

Observation concept

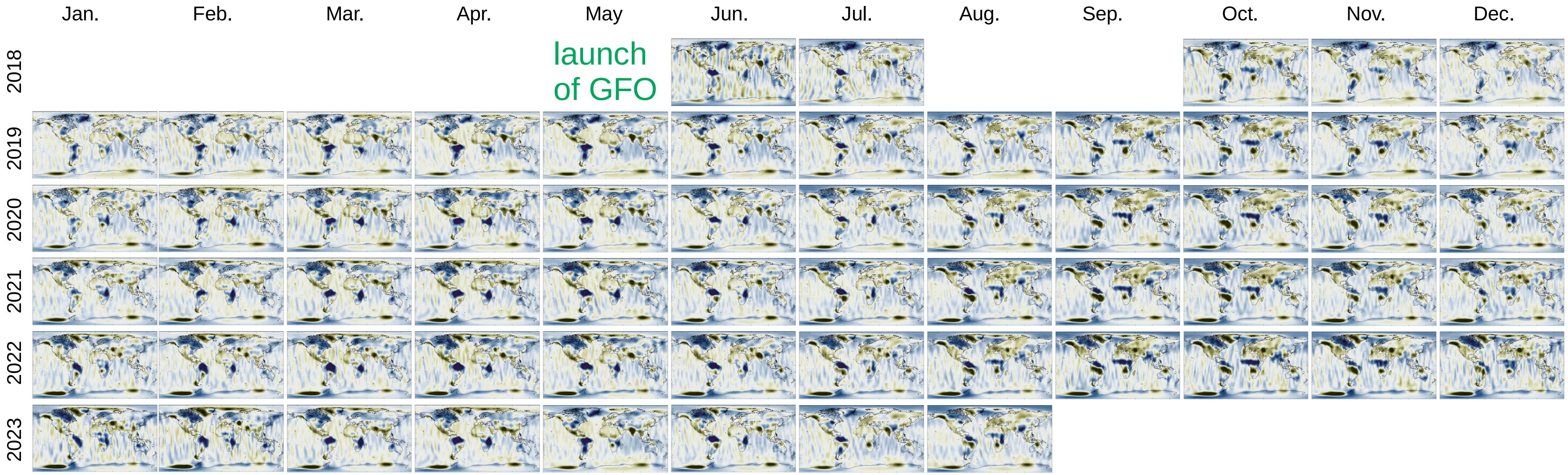


GRACE Follow-On
Observation concept



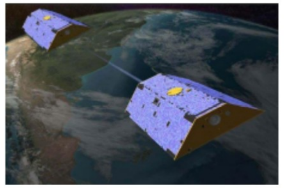
Operational GRACE Follow-On Solution

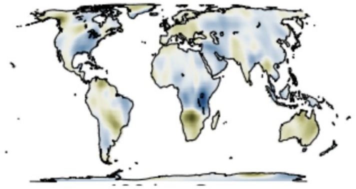
Mosaic Jun 2018 – Aug 2023



Operational GRACE Follow-On Solution

Monthly gravity fields – parametrisation





Force models	
Gravity field	AIUB-GRACE03S static
Astronomic bodies	JPL DE421 (all planets)
Mean pole	Linear
Solid Earth tides	IERS2010
Solid Earth pole tides	IERS2010
Ocean tides	FES2014b (+ admittances from TUG)
Ocean pole tides	Desai
Atmospheric tides	AOD RL06
Atmospheric & oeanic dealiasing	AOD RL06
Relativistic effects	IERS2010

Basic parametrisation

- initial conditions 2x[6]
- accelerometer bias 2x[3]
- accelerometer scaling 2x[3]

parameters per arc 24

Additional parameters

- 15 min PCA per satellite in
  - → radial 2x[96]
  - → along-track 2x[96]
  - → cross-track 2x[96]

parameters per arc 576

in daily arcs (30 days):
18000 ‹orbit› parameters
+ 9405 gravity field d/o=2..96

Processing
Least-squares adjustment

$$\boldsymbol{l} = f(\boldsymbol{x}, x_{AOD}, x_{Tides}, \ldots, x_{ACC})$$

observations introduced like a model

models

unknown parameters (gravity field, orbit, acc)

vector of observations

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A})$$

and

$$\boldsymbol{b} = \mathbf{A}^T \mathbf{P} \boldsymbol{l}$$

$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$$

vector of observations

design matrix

weight matrix

normal equations

Operational GRACE Follow-On Solution
Mosaic Jun 2018 – Aug 2023

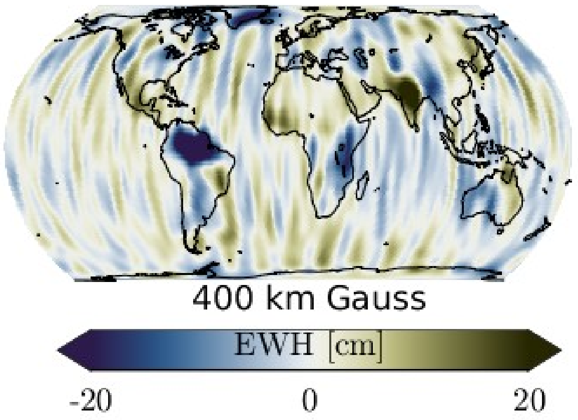
$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A})$$
$$\boldsymbol{b} = \mathbf{A}^T \mathbf{P} \boldsymbol{l}$$

$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$$

Jan. Feb. Mar. Apr. May Jun. Jul. Aug. Sep. Oct. Nov. Dec.

2018
2019
2020
2021
2022
2023

launch
of GFO



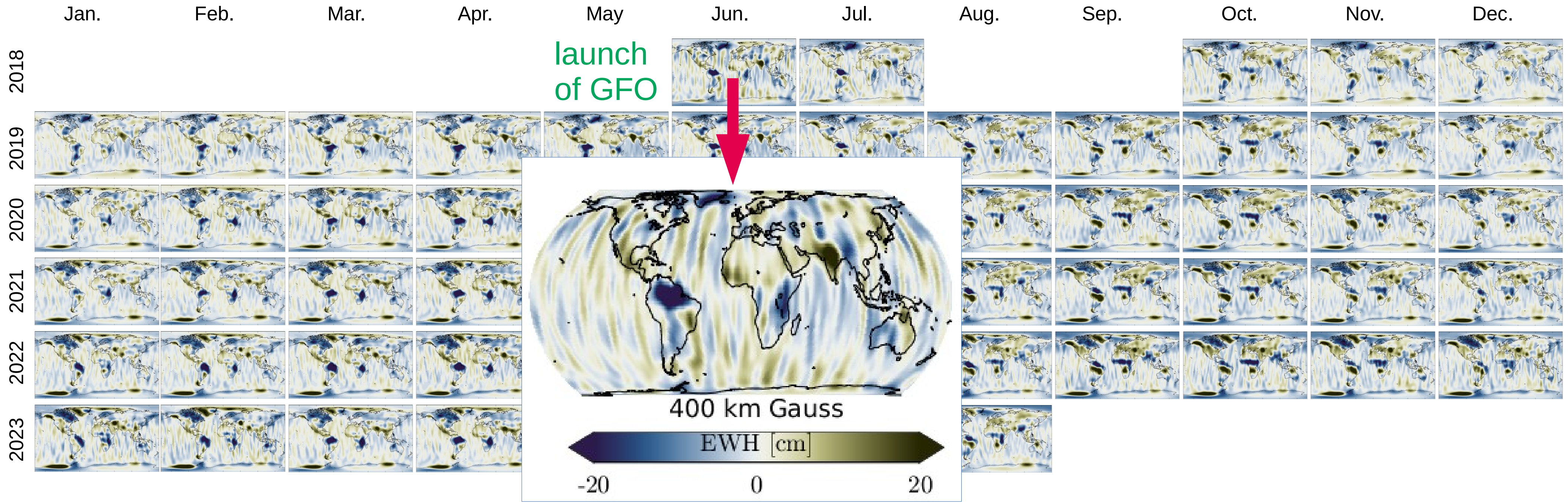
Operational GRACE Follow-On Solution

Mosaic Jun 2018 – Aug 2023

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P}\,\mathbf{A})$$

$$\boldsymbol{b} = \mathbf{A}^T \mathbf{P}\,\boldsymbol{l}$$

$$\Rightarrow \quad \hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$$



Least-squares adjustment

What do we estimate?

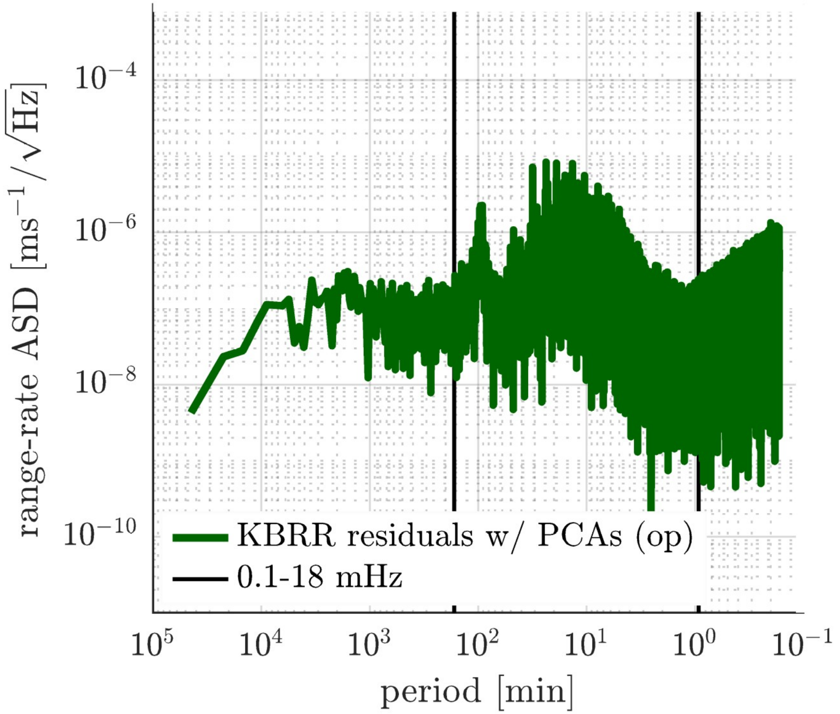
Least-squares

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A})$$

$$\boldsymbol{b} = \mathbf{A}^T \mathbf{P} \boldsymbol{l}$$

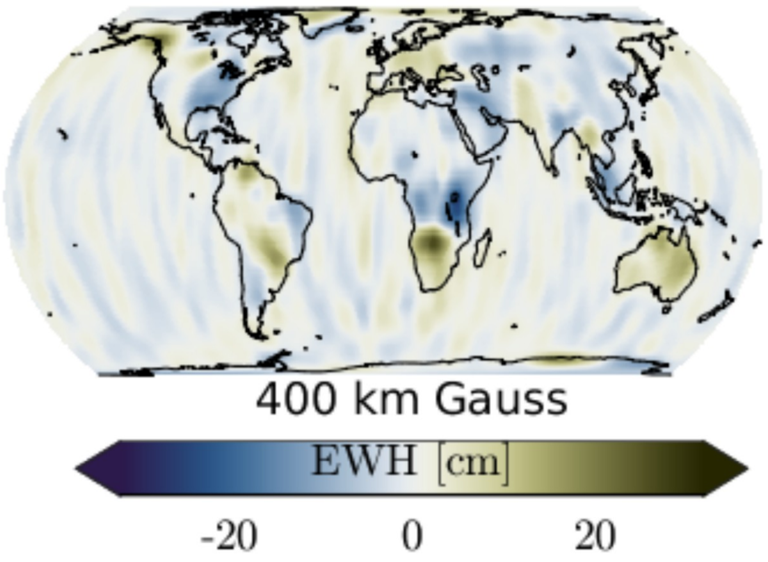
$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$$

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A} \hat{\boldsymbol{x}}$$



- information in the residuals (noise?)

$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$$



- information in the parameters (signal?)

Post-fit residuals

What do we expect?

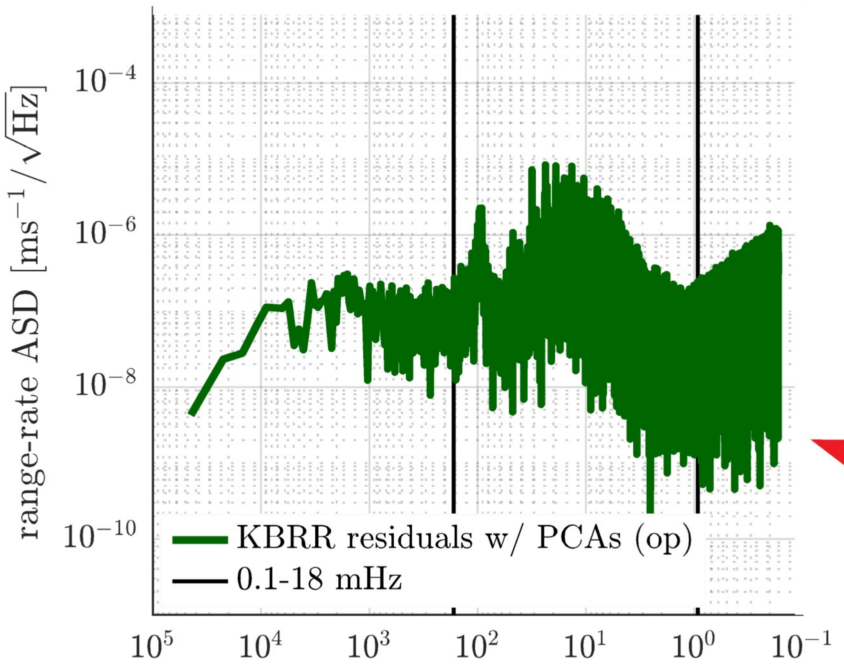
Least-squares

$\mathbf{N}=\left(\mathbf{A}^T\mathbf{P}\,\mathbf{A}\right)$

$\boldsymbol{b}=\mathbf{A}^T\mathbf{P}\,\boldsymbol{l}$

$\hat{\boldsymbol{x}}=\mathbf{N}^{-1}\boldsymbol{b}$

$$\hat{\boldsymbol{e}}=\boldsymbol{l}-\mathbf{A}\,\hat{\boldsymbol{x}}$$



$\hat{\boldsymbol{x}}=\mathbf{N}^{-1}\boldsymbol{b}$ unbiased

$\mathbf{C}_{\hat{x}\hat{x}}=\sigma_0^2\left(\mathbf{A}^T\mathbf{P}\,\mathbf{A}\right)^{-1}$ biased

The estimator is BLUE
(best – linear – unbiased) if

- $\mathrm{E}\left(\hat{\boldsymbol{x}}\right)=\boldsymbol{x}$
- $\mathrm{E}\left(\boldsymbol{e}|\boldsymbol{x}\right)=\mathrm{E}\left(\hat{\boldsymbol{e}}\right)=\mathbf{0}$
- $\mathrm{D}\left(\boldsymbol{e}|\boldsymbol{x},\sigma_0^2\right)=\mathrm{D}\left(\boldsymbol{l}|\sigma_0^2\right)=\sigma_0^2\,\mathbf{P}^{-1}$
  $=\sigma_0^2\,\mathbf{Q}_{ee}=\sigma_0^2\,\mathbf{Q}_{ll}$

$$\mathbf{P}=\frac{1}{\sigma_0^2}\mathbf{I}$$

Post-fit residuals

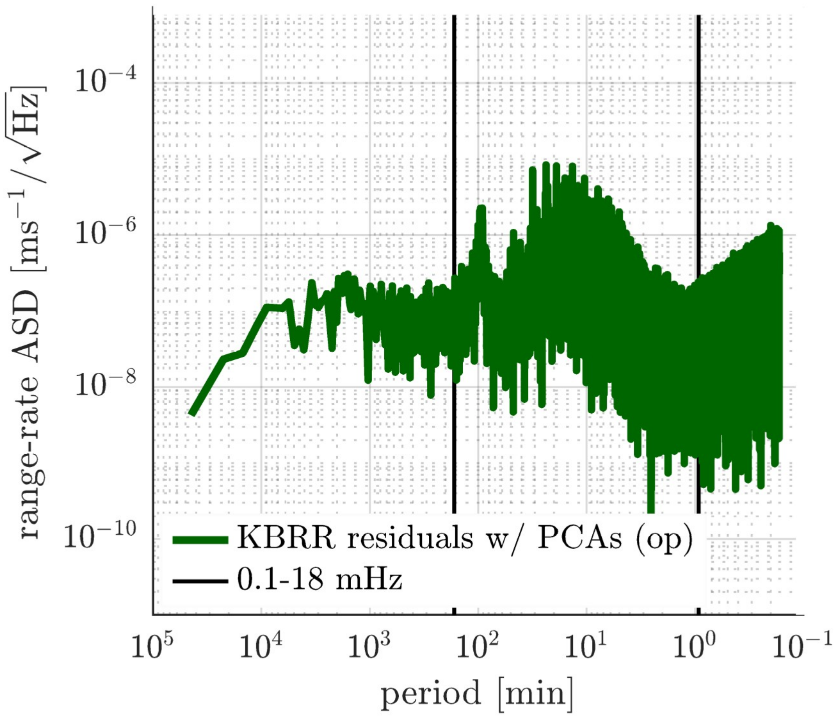
A measure of noise

Least-squares

$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A})$

$\boldsymbol{b} = \mathbf{A}^T \mathbf{P} \boldsymbol{l}$

$\hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$



$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\hat{\boldsymbol{x}}$ (post-fit residuals)

$$\mathrm{cov}(\Delta t_k) = \frac{1}{N} \sum_{i=0}^{N} \hat{e}(t_i)\hat{e}(t_i + \Delta t_k)$$

- stationarity assumed
- biased estimation of auto-covariance
  → covariance matrix nondegenerate

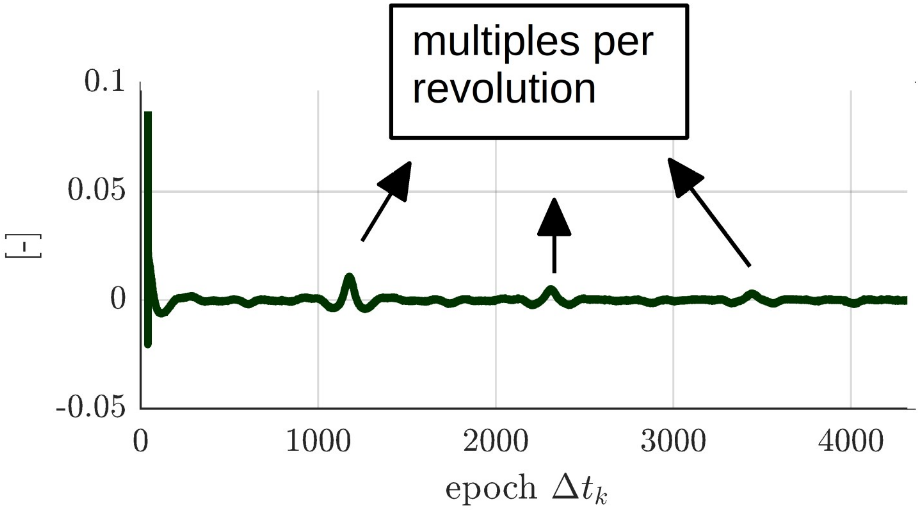
Post-fit residuals
Serial correlation

Least-squares

$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A})$

$\boldsymbol{b} = \mathbf{A}^T \mathbf{P} \boldsymbol{l}$

$\hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$

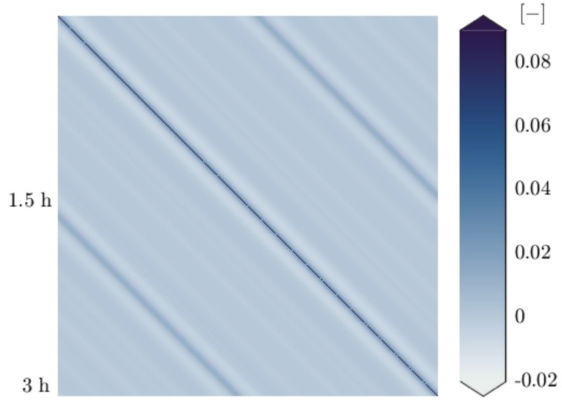


$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\hat{\boldsymbol{x}}$ (post-fit residuals)

$$\mathrm{cov}(\Delta t_k) = \frac{1}{N} \sum_{i=0}^{N} \hat{e}(t_i)\,\hat{e}(t_i + \Delta t_k)$$

- stationarity assumed
- biased estimation of auto-covariance
  → covariance matrix nondegenerate

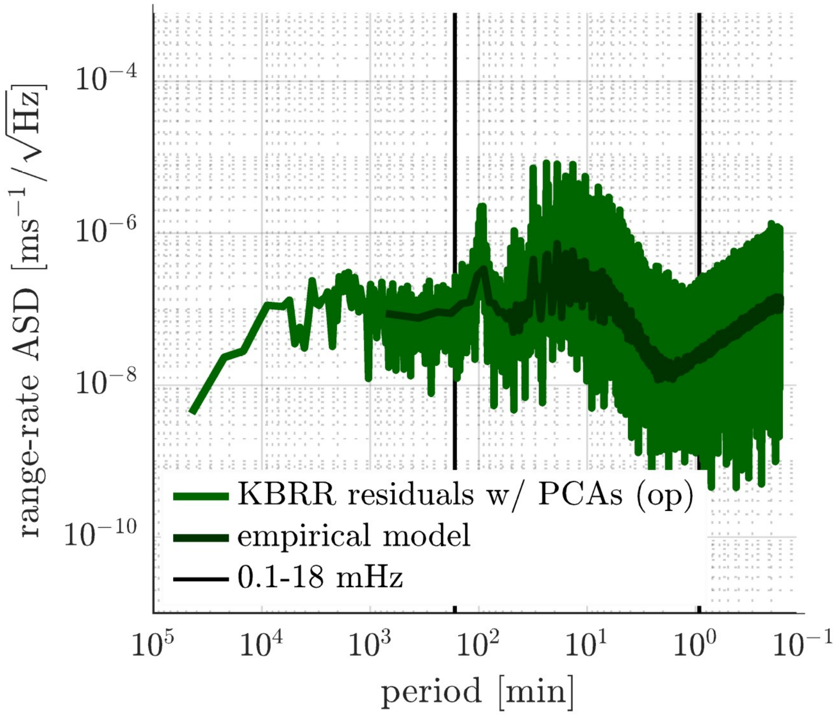
block Toeplitz matrix



→ **P**

Post-fit residuals

The estimated noise model

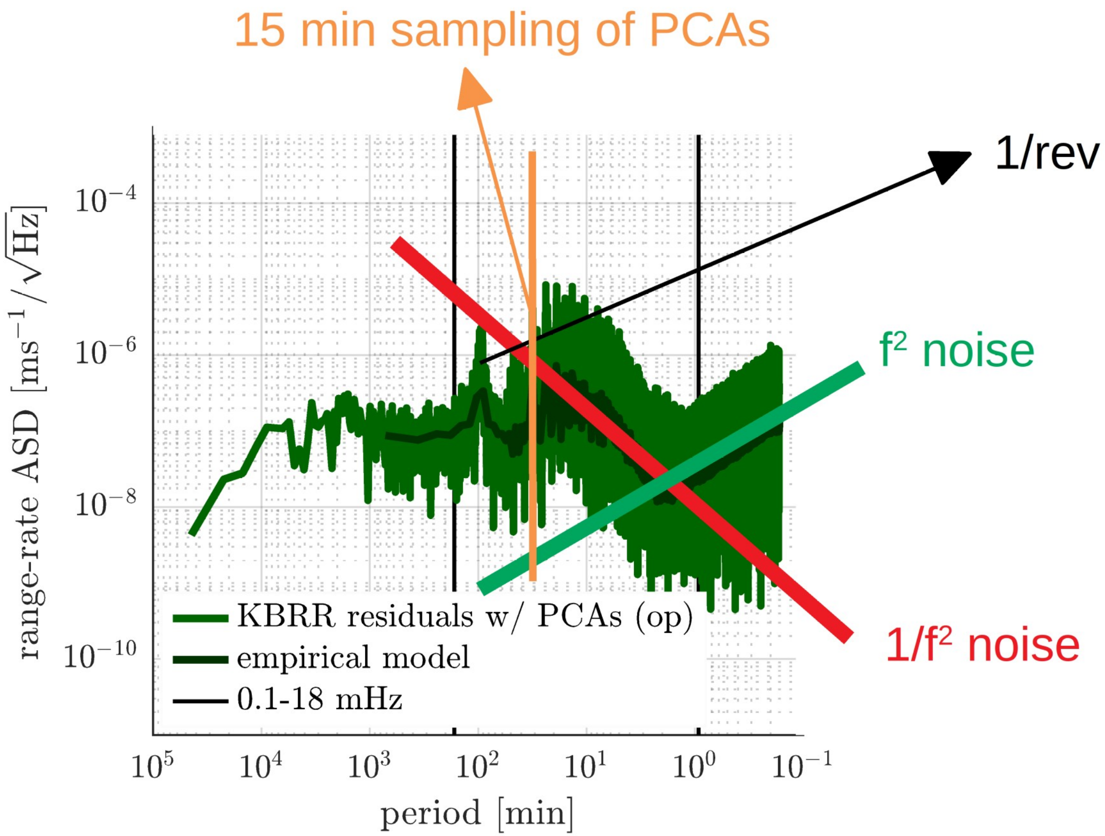


$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\hat{\boldsymbol{x}}$$

$$\text{cov}(\Delta t_k) = \frac{1}{N} \cdot \sum_{i=0}^{N} \hat{e}(t_i)\hat{e}(t_i + \Delta t_k)$$

Post-fit residuals

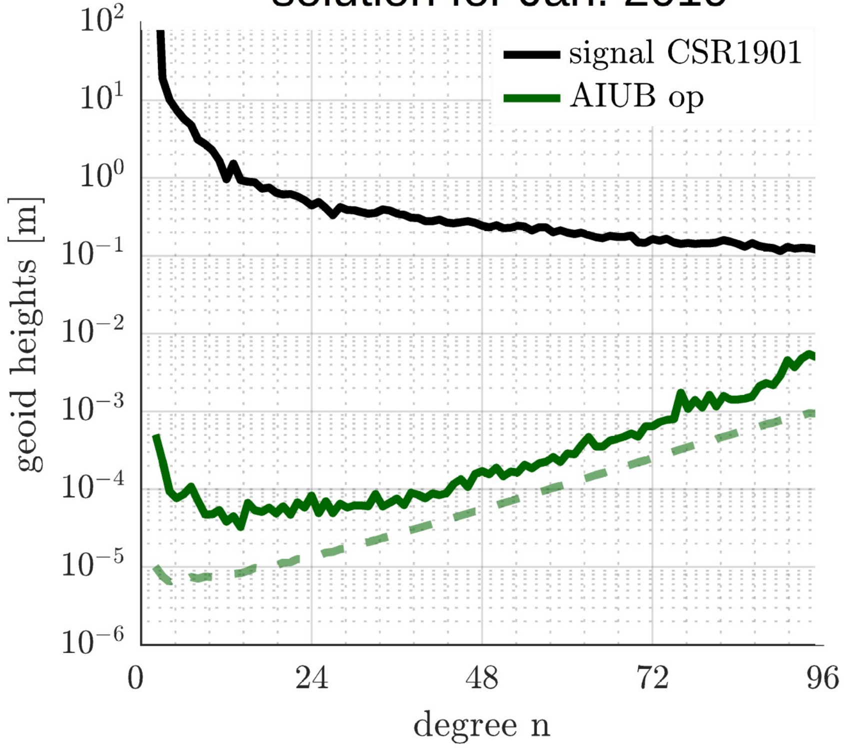
What we see



Results

Difference degree amplitudes

solution for Jan. 2019



- formal errors too optimistic compared to the assessed noise

Gravity field representation in spherical harmonics

unknown coefficients

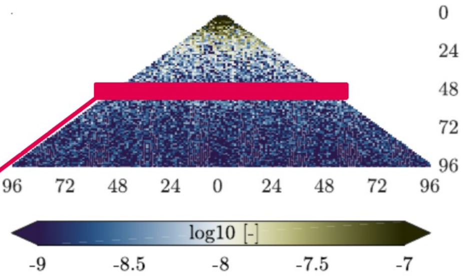
$$V=\frac{GM}{R}\sum_{n=0}^{N}\left(\frac{a_e}{r}\right)^{n}\sum_{m=0}^{n}P_{nm}(\sin(\varphi))[c_{nm}\cos(m\lambda)+s_{nm}\sin(m\lambda)]$$

degree N point of evaluation (r,φ,λ)

↓ degree

			c_{00}			
		s_{11}	c_{10}	c_{11}		
	s_{22}	s_{21}	c_{20}	c_{21}	c_{22}	
s_{33}	s_{32}	s_{31}	c_{30}	c_{31}	c_{32}	c_{33}
			⋮			

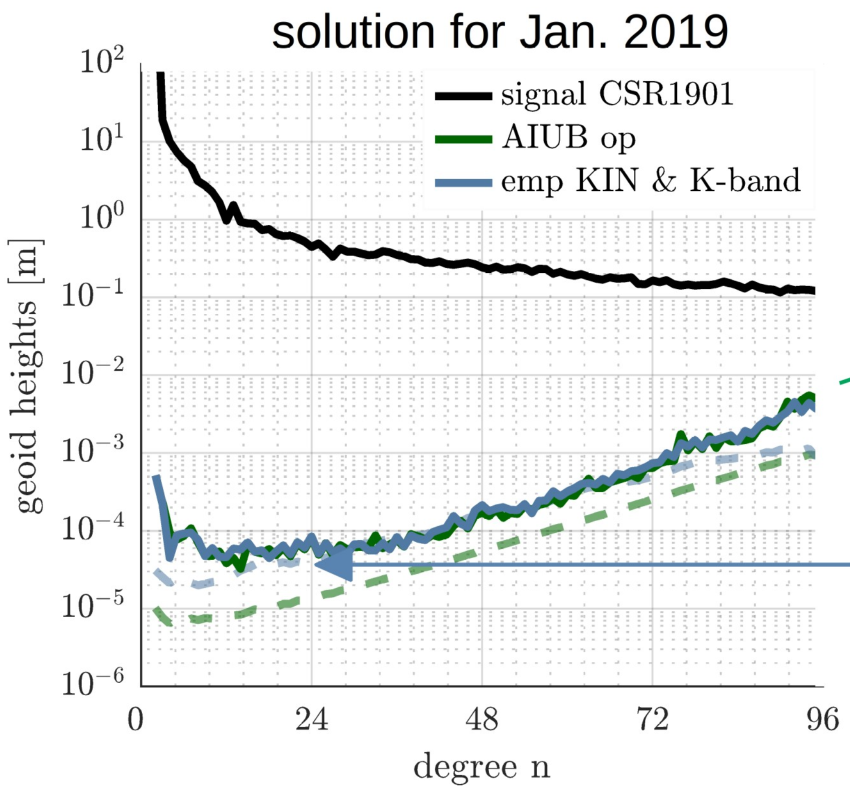
← order →



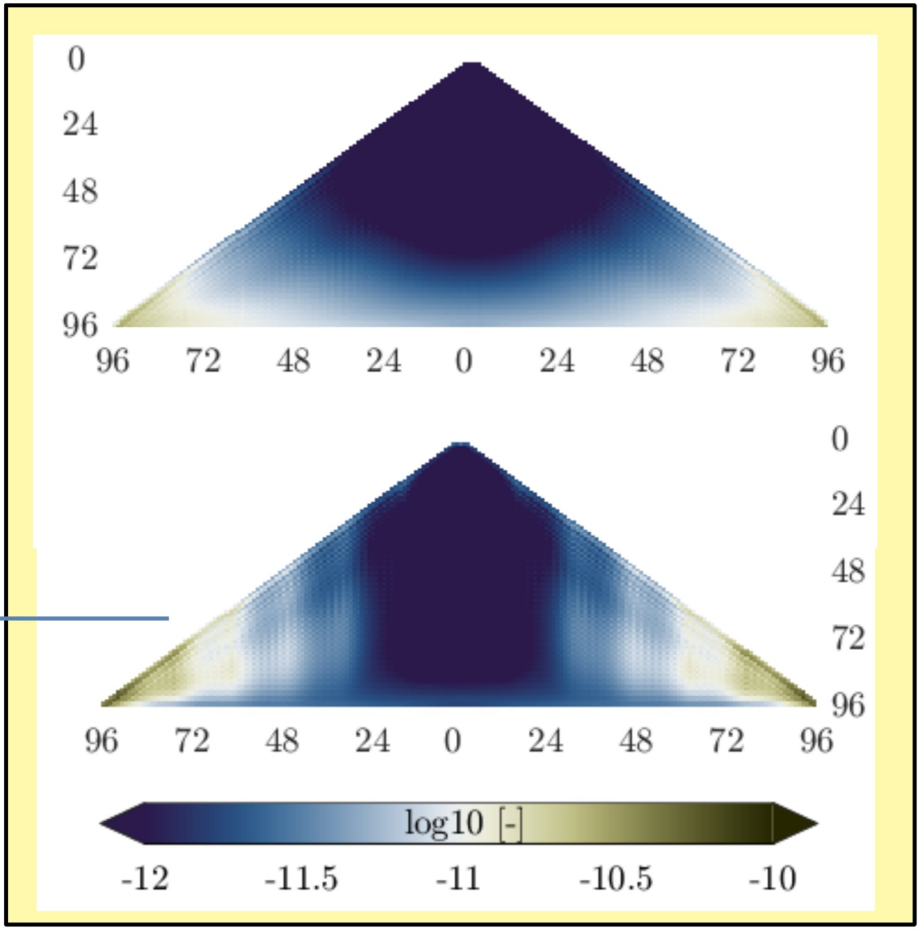
$$\sigma_n=\sqrt{\sum_{m=0}^{n}c_{nm}^2+s_{nm}^2}$$

Results

Spectral domain

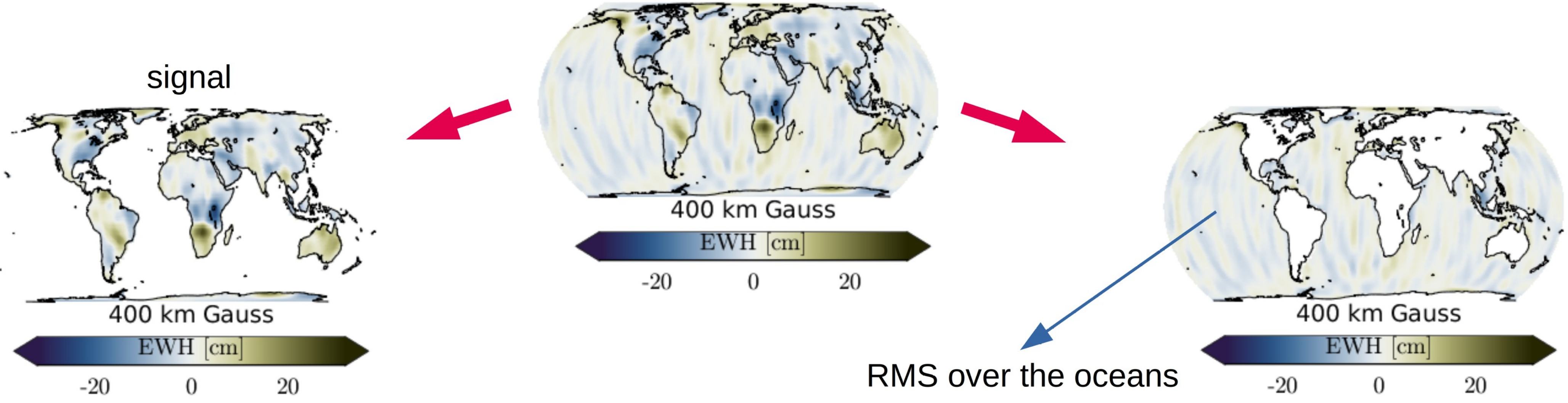


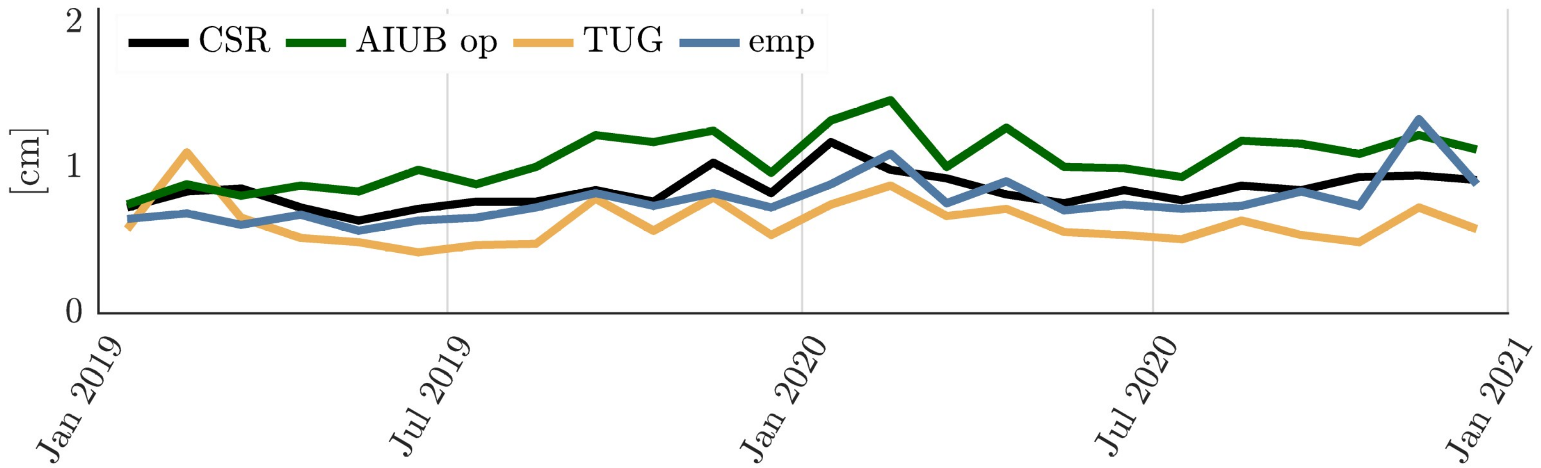
- formal errors reflect assessed noise very well
- including features of resonance orders



Noise evaluation

RMS over the oceans





Thank you for your attention

Contact

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