

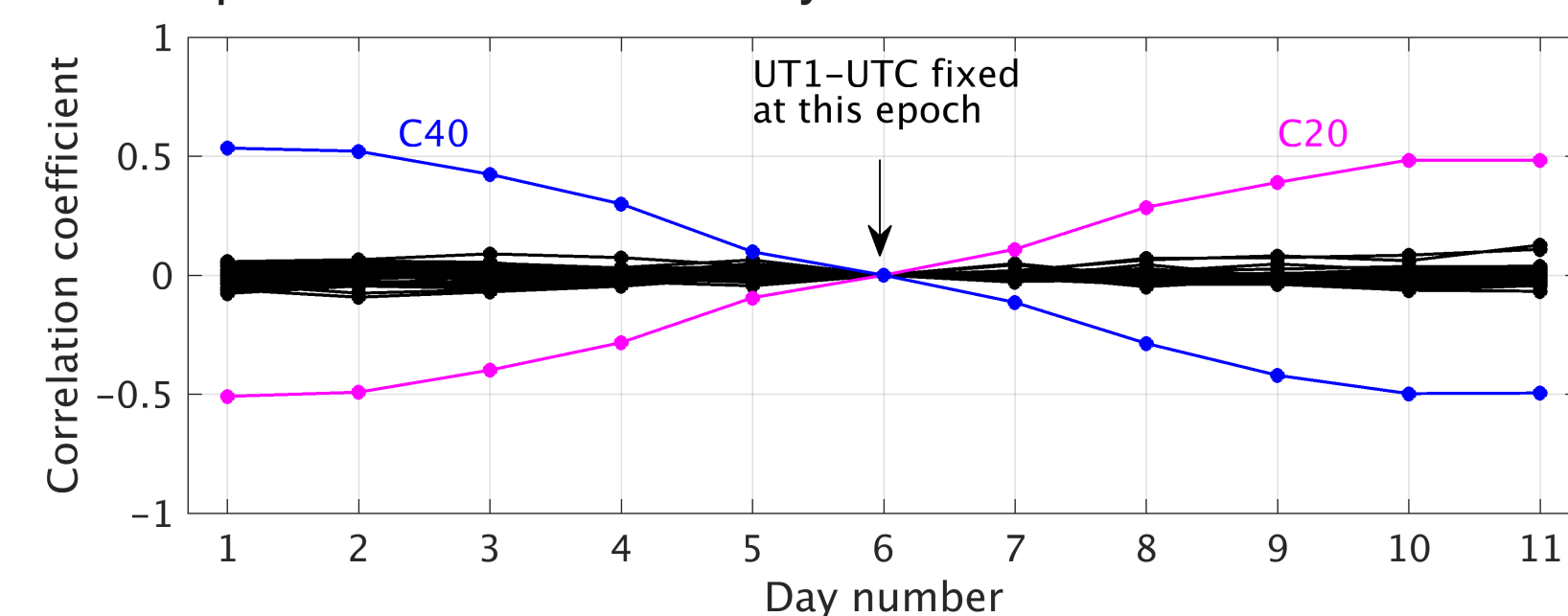
# Impact of estimating geodetic parameters on gravity field coefficients

## Introduction

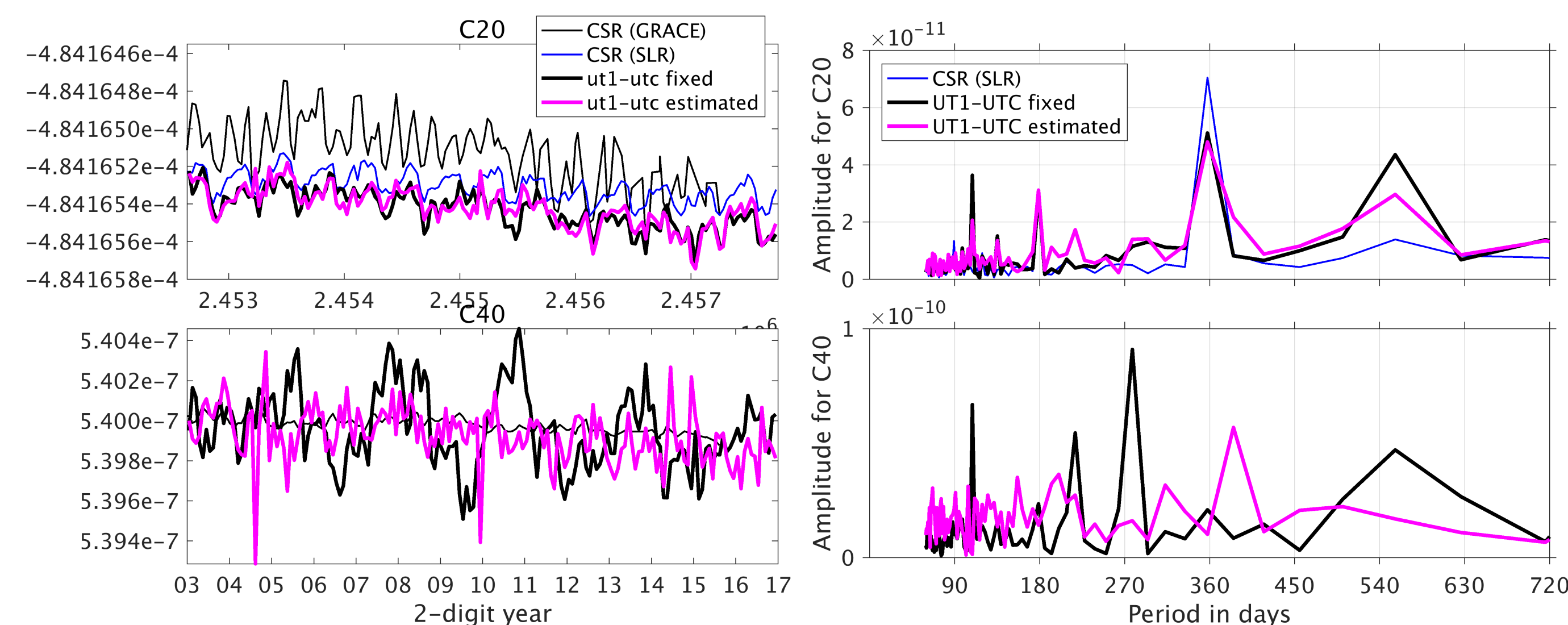
Satellite laser ranging (SLR) delivers precise and unambiguous 2-way optical range measurements to geodetic satellites. The International Laser Ranging Service (ILRS, Pearlman et al. 2002) coordinates all activities. SLR plays a crucial role in generating International Terrestrial Reference Frames (ITRFs). The origin of the ITRFs (the most recent one is ITRF2014) is exclusively defined by SLR observations to the two LAGEOS satellites (Altamimi et al., 2016). Moreover, SLR contributes to the definition of the scale of the ITRF together with Very Long Baseline Interferometry measurements. In this contribution we want to elaborate on the impact of simultaneously estimated geodetic parameters (Earth rotation parameters, station coordinates) on the recovered gravity field coefficients.

## Correlation of UT1-UTC with even-degree zonals

The correlation of UT1-UTC with the osculating ascending nodes at the initial epoch of the satellite orbits causes a rank deficiency that can be circumvented by fixing one UT1-UTC value to an a priori value. The estimated even-degree zonal spherical harmonic coefficients of the Earth's gravity field ( $C_{20}$  and  $C_{40}$ ) show a higher correlation with the estimated daily UT1-UTC values than all other coefficients that were set up (i.e., up to degree and order 5 without  $C_{50}$  plus  $C_{61}$  and  $S_{61}$ , see Fig. 1). When fixing UT1-UTC not only at one epoch but for all epochs to the IERS-08-C04 series,  $C_{40}$  shows very large variations compared to values derived from GRACE data (see Fig. 2, left side, bottom). The spectrum shows that estimating UT1-UTC values for all epochs except for one successfully reduces peaks in the spectrum: for  $C_{20}$  the peak at  $\sim 560$  days, and for  $C_{40}$  the peaks at  $\sim 220$  days and  $\sim 560$  days that correspond to the draconitic years of LAGEOS-2 and LAGEOS-1, respectively.



**Figure 1:** Example for the correlation within a 10-day arc between UT1-UTC values set up at midnight and the estimated gravity field coefficients. The middle epoch was fixed to the a priori value.



**Figure 2:** Left:  $C_{20}$  (top) and  $C_{40}$  coefficients (bottom) computed over 30 days from SLR data to seven geodetic satellites over a time span of 14 years. To compare with, the temporal variations computed by the Center for Space Research (CSR) based on GRACE data and on SLR data are shown as well. Right: the corresponding amplitude spectra.

Due to correlations between some gravity field coefficients and UT1-UTC it is advantageous for the quality of the gravity field coefficients to estimate UT1-UTC parameters as well!

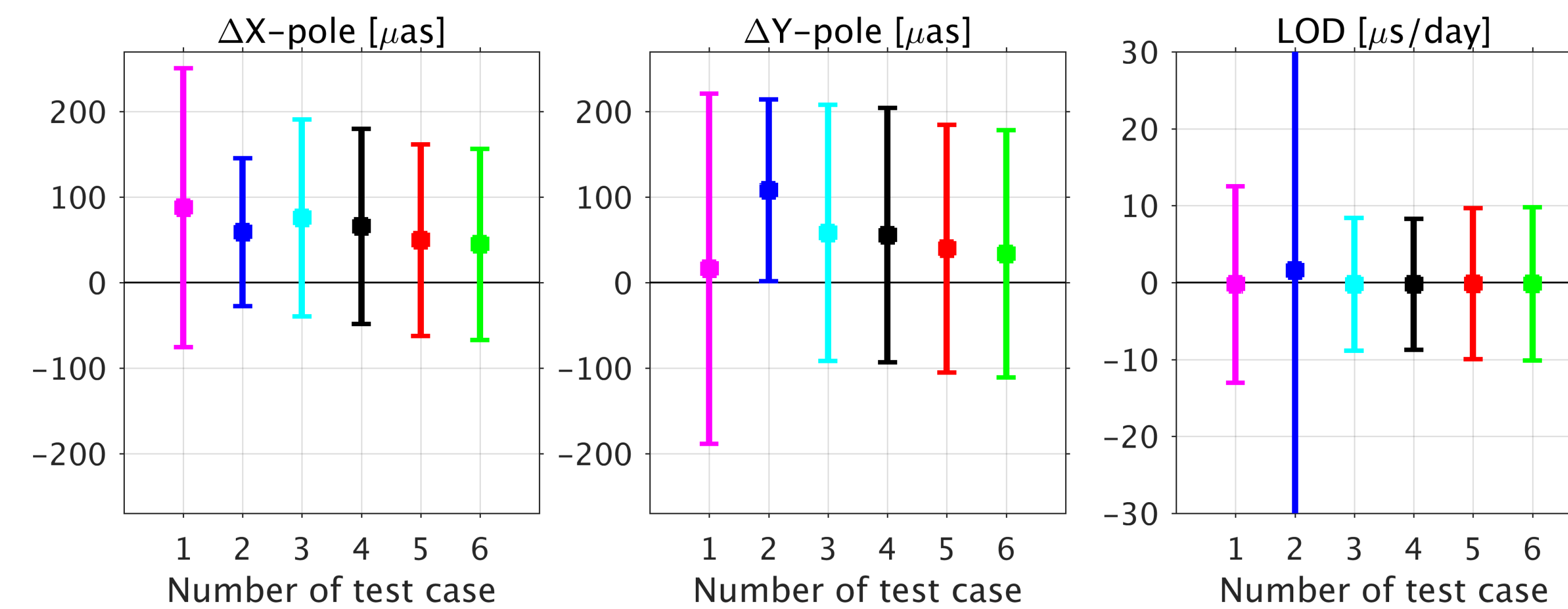
## Effect of estimating gravity field coefficients on geodetic parameters

We investigate the impact of estimating gravity field coefficients up to a certain degree and order (d/o) on the quality of simultaneously estimated geodetic parameters such as station coordinates, earth orientation parameters (EOPs), and geocenter motion (i.e., the variation of the Earth's center of mass w.r.t. the origin of the ITRF). Over 2010 and 2011 laser ranges to LAGEOS-1 and -2, Ajisai, Stella, and Starlette were analyzed. The following six test cases were set up: estimating

1. no gravity field coefficients and no 1/rev cross track acceleration,
2. no gravity field coefficients but a 1/rev cross track acceleration,
3. gravity field coefficients up to d/o 2 but no 1/rev cross track acceleration,
4. gravity field coefficients up to d/o 3 but no 1/rev cross track acceleration,
5. gravity field coefficients up to d/o 4 but no 1/rev cross track acceleration,
6. gravity field coefficients up to d/o 5 but no 1/rev cross track acceleration.

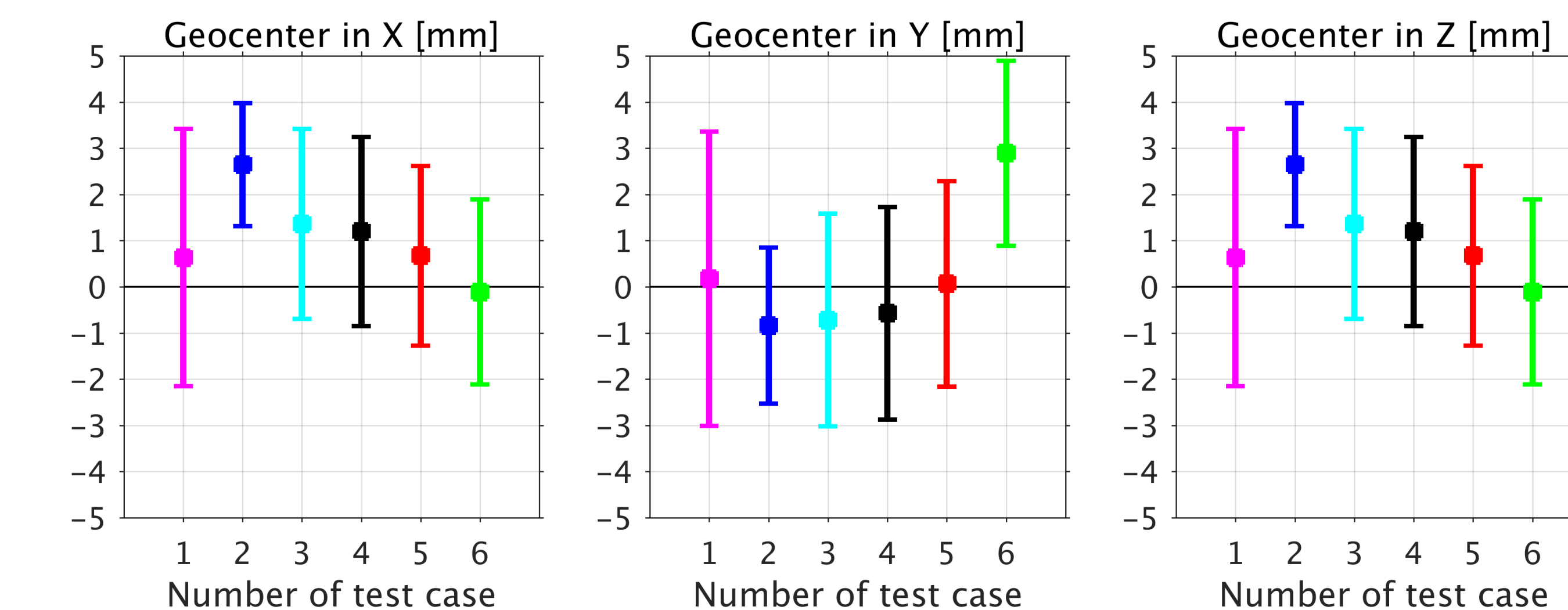
When fixing the gravity field to the a priori model, once-per-revolution (1/rev) empirical orbit parameters are typically set up in the cross track direction to absorb large variations of  $C_{20}$  (case 2). In case these terms are omitted (case 1), the results depend mainly on the quality of the a priori  $C_{20}$  value. EGM2008 (Pavlis et al., 2008) was used as a priori model.

Fig. 3 shows the mean values and the mean formal errors of the estimated EOPs w.r.t. the IERS-08-C04 series. The largest formal errors regarding the pole coordinates we see for case 1. They get significantly smaller when adding 1/rev terms in the cross track component (case 2). In that case, however, the formal error of the Length-of-Day (LOD) parameter grows considerably. Estimating a larger number of gravity field coefficients has a positive influence on the mean value of the pole coordinates. The largest improvement when setting up gravity field coefficients can be observed for the LOD values (cf. Fig.3, right). This result agrees well with the findings of Sošnica (2015).



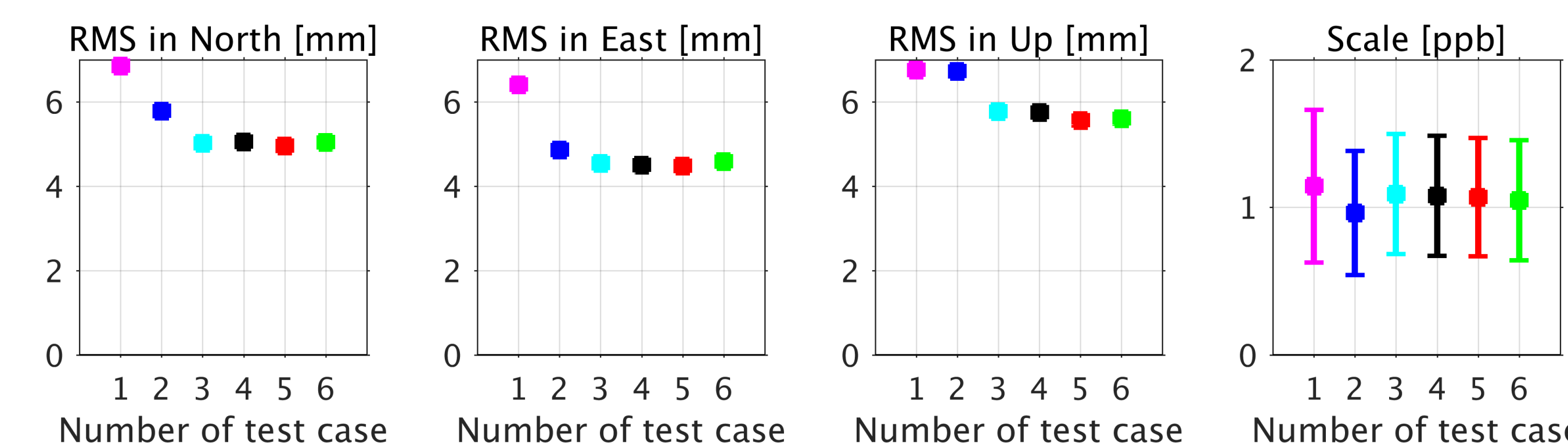
**Figure 3:** Mean values and mean formal errors of the estimated EOPs w.r.t. the IERS-08-C04 series for the six test cases described above. For reason of visibility the formal error of LOD for case 2 is not fully shown.

The effect on the geocenter motion is depicted in Fig. 4. Apart from the y-component, the mean value is closest to zero for case 6, i.e. when estimating coefficients up to d/o 5. Estimating no gravity field at all has a negative impact. Either the RMS is significantly larger compared to all other solutions (case 1) or the RMS is small but the mean value deviates considerably from zero (case 2).



**Figure 4:** Mean and RMS values of the estimated geocenter motion for the six test cases described above.

Fig. 5 summarizes the 10-day coordinate comparison with SLRF2008 using a Helmert transformation (3 translations, 3 rotations, 1 scale). When gravity field coefficients are estimated, the agreement is at the level of 4-5 mm for the horizontal components and 5-6 mm for the vertical component. Fixing the gravity field leads to increased RMS values in all three components (6-7 mm). The scale w.r.t. SLRF2008 amounts for all cases to  $\sim 1$  ppb. This may be explained by the fact that the latest data included in SLRF2008 is from 2008. A degradation after 2008 may therefore be expected.

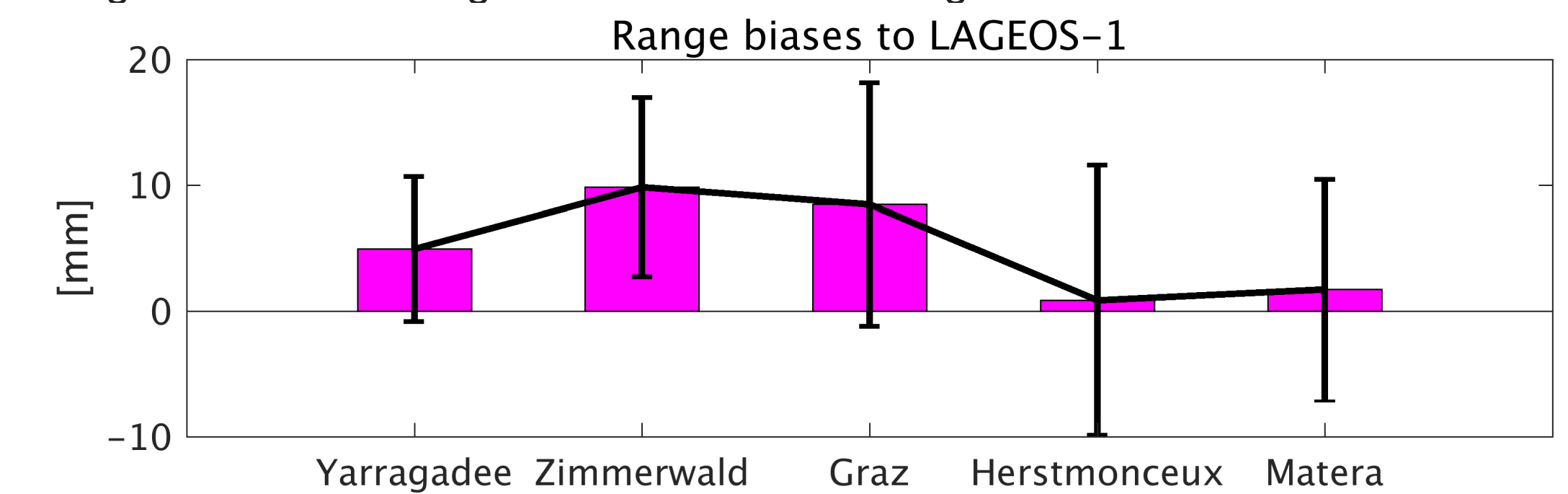


**Figure 5:** Helmert transformation between the different solutions and SLRF2008. The RMS values of the coordinate differences after the Helmert transformation is given averaged over all 10-day arcs in North, East, and Up direction as well as the scale.

From all geodetic parameters investigated in this section, the LOD parameter benefits most from simultaneously estimating gravity field coefficients!

## Setting up range biases from all stations to LAGEOS

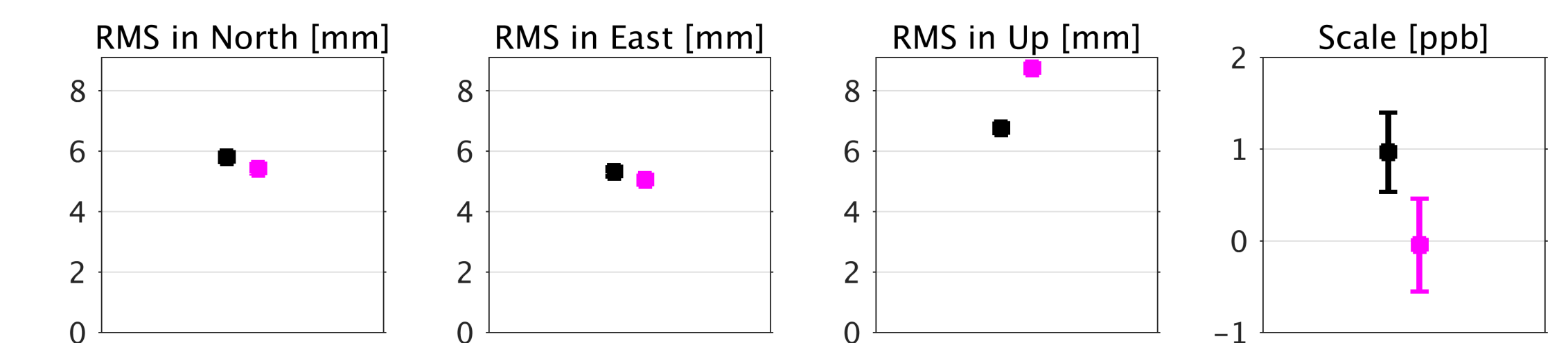
Station-dependent range biases are a frequently discussed topic in the laser ranging community. Appleby et al. (2016) conducted a number of experiments focusing on range biases. They are difficult to determine with high precision and are not always reported by the stations. Range biases are highly correlated with the station height over short arcs which is why estimating range biases for all stations should be avoided. To investigate the impact of setting up additional range biases on other geodetic parameters in more detail, we conducted an experiment where range biases to LAGEOS are set up not only for a few specific stations as suggested by the ILRS, but for all stations. The two sets of LAGEOS normal equations were then combined with normal equations based on Ajisai, Stella, and Starlette observations (where range biases are set up for all stations) to get combined 10-day solutions. Fig. 6 shows the magnitude of estimated range biases to LAGEOS-1 for some stations.



**Figure 6:** Estimated range biases to LAGEOS-1 for five highly productive laser stations.

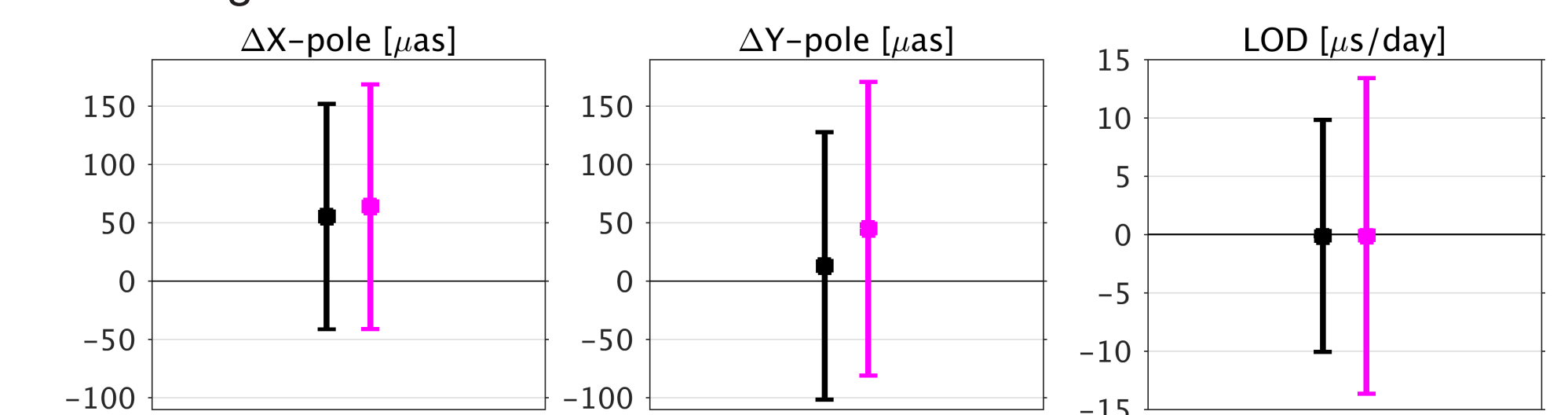
Concerning the estimated gravity field coefficients, no significant difference could be observed when setting up the full set of range biases to LAGEOS or not. The formal errors of the coefficients are not affected either. In contrast to the gravity field, the formal errors of the geocenter coordinates are about twice as large for all three components when setting up range biases for all stations.

The agreement of the two solutions concerning station coordinates is good in the horizontal components (cf. Fig. 7). The RMS of the height component, however, is two millimeters larger when all range biases are estimated. This is due to the correlation between range biases and station heights as mentioned above. As in the experiments by Appleby et al. (2016), the scale changes significantly when setting up range biases to all satellites.



**Figure 7:** Helmert transformation between the two solutions and SLRF2008 (black: setting up biases to LAGEOS as recommended by the ILRS, magenta: setting up biases to LAGEOS for all stations). The RMS values of the coordinate differences after the Helmert transformation is given averaged over all 10-day arcs in North, East, and Up direction as well as the scale.

The formal errors of the estimated EOPs increase when setting up additional range biases to LAGEOS (cf. Fig. 8). Moreover, the offset of the y-pole coordinate deviates significantly from zero compared to estimating a reduced set of biases.



**Figure 8:** Mean values and mean formal errors of the estimated EOPs w.r.t. the IERS-08-C04 series (black: setting up biases to LAGEOS as recommended by the ILRS, magenta: setting up biases to LAGEOS for all stations).

Setting up range biases to LAGEOS for all stations changes the scale significantly from  $\sim 1$  ppb to  $\sim 0$  ppb. It has also a negative effect on the geocenter and on the EOPs.

## References

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