

## Impact of Earth Radiation Pressure on LAGEOS Orbits and on the Global Scale

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**Abstract.** *The indirect solar radiation pressure caused by reflected or re-emitted radiation by the Earth's surface is an important non-gravitational force perturbing the orbits of geodetic satellites (Rubincam and Weiss, 1986; Martin and Rubincam, 1996). In the case of LAGEOS this acceleration is of the order of 15% of the direct solar radiation pressure. Therefore, Earth radiation pressure has a non-negligible impact not only on LAGEOS orbits, but also on the SLR-derived terrestrial reference frame. We investigate the impact of the Earth radiation pressure on LAGEOS orbits and on the SLR-derived parameters.*

*Earth radiation pressure has a remarkable impact on the semi-major axes of the LAGEOS satellites, causing a systematic reduction of 1.5 mm. The infrared Earth radiation causes a reduction of about 1.0 mm and the Earth's reflectivity of 0.5 mm of the LAGEOS' semi-major axes. The global scale defined by the SLR network is changed by 0.07 ppb, when applying Earth radiation pressure. The resulting station heights differ by 0.5-0.6 mm in the solution with and without Earth radiation pressure. However, when range biases are estimated, the height differences are absorbed by the range biases, and thus, the station heights are not shifted.*

### Introduction

We consider independently two types of accelerations (see Figure 1):

- Reflected visible radiation (reflectivity/albedo),
- Emitted infrared radiation (emissivity).

We make use of the monthly global maps of Earth reflectivity and emissivity from the CERES project (Clouds and the Earth's Radiant Energy System, Wielicki et al. 1996). The largest reflectivity is found in the Polar Regions; whereas the largest emissivity can be found in the tropic areas (see Figure 2). More than 60% of the total Earth radiation pressure forces is due to infrared emissivity.

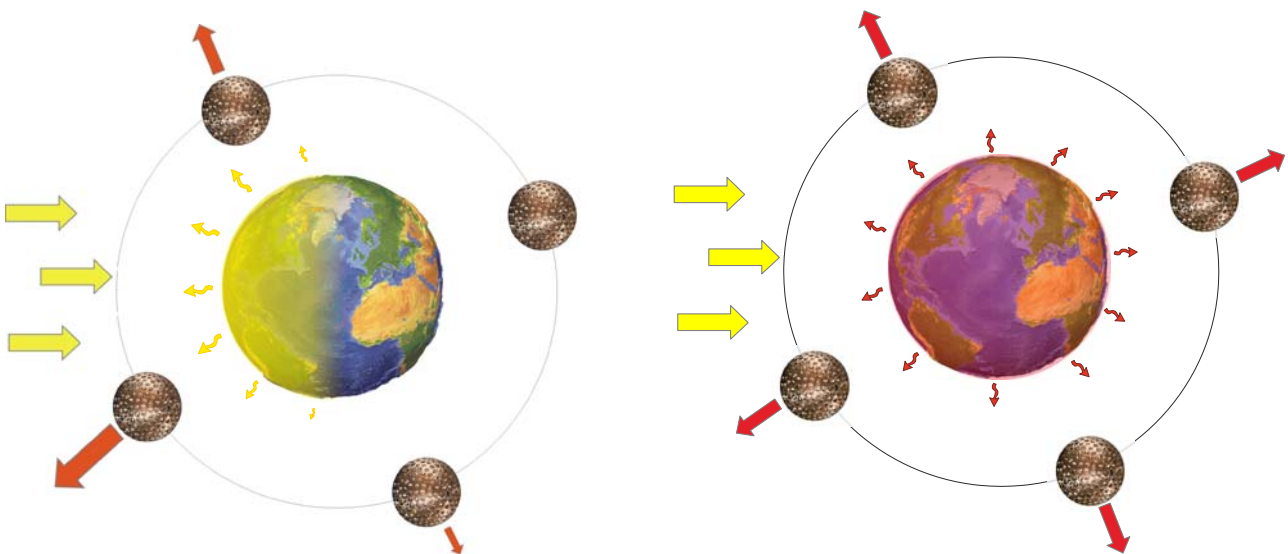
A mathematical model of Earth radiation pressure (Knocke et al., 1988; Rodríguez-Solano, 2009) considers the infrared emissivity and the reflectivity as purely diffusive, like a Lambertian sphere. The model assumes that the specular reflections have nearly a negligible impact on satellite orbits. Three assumptions for Earth radiation pressure are considered (following Rodríguez-Solano, 2009):

- the Earth has the same reflective properties as a Lambertian sphere (the specularity from the oceans is neglected),
- the radiation is reflected or emitted by the Earth's surface or the surface of the highest clouds,
- all energy received by the Earth from the Sun has to leave it (i.e., there is a global conservation of energy).

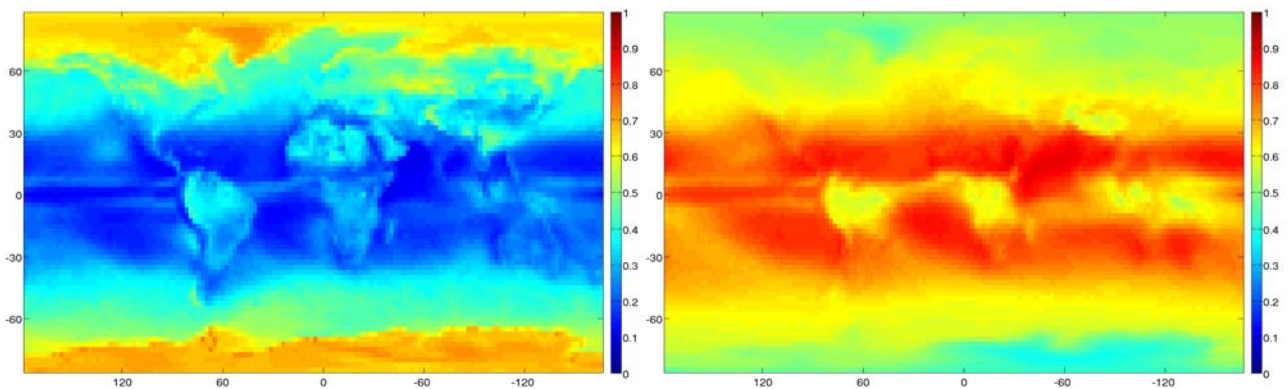
## A priori accelerations

Figure 3 shows the accelerations acting on LAGEOS-2 due to the reflectivity (left) and reflectivity and emissivity (right) in the radial direction ( $R$ ), whereas Figure 4 shows the decomposition of the reflectivity and emissivity accelerations in the along-track ( $S$ , left) and out-of-plane ( $W$ , right) directions. Maximum acceleration, amounting  $4.4 \times 10^{-10} \text{ m/s}^2$ , is in the radial direction. Albedo reflectivity depends on the relative position of the Sun, whereas the emissivity imposes a rather constant acceleration regardless of the relative Sun-Earth-satellite configuration. The Earth radiation pressure in  $S$  and in  $W$  is a factor of fourteen smaller than in  $R$ .

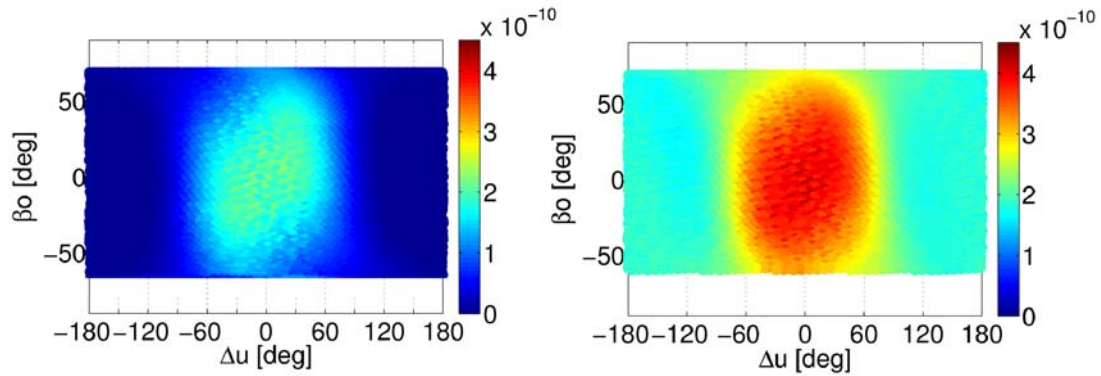
The constant radial force is acting in the opposite direction to the gravitational attraction of the Earth. Hence, this radial acceleration has an impact on the dynamical global scale, defined as  $GM$  (the product of the gravitational constant  $G$  and the mass  $M$  of the Earth).  $GM$  is of crucial interest in SLR data analyses, because the currently accepted conventional value of  $GM$  was derived using SLR tracking of LAGEOS.



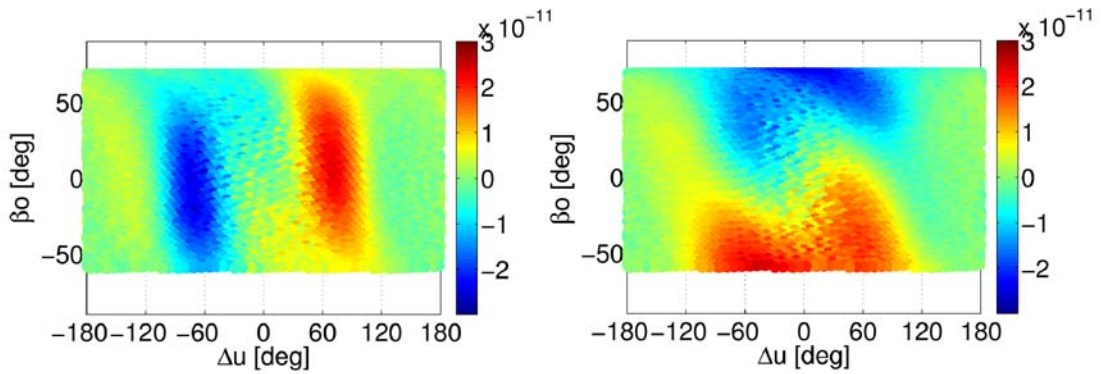
**Figure 1. Left:** General concept of the Earth's reflectivity (albedo).  
**Right:** General concept of the Earth's emissivity.



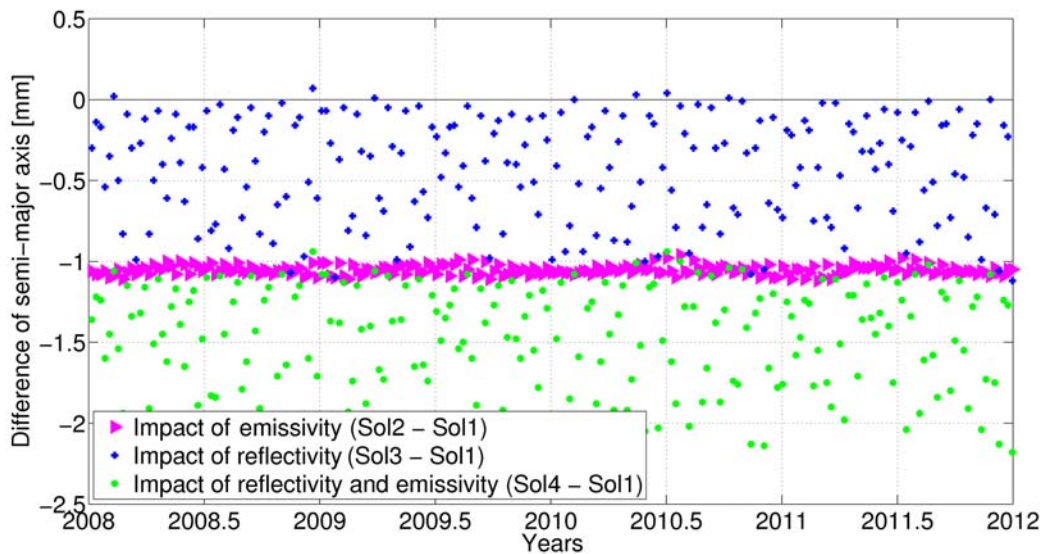
**Figure 2. Left:** Map of mean Earth's reflectivity in April from CERES,  $2.5^\circ \times 2.5^\circ$  grid.  
**Right:** Map of mean Earth's emissivity in April from CERES,  $2.5^\circ \times 2.5^\circ$  grid.



**Figure 3.** LAGEOS-2 acceleration due to reflectivity (**left**) and reflectivity + emissivity (**right**) for the radial direction where  $\beta_0$  is the elevation angle of the Sun over orbital plane and  $\Delta u$  is argument of latitude w.r.t. the Sun (see Fig. 2 from Sośnica et al., 2014). Units:  $\text{m/s}^2$ .



**Figure 4.** LAGEOS-2 acceleration due to reflectivity and emissivity in the along-track (**left**) and out-of-plane (**right**) directions. Units:  $\text{m/s}^2$ .



**Figure 5.** Differences of LAGEOS-2 semi-major axis between solution 1 (without Earth radiation pressure) and solutions 2-4.

## Description of the solutions

Four years of SLR data are processed (2008.0-2012.0) using a development SLR version of the Bernese GNSS Software (Thaller et al., 2012) to evaluate the impact of Earth radiation pressure on LAGEOS solutions. The orbit modeling of LAGEOS satellites has been described in Sošnica et al., (2012), whereas the datum definition and parameterization of the solution has been described in Thaller et al., (2013), Sošnica et al., (2013), and Sošnica (2014). The identical set of screened observations was used in four solutions (4.3% of normal points were removed in total).

Four independent LAGEOS solutions are processed (see Table 1): without considering Earth radiation pressure (solution 1), with infrared emissivity (solution 2), with reflectivity (solution 3), and with both effects (solution 4). All other processing options are identical to guarantee the comparability of the four solutions.

**Table 1.** List of Solutions processed for validating the Earth radiation pressure.

	<b>Reflectivity</b>	<b>Emissivity</b>
<b>Solution 1</b>	NO	NO
<b>Solution 2</b>	NO	<b>YES</b>
<b>Solution 3</b>	<b>YES</b>	NO
<b>Solution 4</b>	<b>YES</b>	<b>YES</b>

## Impact on LAGEOS orbits

Figure 5 shows a reduction of the osculating semi-major axis due to the infrared emissivity of 1 mm, and due to the reflectivity from 0 mm to 1 mm (0.5 mm on average). In total, orbits of both LAGEOS are lowered due to the Earth radiation pressure by about 1.5 mm. The perturbing acceleration due to the Earth radiation pressure is largest in the radial direction. Assuming a fixed value of the  $GM$  and no estimated empirical acceleration in  $R$ , the semi-major axis  $a$  of LAGEOS is reduced by a radial acceleration  $R_a$  due to Earth radiation pressure by:

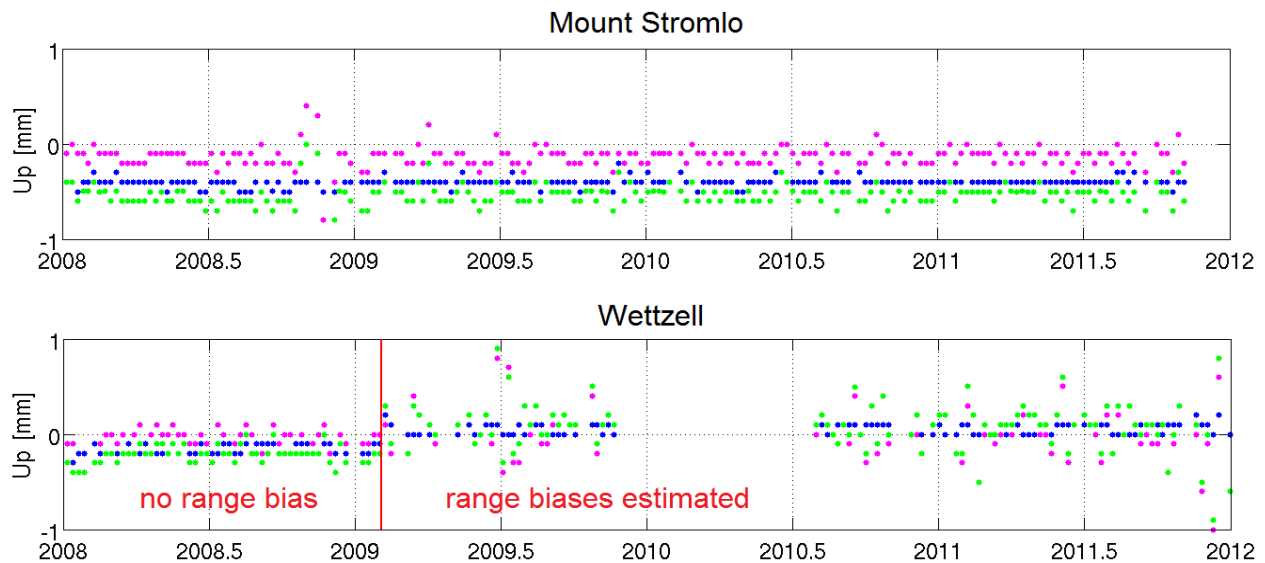
$$\Delta a = -\frac{4 R_a a^3}{3 GM}.$$

This equation explains the observed changes in the LAGEOS semi-major axis. Assuming a mean radial acceleration  $R_a=2.8 \cdot 10^{-10} \text{ m/s}^2$ , the resulting  $\Delta a$  is  $-1.6 \text{ mm}$ , which agrees very well with the observed differences.

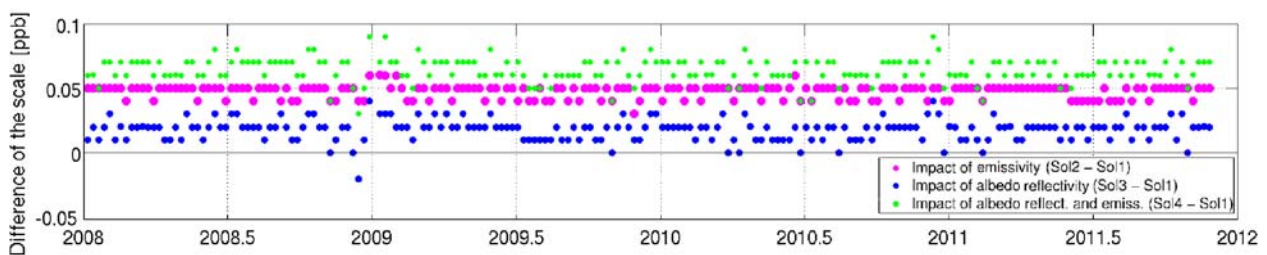
## Impact on station coordinates

Figure 6 shows the differences in time series of SLR station height components. The station height components for Mount Stromlo in Australia and for Wettzell in Germany are presented as examples of the SLR core stations. The height component of Mount Stromlo is systematically shifted by  $-0.2 \text{ mm}$  due to the emissivity and  $-0.4 \text{ mm}$  due to the reflectivity. The total shift of the height component is on average  $-0.6 \text{ mm}$  for all SLR stations in the tropic areas. For SLR stations located in the middle latitudes, e.g., for European SLR stations, the total shift of the height component is slightly smaller, i.e.,  $-0.4 \text{ mm}$  on average. The mean reduction of the station heights corresponds approximately to a half of the value of the LAGEOS-1/2 semi-major axes' lowering ( $-1.5 \text{ mm}$ ), which is related to the ratio between the Earth radius (6.378 km) and the semi-major axis of LAGEOS ( $a=12.158 \text{ km}$ ).

Figure 6 (bottom) shows the differences of station coordinate time series for Wettzell in Germany. The height component shows a systematic shift, similar to Mount Stromlo, due to the infrared emissivity and reflectivity only till January 2009. After January 2009 the systematic differences between solutions are not distinguishable, but the scatter of the height component is significantly increased. It related to the estimation of the range bias. Before January 2009 no range bias is estimated for Wettzell, whereas after January 2009 one value of a range bias per week for each LAGEOS is solved for. The impact of the Earth radiation pressure on station coordinates is accumulated in the weekly time series of estimated range biases, whereas the height component remains almost unaffected. The estimation of range biases, thus, accounts for the neglected Earth radiation pressure modeling.



**Figure 6.** Differences of station heights between solution 1 and solutions 2-4 for:  
**Top:** Mount Stromlo (Australia) with no estimated range biases,  
**Bottom:** Wettzell (Germany) with estimated range biases after January 2009.



**Figure 7.** Differences of the scale between solution 1 and solutions 2-4 from the Helmert transformation of SLR station coordinates.

### Impact on global scale

Figure 7 shows the difference between the geometric scale in the solution without applying the Earth radiation pressure and solutions with the models applied. The infrared emissivity imposes a scale difference of 0.05 ppb, whereas the albedo imposes an additional scale difference of 0.02 ppb. The total difference of 0.07 ppb corresponds to a reduction of network size of 0.5 mm w.r.t. the Earth radius.

## Summary

Earth radiation pressure has an impact on semi-major axes of LAGEOS satellites. When the Earth radiation pressure modeling is applied, the semi major axis is reduced by 1.5 mm. Earth radiation pressure affects the global scale by 0.07 ppb, which corresponds to 0.5 mm w.r.t. the Earth's radius. Station heights are systematically shifted by about  $-0.5$  mm on average due to applying Earth radiation pressure modeling when range biases are not estimated. The estimation of range biases absorbs the modeling differences and removes the systematic shift in station heights.

## References

- Knocke P, Ries J, Tapley B, *Earth radiation pressure effects on satellites*. In: AIAA/AAS Astrodynamics Conference, pp. 577-587, 1988.
- Martin C, Rubincam D, *Effects of Earth albedo on the LAGEOS I satellite*. J Geophys Res 101(B2): 3215-3226, 1996.
- Rodriguez-Solano CJ, Impact of albedo modelling on GPS orbits. Master thesis, Technische Universität München, <https://mediatum2.ub.tum.de/doc/1083571/1083571.pdf>, 2009.
- Rubincam D, Weiss N, *Earth albedo and the orbit of Lageos*. Celest Mech Dyn Astron 38(3), pp. 233-296, 1986.
- Sośnica K, Thaller D, Jäggi A, Dach R, Beutler G, *Sensitivity of Lageos Orbits to Global Gravity Field Models*. Artif Sat 47(2), pp. 35-79, 2012. doi:10.2478/v10018-012-0013-y
- Sośnica K, Thaller D, Dach R, Jäggi A, Beutler G, *Impact of loading displacements on SLR-derived parameters and on the consistency between GNSS and SLR results*. J Geod 87(8), pp. 751-769, 2013.
- Sośnica K, *Determination of Precise Satellite Orbits and Geodetic Parameters using Satellite Laser Ranging*. PhD thesis of the Philosophisch--naturwissenschaftlichen Fakultät of the University of Bern (in review), 2014.
- Sośnica K, Jäggi A, Thaller D, Meyer U, Baumann C, Dach R, Beutler G, *Earth gravity field recovery using GPS, GLONASS, and SLR satellites*. Proceedings of the 18th International Workshop on Laser Ranging, 11-15 November 2013, Fujiyoshida, Japan, 2014.
- Thaller D, Sośnica K, Mareyen M, Dach R, Jäggi A, Beutler G, *Geodetic parameters estimated from LAGEOS and Etalon data and comparison to GNSS-estimates*. J Geod (submit.), 2013.
- Thaller D, Sośnica K, Dach R, Jäggi A, Beutler G, *Lageos-Etalon solutions using the Bernese Software*. In: Proceedings of 17th International Workshop on Laser Ranging, Mitteilungen des BKG, vol 48, pp.333-336, 2012.
- Wielicki B, Barkstrom B, Harrison E, Lee R, Smith G, Cooper J, *Clouds and the Earth's Radiant Energy System (CERES): an Earth observing system experiment*. Bulletin of the American Meteorological Society 77(5), pp.853-868, 1996.