

# Validation and estimation of low-degree gravity field coefficients using LAGEOS

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## ABSTRACT

Precise orbit determination is an essential task for analyzing satellite laser ranging (SLR) data. The quality of the satellite orbits directly depends on the background models used for dynamic orbit determination, e.g., on the underlying model of the Earth's gravity field. We investigate the influence of more than ten recent and well known gravity field models on the quality of a combined LAGEOS-1 and LAGEOS-2 orbit determination by analyzing orbital fits. For this purpose we process the SLR data collected by the stations of the International Laser Ranging Service (ILRS) to both LAGEOS satellites in 2008 and show that not only the type and maximum degree of the underlying gravity field model is essential, but also the proper choice of a limited number of empirical orbit parameters that have to be co-estimated with all other relevant parameters like station coordinates, Earth orientation parameters, and the satellite's initial conditions on a weekly basis. Based on the experience gained from such validations, the LAGEOS SLR data collected by the ILRS in 2009 are used to estimate weekly corrections to the  $C_{20}$  values of the underlying a priori gravity field model, and to accumulate the estimates to monthly corrections.

## 1 Introduction

The satellite laser ranging data (SLR) to both LAGEOS satellites are processed in a combined analysis based on 7-day arcs using the gravity field models listed in Table 1 (ICGEM, 2011) according to two different solution strategies. For solution (a) one constant empirical acceleration is estimated per 7-day arc for each LAGEOS satellite in the along-track direction in addition to the initial conditions, as well as once-per-revolution (OPR) accelerations in the along-track and cross-track directions. The OPR accelerations in the respective directions are set up as coefficients scaling the cosine and sine of the argument of latitude, i.e., of the angle between the nodal line and the satellite's geocentric position vector as measured from the ascending node. For solution (b) essentially the same parametrization is used, but without estimating the coefficients of the OPR cross-track accelerations. For both solutions the coordinates of the ILRS tracking stations and the Earth orientation parameters are co-estimated on the same weekly basis.

Gravity field model	Year	Max. degree	Drift	SLR	CHAMP	GRACE	GOCE	Ground data
JGM3	1994	70		X				X
EGM96	1996	360		X				X
EIGEN-GL04C	2006	360	4	X		X		X
EGM2008	2008	2190				X		X
EIGEN51C	2010	359	4	X	X	X		X
ITG-GRACE2010	2010	180				X		
AIUB-CHAMP03S	2010	100			X			
AIUB-GRACE03S	2011	160	30			X		
GO-CONS-2-DIR-R2	2011	240					X	
GOCO02S	2011	250		X	X	X	X	
AIUB - SST - only	2011	120			X		X	

Table 1: Gravity field models and their characteristics

## 2 Validation of gravity field models

The LAGEOS orbits are sensitive only up to about degree and order 14 of the Earth's gravity field. Orbit solutions only differ slightly when the gravity field coefficients are taken into account up to higher degrees and orders, e.g., on a level of about 0.5 mm for a spherical harmonic expansion up to degree 20. Coefficients above degree 20 do not significantly impact the LAGEOS trajectories.

### 2.1 Standard solution

Figure 1 shows the root-mean-square (RMS) error of the SLR residuals obtained from the weekly solutions when using the gravity field models listed in Table 1 and adopting the solution strategy (a). Similar results of good quality are obtained for the majority of the models, apart from EGM96 showing a slightly inferior performance. JGM3 and ITG-GRACE2010 also show a very small degradation with respect to other models. Smallest RMS values are obtained for EGM2008, GO-CONS-2-DIR-R2, AIUB-GRACE03S, EIGEN-51C, and EIGEN-GL04C (7.13, 7.14, 7.15, 7.16, and 7.17mm, respectively). The RMS of ITG-GRACE2010 may be reduced to 7.18 mm as well, provided that the degree-one coefficients are set to zero. This modified model is labeled as "ITG-GRACE2010 mod" in Fig. 1.

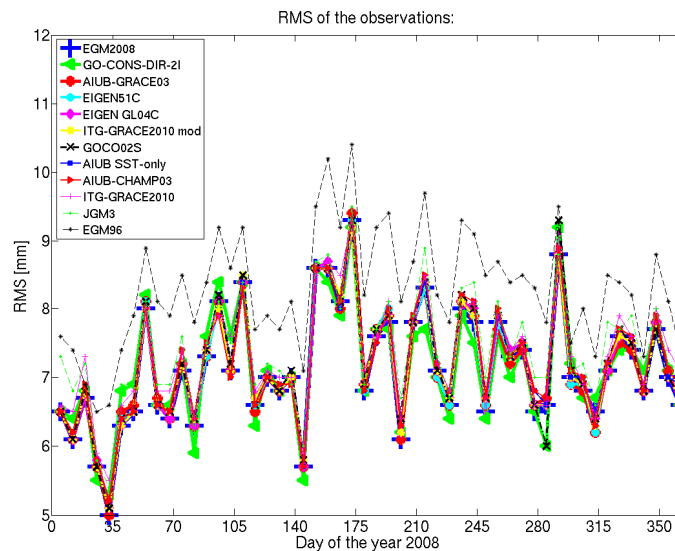
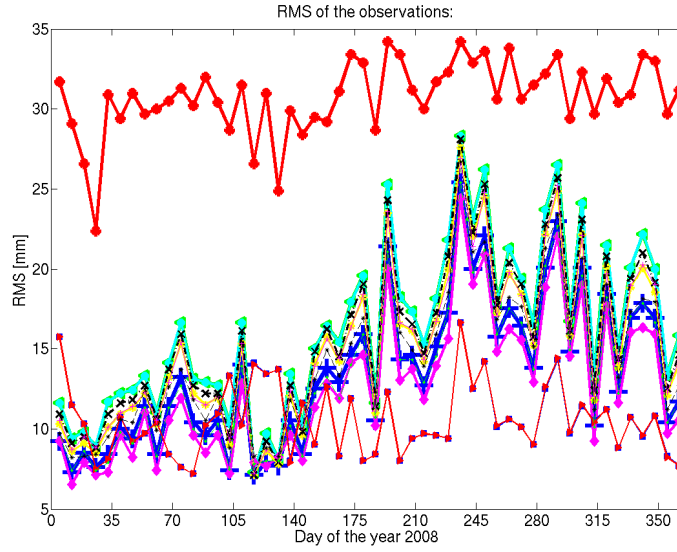


Figure 1: RMS of weekly LAGEOS solutions with the full set of OPR accelerations estimated

### 2.2 Omission of OPR cross-track accelerations

Figure 2 shows the RMS error of the SLR residuals obtained from the weekly solutions when using the gravity field models listed in Table 1 and adopting the solution strategy (b). A very pronounced discrimination between the different models is obvious. AIUB-GRACE03S, among the best models when adopting solution strategy (a) (see Fig. 1), is now showing an exceptionally poor performance. Smallest RMS values are obtained for the GPS-only models AIUB-CHAMP03S and AIUB-SST-only (10.51 and 10.52 mm, respectively), the latter being an extension of the CHAMP-based model with GPS data from GOCE. The best performance of the GRACE-based models is obtained for EIGEN-GL04C with an RMS of 12.56 mm.



**Figure 2: RMS of weekly LAGEOS solutions without OPR cross-track accelerations estimated**

### 2.2.1 Correlation of OPR accelerations with $C_{20}$

Equation 1 shows the acceleration due to  $C_{20}$  in the radial ( $R$ ), along-track ( $A$ ), and cross-track ( $C$ ) directions as a function of the argument of latitude  $u$ , the geocentric distance  $r$ , and the orbital inclination  $i$ :

$$\begin{pmatrix} R \\ A \\ C \end{pmatrix} = \frac{3}{2} \frac{GMa_e C_{20}}{r^4} \begin{pmatrix} 1 - \frac{3}{2} \sin^2 i + \frac{3}{2} \sin^2 i \cos 2u \\ \sin^2 i \sin 2u \\ \sin 2i \sin u \end{pmatrix}$$

**Equation 1: Acceleration due to  $C_{20}$**

Since only the cross-track component is governed by a OPR periodicity, Eq. 1 illustrates a full correlation between  $C_{20}$  and the sine coefficient of an empirically determined OPR cross-track acceleration per arc. The results of the solution strategy (a) are thus almost insensitive to the quality of the  $C_{20}$  coefficient of the used gravity field model. Deficient  $C_{20}$  estimates, such as for AIUB-GRACE03S where  $C_{20}$  is derived from GRACE-only, may be perfectly absorbed by the sine coefficient of the empirical OPR cross-track acceleration. Solution strategy (b) is thus well suited to mainly validate the quality of the  $C_{20}$  estimates, whereas solution strategy (a) is well suited to essentially “exclude”  $C_{20}$  from the analysis. Obviously, solution strategy (a) cannot be used to estimate  $C_{20}$  from SLR data on a weekly basis as it is performed in Sect. 3.

## 3 Estimation of low-degree gravity field coefficients

Figure 3 shows normalized and unconstrained weekly estimates of  $C_{20}$  when using the a priori gravity field model GGM02S and when adopting the solution strategy (b). For comparison with the monthly series from the Center for Space Research (CSR), the weekly estimates are accumulated to monthly solutions as well, and the a priori values of GGM02S are shown as reference. The first result of  $C_{20}$  estimates obtained with the Bernese Software (Dach et al., 2007) shows a fair agreement with the series from CSR.

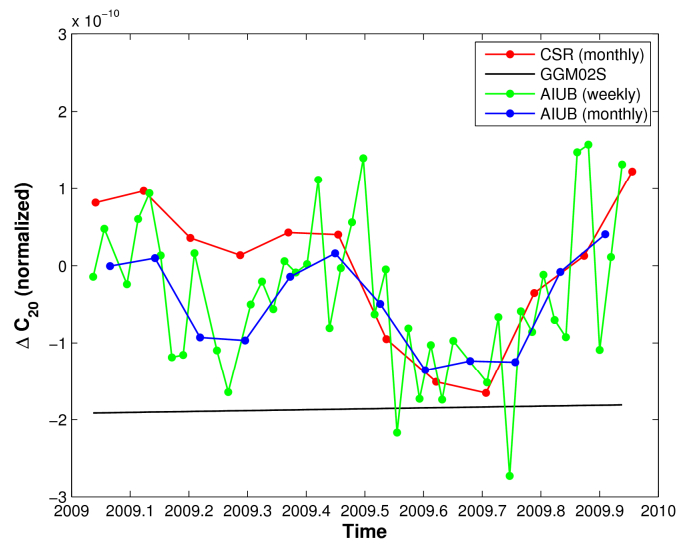


Figure 3: Weekly and monthly estimates of  $C_{20}$  w.r.t.  $4.84169411 \cdot 10^{-4}$

## 4 Conclusions

The LAGEOS satellites are sensitive up to about degree and order 14 of the Earth's gravity field. The smallest RMS of fit to the SLR data from LAGEOS-1 and LAGEOS-2 are obtained for the gravity field models EGM2008, GO-CONS-2-DIR-R2, AIUB-GRACE03S, EIGEN-51C, and EIGEN-GL04C when estimating the full set of OPR accelerations. Without estimating OPR cross-track accelerations, the validation results are mainly dominated by the quality of the  $C_{20}$  estimates, e.g., revealing an exceptionally poor quality of  $C_{20}$  for AIUB-GRACE03S and best results for the GPS-only models AIUB-CHAMP03S and AIUB-SST-only. First results of  $C_{20}$  estimates obtained with the Bernese Software show a fair agreement with the series from CSR when omitting OPR cross-track accelerations. Longer data series will be processed in the near future.

## References

Dach, R, Hugentobler U, Fridez P., Meindl M., 2007: Bernese GPS Software Version 5.0. Astronomical Institute, University of Bern.

ICGEM, 2011: International Centre for Global Earth Models. GFZ Helmholtz Centre Potsdam, <http://icgem.gfz-potsdam.de/ICGEM/>

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