Impact of Earth rotation on the signal propagation between CERN and Gran Sasso

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G. Beutler, A. Jäggi, T. Schildknecht, R. Dach
Astronomical Institute, University of Bern,
Sidlerstrasse 5, 3012 Bern, Switzerland

1 Introduction

The slant range d_1 associated with a signal traveling with speed of light c from P_1 at time t to P_2 is the distance between P_1 at emission time t and P_2 at reception time $t + \Delta t$, where $\Delta t = d_1/c$ is the propagation time of the signal between P_1 and P_2.

P_1 and P_2 are assumed to be at rest in the Earth-fixed coordinate system and to have the geographic coordinates r_i, λ_i, ϕ_i, $i = 1, 2$. d_1 is calculated in two different ways, which both give the same result.

2 Geographic and Quasi-inertial Coordinates

Let:

$$r_i,\ \phi_i,\ \lambda_i,\ i = 1, 2 \tag{1}$$

the spherical geographic coordinates, namely geocentric radius vector, latitude, and longitude, of the signal transmitter at P_1 and the signal detector at P_2 in the Earth-fixed coordinate system.

The corresponding rectangular coordinates are:

$$\boldsymbol{r}_{Ei} = \begin{pmatrix} x_{Ei} \\ y_{Ei} \\ z_{Ei} \end{pmatrix} = r_i \begin{pmatrix} \cos\phi_i \cos\lambda_i \\ \cos\phi_i \sin\lambda_i \\ \sin\phi_i \end{pmatrix} . \tag{2}$$

The distance in the Earth-fixed system between P_1 and P_2 is:

$$d_0 = \sqrt{(x_{E2} - x_{E1})^2 + (y_{E2} - y_{E1})^2 + (z_{E2} - z_{E1})^2} . \tag{3}$$

Assuming a simplified model for the rotation of the Earth (neglecting polar motion, precession and nutation) the Earth-fixed coordinates may be easily transformed into a quasi-inertial coordinate system, see (Beutler, 2005):

$$\boldsymbol{r}_{Ii} = \mathbf{R}_3(-\Theta)\, \boldsymbol{r}_{Ei}\ , i = 1, 2\ , \tag{4}$$

where $\boldsymbol{r}_{Ii}$ are the coordinates in the quasi-inertial system, Θ is the Greenwich sidereal time, and $\mathbf{R}_3(\alpha)$ describes the particular rotation about the third coordinate axis and angle α. The resulting system is called *quasi*-inertial, because the geocenter, the origin of the system is in accelerated motion about the Sun. Apart from that the coordinate array $\boldsymbol{r}_{Ii}$ refers to the geocentric, equatorial system.

3 Determining the Slant Range d_1: Method 1

3.1 General Relation

The slant range between between P_1 and P_2 is defined as:

$$d_1 = |\boldsymbol{r}_{I2}(t + \Delta t) - \boldsymbol{r}_{I1}(t)|\ . \tag{5}$$

d_1 thus is the distance in the quasi-inertial system between P_1 at transmission time and P_2 at reception time. Δt is the signal traveling time. Assuming an experiment in vacuum and neglecting the influence of the geopotential we may approximate:

$$\Delta t = d_0/c\ , \tag{6}$$

where $c = 299792458$ m/s is the speed of light in vacuo.

Let us now adopt a special quasi-inertial coordinate system with its coordinate plane (x_{I1}, x_{I3}) coinciding with the meridian of the transmitter at emission time t. For this special selection we have $\Theta(t) = 0$ and therefore

$$\boldsymbol{r}_{I1}(t) = \boldsymbol{r}_{E1}(t)\ . \tag{7}$$

We have moreover

$$\Theta(t + \Delta t) = \omega \cdot \Delta t\ , \tag{8}$$

where $\omega = 7.292115 \cdot 10^{-5}$ rad/s is the angular velocity of Earth rotation. We may thus establish the coordinate transformation for P_2 at $t + \Delta t$ into the inertial system as:

$$\boldsymbol{r}_{I2}(t + \Delta t) = \begin{pmatrix} \cos\omega\Delta t & -\sin\omega\Delta t & 0 \\ \sin\omega\Delta t & \cos\omega\Delta t & 0 \\ 0 & 0 & 1 \end{pmatrix} \boldsymbol{r}_{E2}\ . \tag{9}$$

As $\omega \Delta t$ is a small angle we may use the approximation:

$$\boldsymbol{r}_{I2}(t+\Delta t) = \begin{pmatrix} 1 & -\omega\Delta t & 0 \\ \omega\Delta t & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \boldsymbol{r}_{E2} \tag{10}$$

or

$$\boldsymbol{r}_{I2}(t+\Delta t) = \begin{pmatrix} x_{E2} - \omega\Delta t\, y_{E2} \\ y_{E2} + \omega\Delta t\, x_{E2} \\ z_{E2} \end{pmatrix} . \tag{11}$$

Introducing this relation into Eq. (5) and neglecting higher order terms we obtain:

$$d_1 = d_0 + \omega\Delta t \cdot \left[x_{E1} y_{E2} - x_{E2} y_{E1}\right] / d_0 \ . \tag{12}$$

Using Eq. (2) to further modify the above formula and well-known trigonometric relations gives the final result:

$$d_1 = d_0 + \omega\Delta t\ \frac{r_1 r_2}{d_0} \cos\phi_1 \cos\phi_2 \sin(\lambda_2 - \lambda_1) \ . \tag{13}$$

Formula (13) shows that $d_1 = d_0$ for $\lambda_2 = \lambda_1$ and that it is maximum if transmitter and detector are located on the equator and separated by a certain longitude difference. It is furthermore clear that $d_1 > d_0$ for $\lambda_2 > \lambda_1$, i.e., if P_2 is located in the East of P_1, we have $d_1 - d_0 > 0$, i.e. the slant range is larger than the geometric distance d_0 between P_1 and P_2.

3.2 Application to the OPERA Experiment

Let us now calculate the difference $d_1 - d_0$, if P_1 is the CERN in Geneva and P_2 the Gran Sasso laboratory in Italy. We assume the following spherical Earth-fixed coordinates for P_1 and P_2:

$$\begin{array}{l} P_1 : r_1 = 6368000 \text{ m } \phi_1 = 46.20^\circ\ \lambda_1 = 06.15^\circ \\ P_2 : r_2 = 6368000 \text{ m } \phi_2 = 42.47^\circ\ \lambda_2 = 13.55^\circ \end{array} . \tag{14}$$

The mean Earth radius was taken for r_i, $i = 1, 2$, and rather approximate coordinates (from the internet) for the geographical coordinates of CERN and Gran Sasso. The distance d_0 corresponding to the adopted and approximate coordinates is $d_0 = 718800$ m. With these values we obtain:

$$d_1 - d_0 = 0.579 \text{ m} \ . \tag{15}$$

This corresponds to a correction of the signal propagation time due to Earth rotation of

$$\Delta t(\text{Earth rotation}) = \frac{d_1 - d_0}{c} = 1.93 \text{ ns} \ . \tag{16}$$

The result corresponds to the crude coordinates (14). Better coordinates have to be used for generating the final result.

4 Determining the Slant Range d_1: Method 2

We introduce an equatorial Earth-fixed coordinate system, with its zero meridian going through the Gran Sasso laboratory. The spherical coordinates of P_1 and P_2 in this system are:

$$\begin{array}{l} P_1 : r_1 = 6368000 \text{ m } \phi = 46.20^\circ\ \tilde{\lambda}_1 = -7.40^\circ \\ P_2 : r_2 = 6368000 \text{ m } \phi = 42.47^\circ\ \ \tilde{\lambda}_2 = \ \ 0.00^\circ \end{array} , \tag{17}$$

where $\tilde{\lambda}_1 = \lambda_1 - \lambda_2$. The corresponding rectangular coordinates are:

$$\boldsymbol{r}_{E1} = r_1 \begin{pmatrix} \cos\phi_1 \cos\tilde{\lambda}_1 \\ \cos\phi_1 \sin\tilde{\lambda}_1 \\ \sin\phi_1 \end{pmatrix} , \boldsymbol{r}_{E2} = r_2 \begin{pmatrix} \cos\phi_2 \\ 0 \\ \sin\phi_2 \end{pmatrix} . \tag{18}$$

In analogy to the deliberations in Sect. 3 we introduce the inertial system coinciding with the new Earth-fixed system at time t. In this system the Gran Sasso laboratory has the velocity vector

$$\boldsymbol{v}_{I2} = r_2\, \omega \begin{pmatrix} 0 \\ \cos\phi_2 \\ 0 \end{pmatrix} . \tag{19}$$

The change of the length of the baseline vector between P_1 at t and P_2 at $t + \Delta t$ consequently is:

$$\begin{aligned} d_1 - d_0 &= \boldsymbol{v}_{I2} \cdot \left\{\boldsymbol{r}_{E2} - \boldsymbol{r}_{E1}\right\} / d_0\ \Delta t \\ &= -r_2\, \omega\ \cos\phi_2\ \left\{r_1 \cos\phi_1\ \sin\tilde{\lambda}_1\right\} / d_0\ \Delta t \ , \\ &= \ \omega\ \Delta t\ \tfrac{r_1 r_2}{d_0} \cos\phi_1 \cos\phi_2 \sin(\lambda_2 - \lambda_1) \end{aligned} \tag{20}$$

which is identical to the result (13) obtained with the first method.

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References

Beutler G (2005) Methods of celestial mechanics. Springer, Berlin Heidelberg New York