

GNSS Estimates of Short-period Nutation



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Introduction

- Nutation offsets can only be determined with VLBI and LLR
- Feasibility of determining short-period nutation from GPS-derived nutation rates already shown a decade ago
- Homogeneously reprocessed GNSS long-time series of nutation rates provide a valuable basis for the determination of short-period nutation

Outline

- Estimation of nutation rates from GNSS observations
- Estimation of short-period nutation models
- Precision and convergence of GNSS nutation models
- Comparisons of different GNSS nutation models with VLBI and IAU2000A

Estimation of Nutation Parameters with GNSS

Determination of nutation offsets with GNSS not possible due to correlations with the orbital elements

$$\Delta \dot{\epsilon} = \cos \Omega \dot{i} + \sin i \sin \Omega \dot{u}_0$$

$$\Delta \dot{\psi} \sin \epsilon_0 = -\sin \Omega \dot{i} + \sin i \cos \Omega \dot{u}_0$$

$\Delta \dot{\epsilon}$ nutation rate in obliquity

$\Delta \dot{\psi}$ nutation rate in longitude

ϵ_0 mean obliquity of the ecliptic

Ω right ascension of the ascending node

i inclination

u_0 argument of latitude

Estimation of Nutation Models

$$\delta\Delta\epsilon(t) = \sum_{j=1}^n \delta\epsilon_j^c \cos \theta_j(t) + \delta\epsilon_j^s \sin \theta_j(t)$$

$$\delta\Delta\psi(t) = \sum_{j=1}^n \delta\psi_j^c \cos \theta_j(t) + \delta\psi_j^s \sin \theta_j(t)$$

$\delta\psi_j^s, \delta\psi_j^c$

$\delta\epsilon_j^s, \delta\epsilon_j^c$

θ_j

n

sine/cosine coefficients of nutation in longitude
sine/cosine coefficients of nutation in obliquity
combination of the five fundamental arguments
number of nutation periods

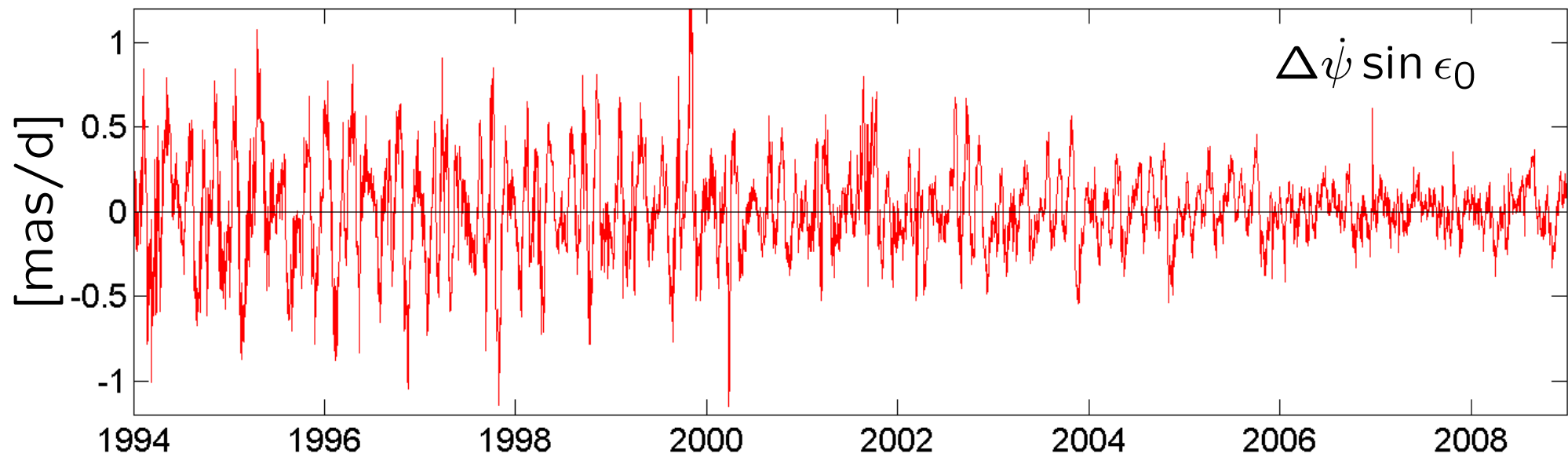
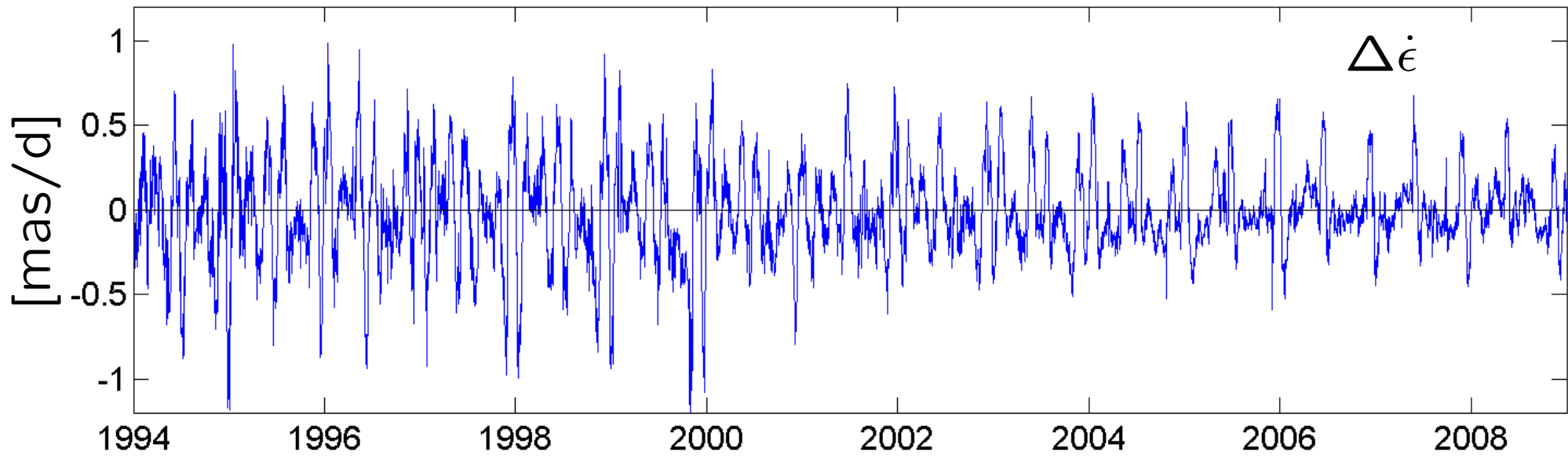
Estimation of $n = 34$ nutation amplitudes between
4 and 16 days in a least squares adjustment

GNSS and VLBI Solutions

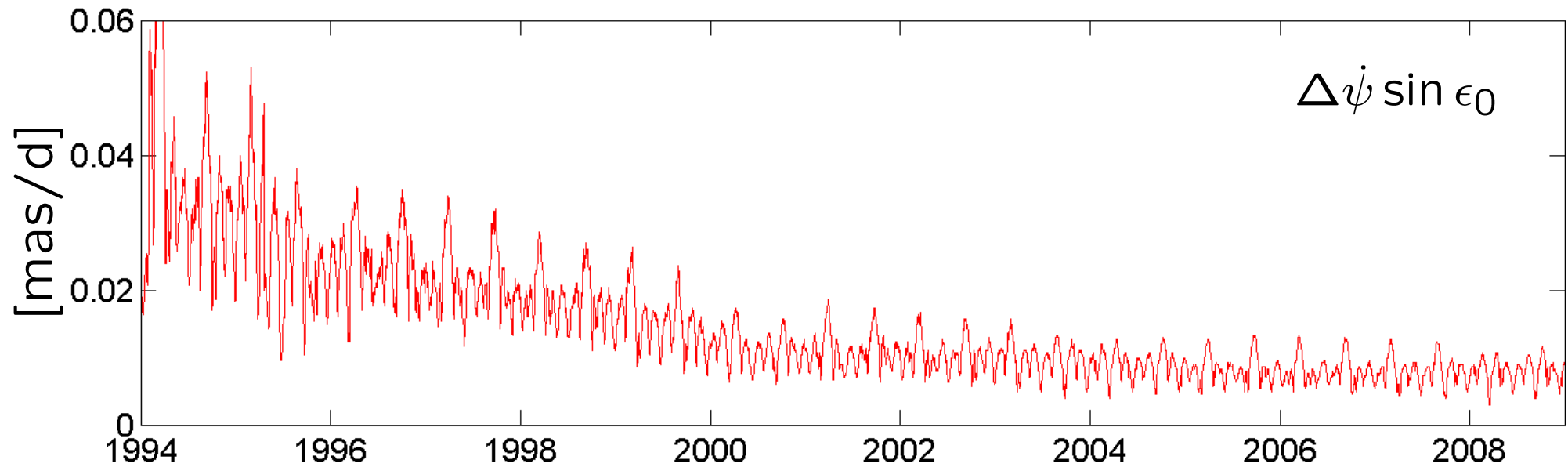
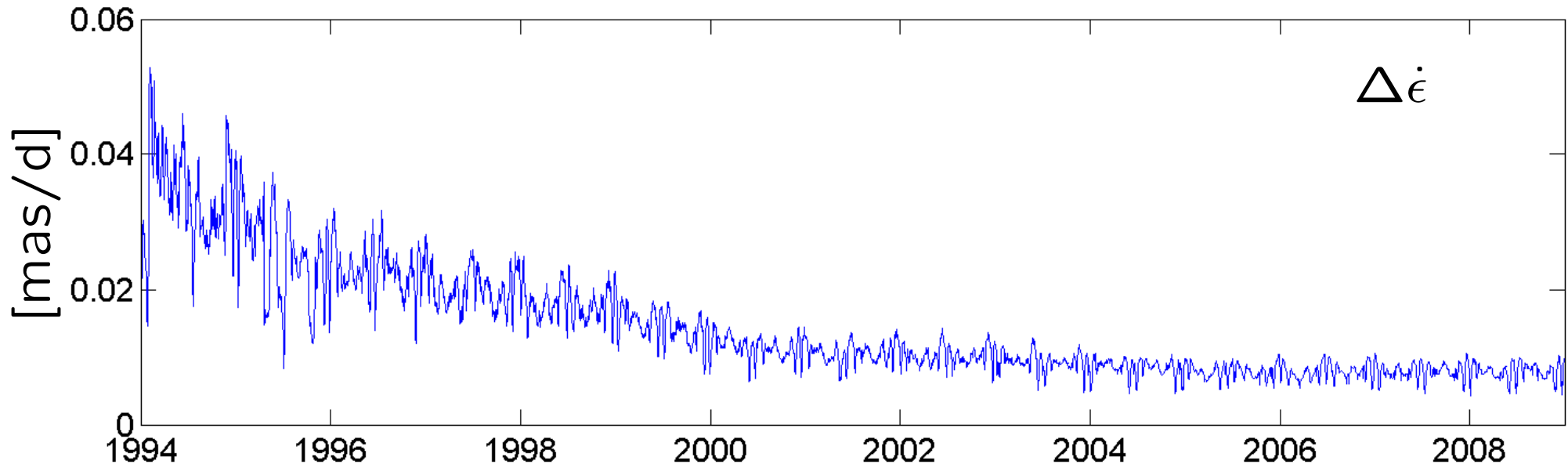
All GPS and GNSS solutions were computed with the Bernese GPS Software

| Solution | Abb. | Technique | Time Interval |
|-----------------|-------------|------------------|----------------------|
| CODE operat. | COD | GPS/GNSS | Jan 1994 – Mar 2009 |
| CODE reproc. | CO1 | GPS | Jan 1994 – Dec 2008 |
| Potsdam Dresden | PD1 | GPS | Jan 1994 – Dec 2008 |
| DGFI | DGF | VLBI | Jan 1984 – Dec 2008 |
| GSFC | GSF | VLBI | Jan 1984 – Jan 2009 |
| GGOS-D | GIC | VLBI comb. | Jan 1984 – Dec 2005 |

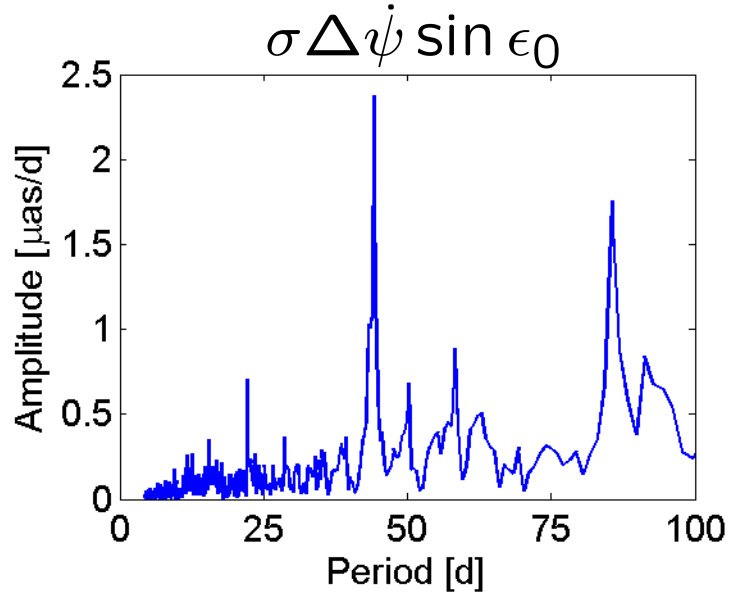
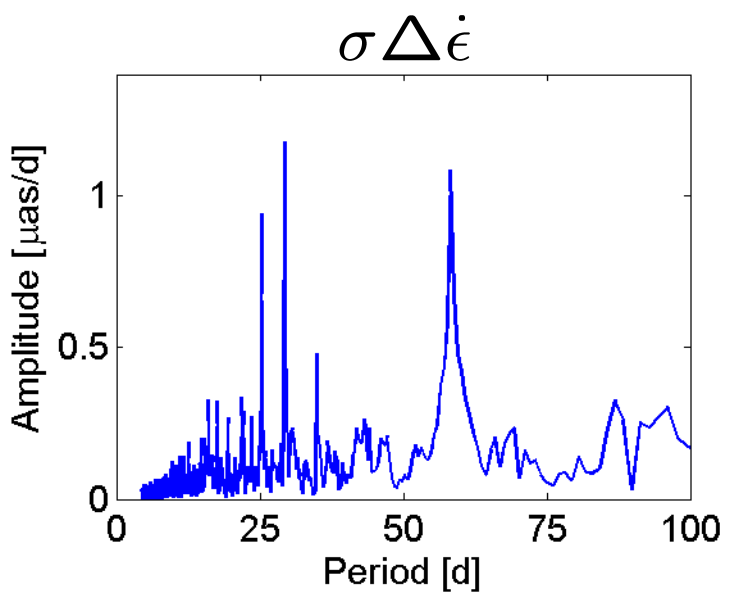
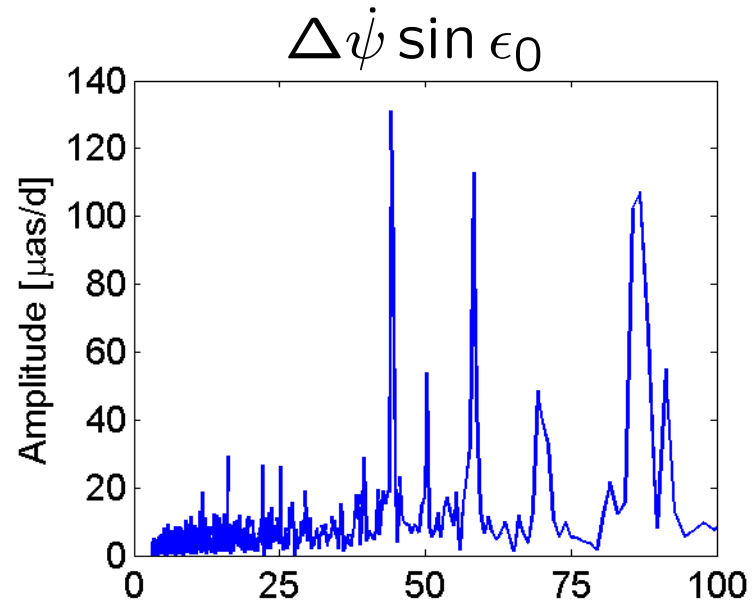
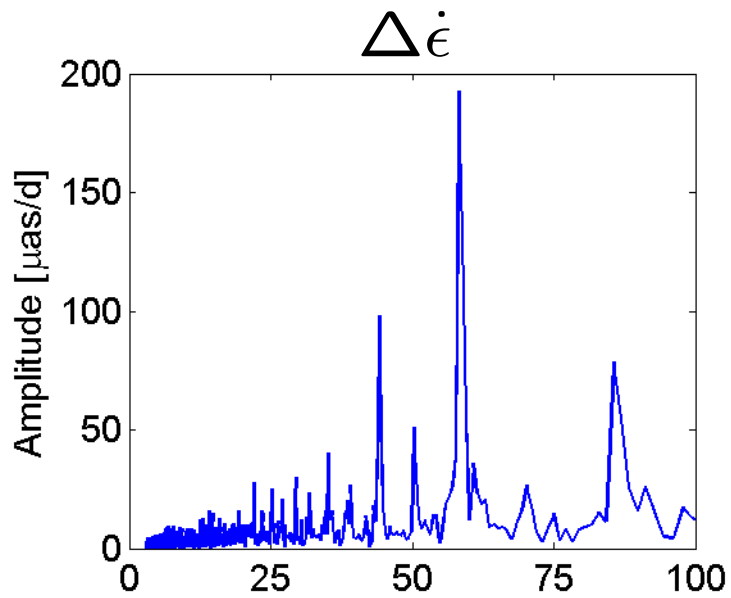
CODE Reprocessed (CO1) Nutation Rates



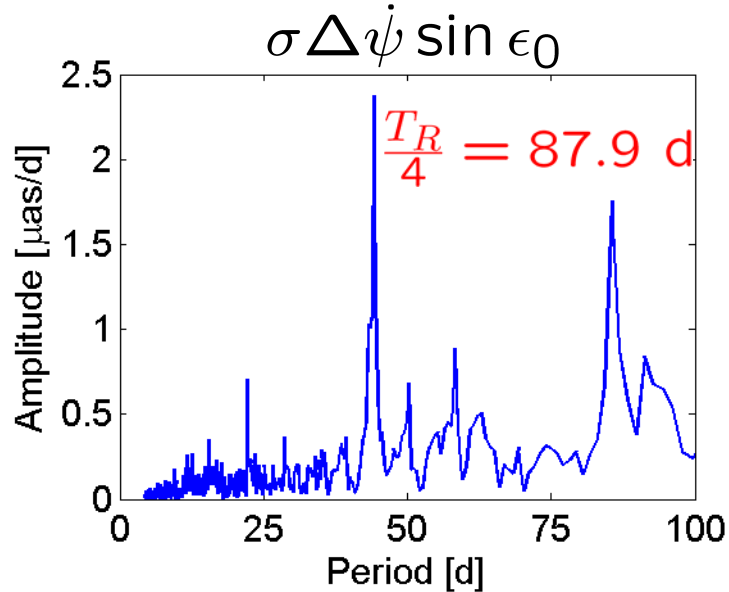
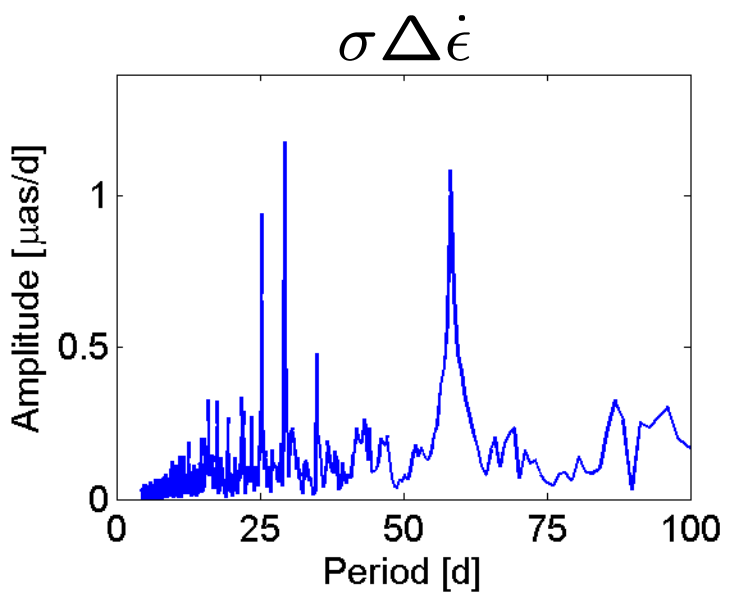
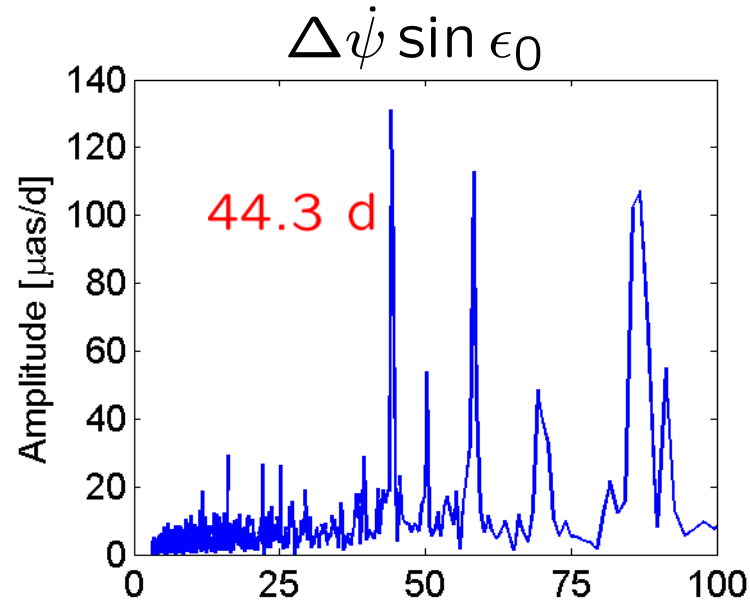
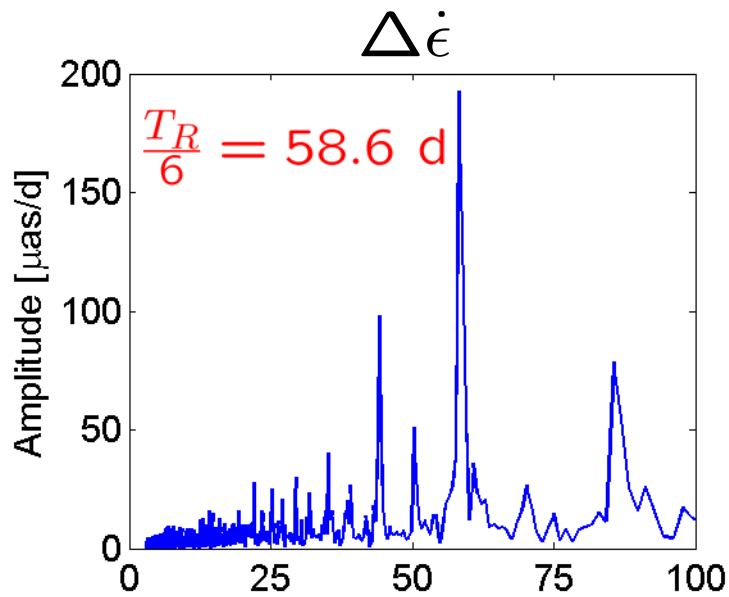
Formal Errors of CO1 Nutation Rates



CO1 Nutation Rate Spectra



CO1 Nutation Rate Spectra

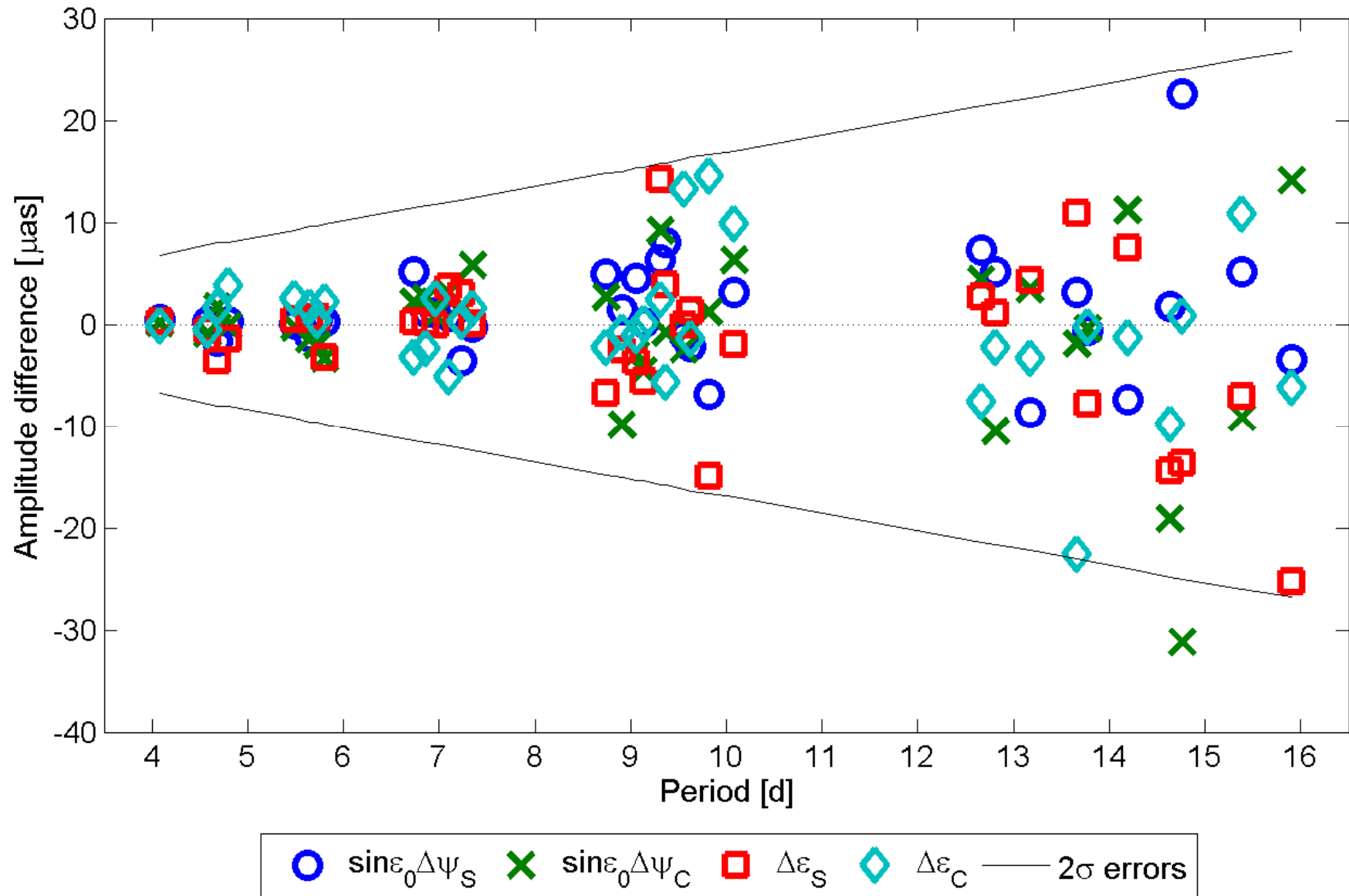


$$T_R = 351.5 \text{ d}$$

Draconitic year
„GPS-year“

Orientation of
the orbital
planes w.r.t.
the Sun

Differences of CO1 w.r.t. IAU2000A

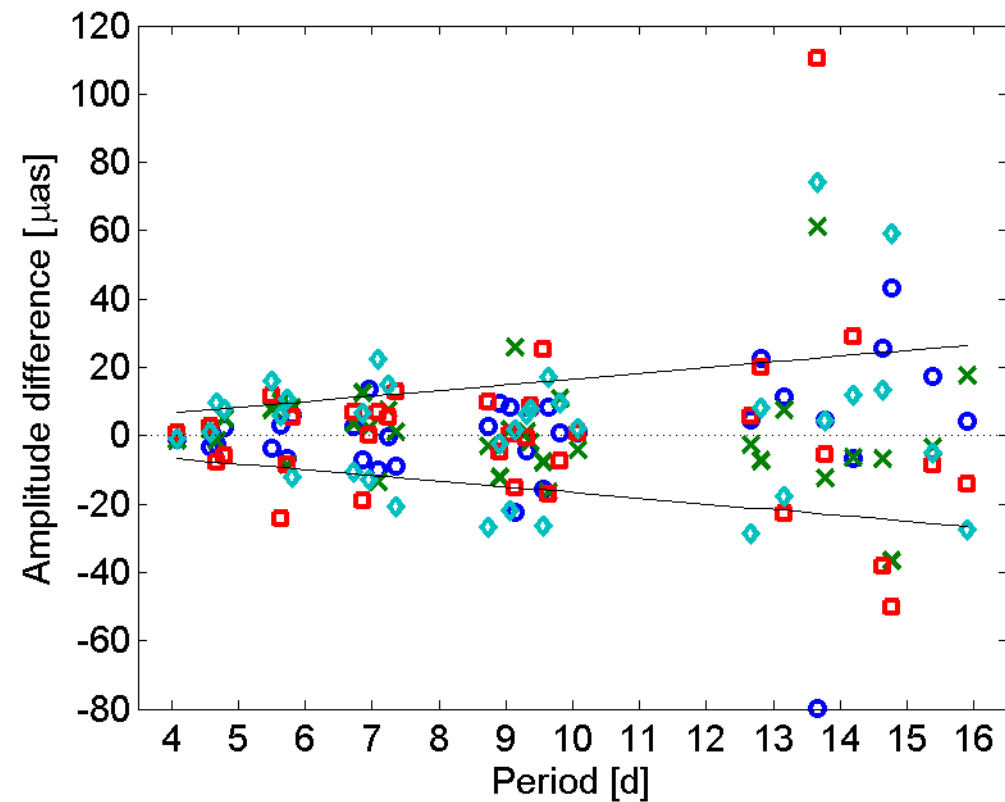
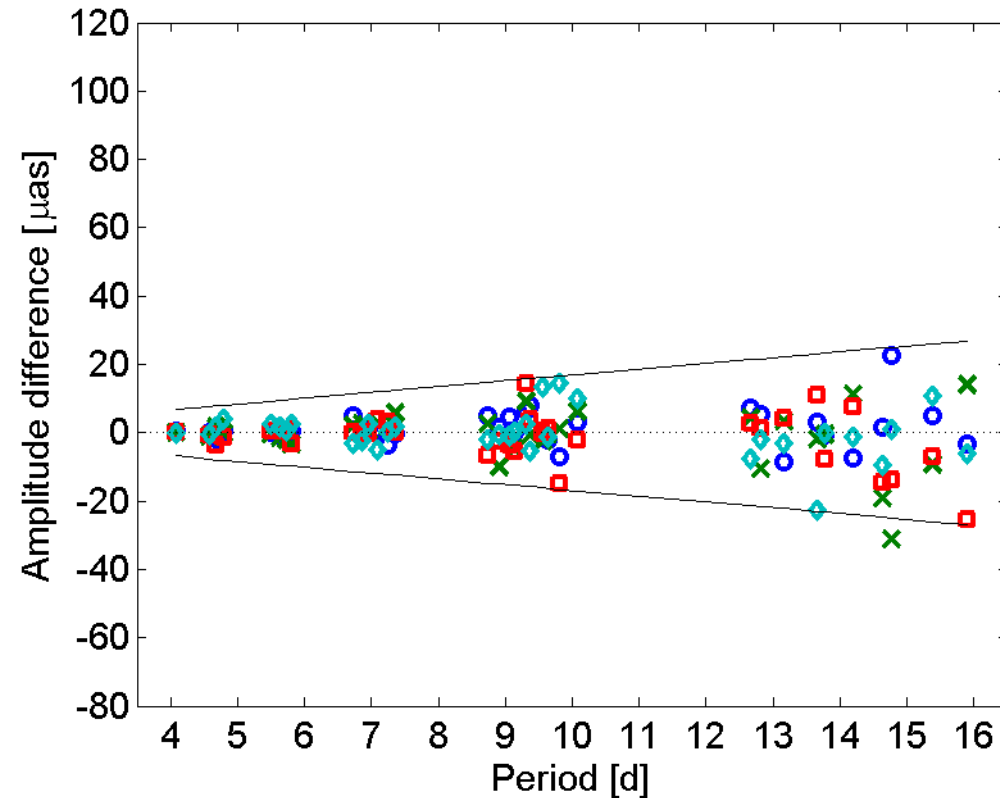


Reprocessed vs. Operational Series

Differences with respect to IAU2000A

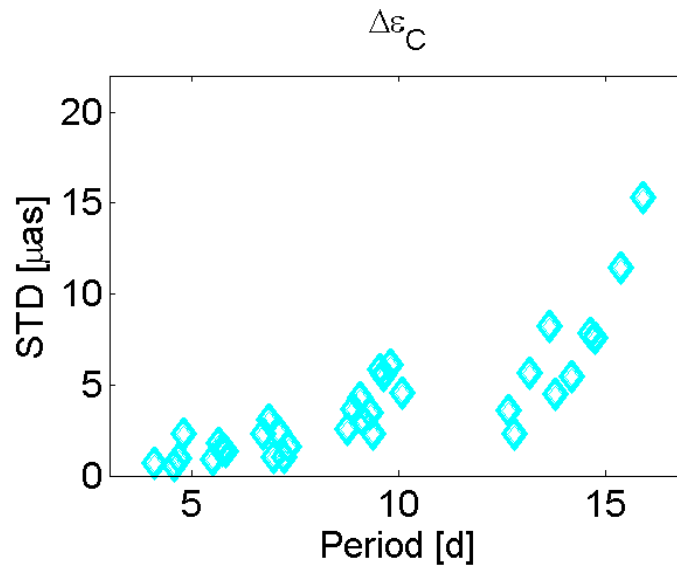
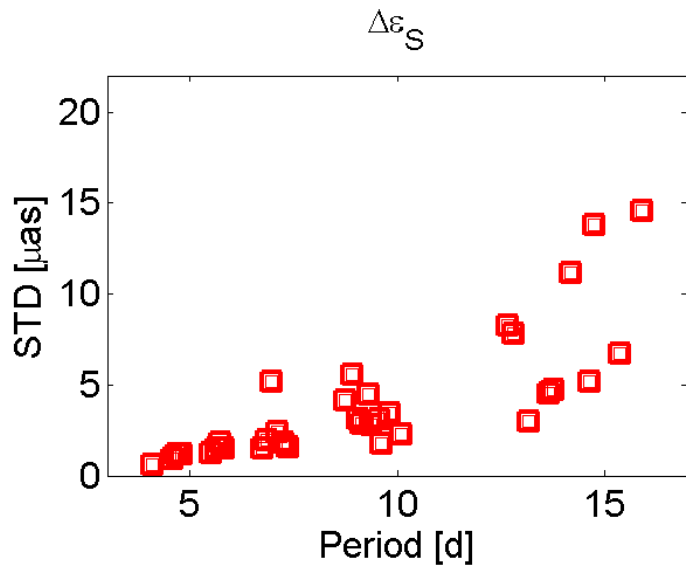
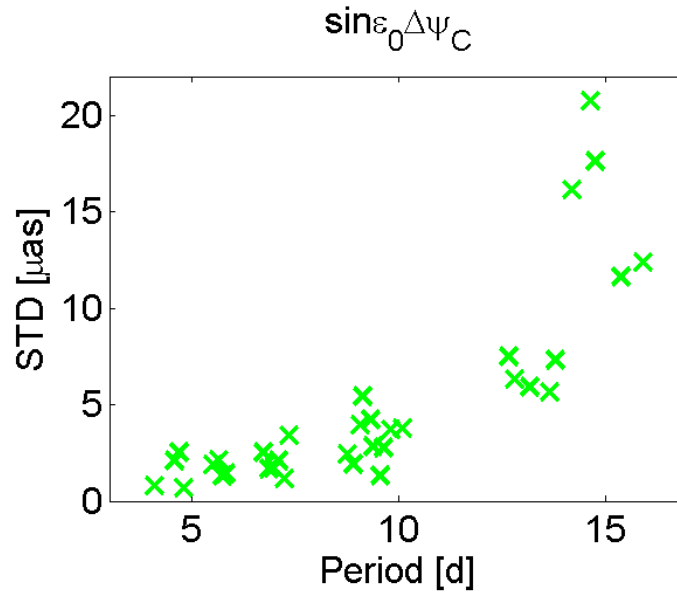
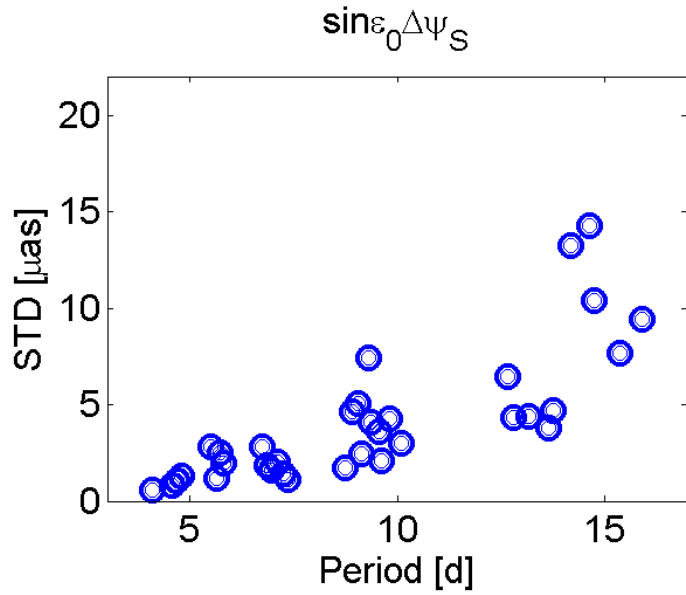
CO1 reprocessed GPS series

COD operational GNSS series



Legend: \circ $\sin \epsilon_0 \Delta \psi_S$ \times $\sin \epsilon_0 \Delta \psi_C$ \square $\Delta \epsilon_S$ \diamond $\Delta \epsilon_C$ — 2σ errors

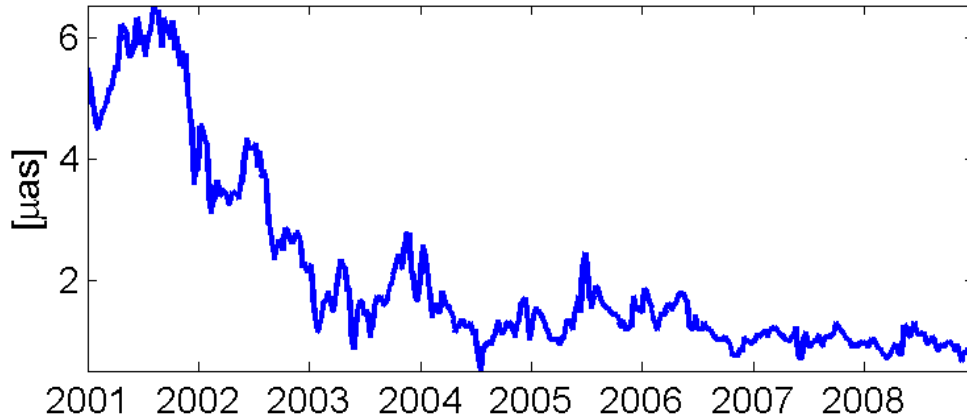
Precision of Nutation Models



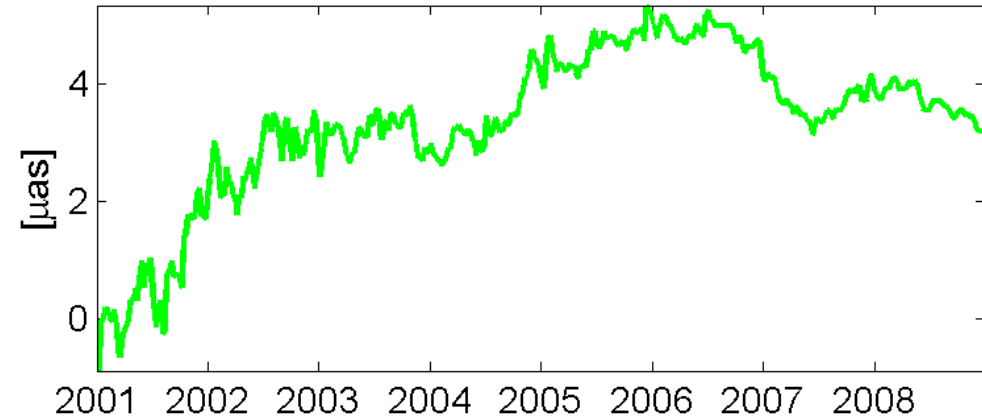
Estimation of 8 nutation models from the CO1 series based on 8 years of data shifted by 1 year (time period 1996-2008)

Convergence of Nutation Coefficients

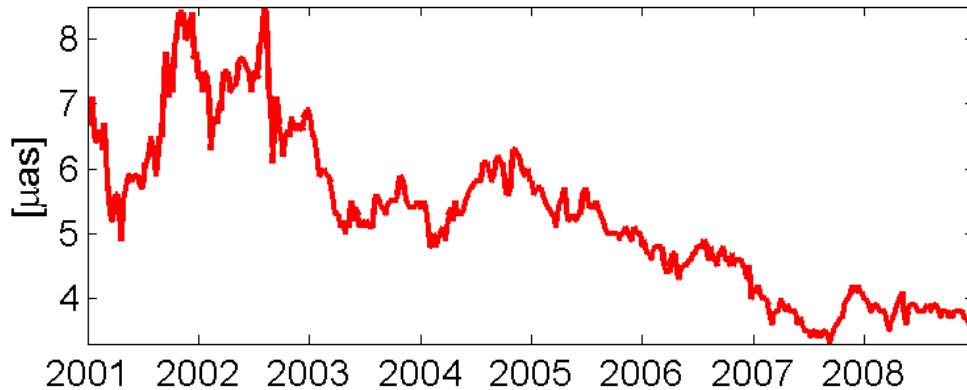
$$\sin \varepsilon_0 \Delta \psi_S$$



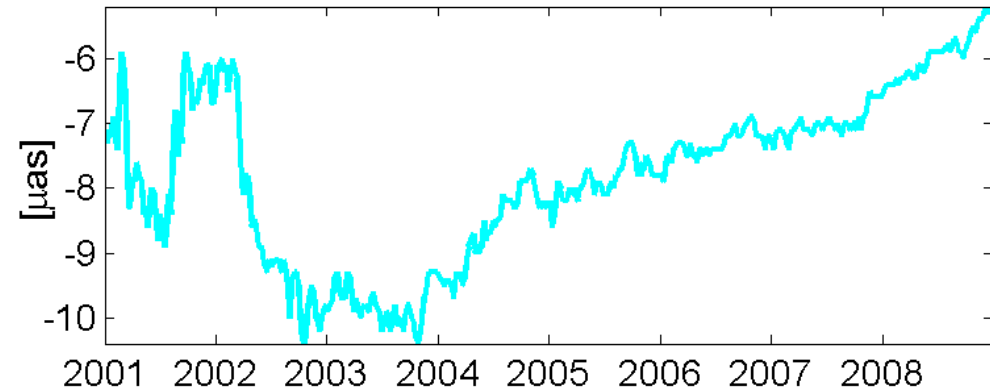
$$\sin \varepsilon_0 \Delta \psi_C$$



$$\Delta \varepsilon_S$$



$$\Delta \varepsilon_C$$



Estimation of nutation models from the CO1 series starting with January 1996 until the time displayed on the x-axis, 7.096 d period

Impact of Different Processing Options

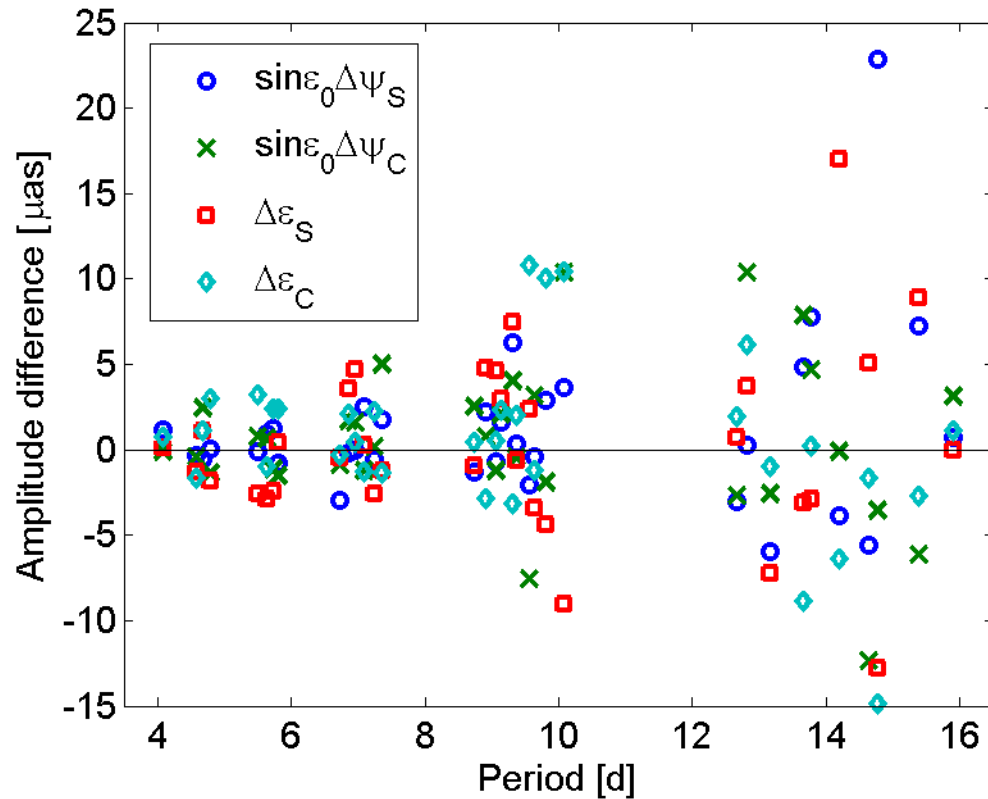
A priori radiation pressure model

- CO1: empirical CODE model
- PD1: physical ROCK model

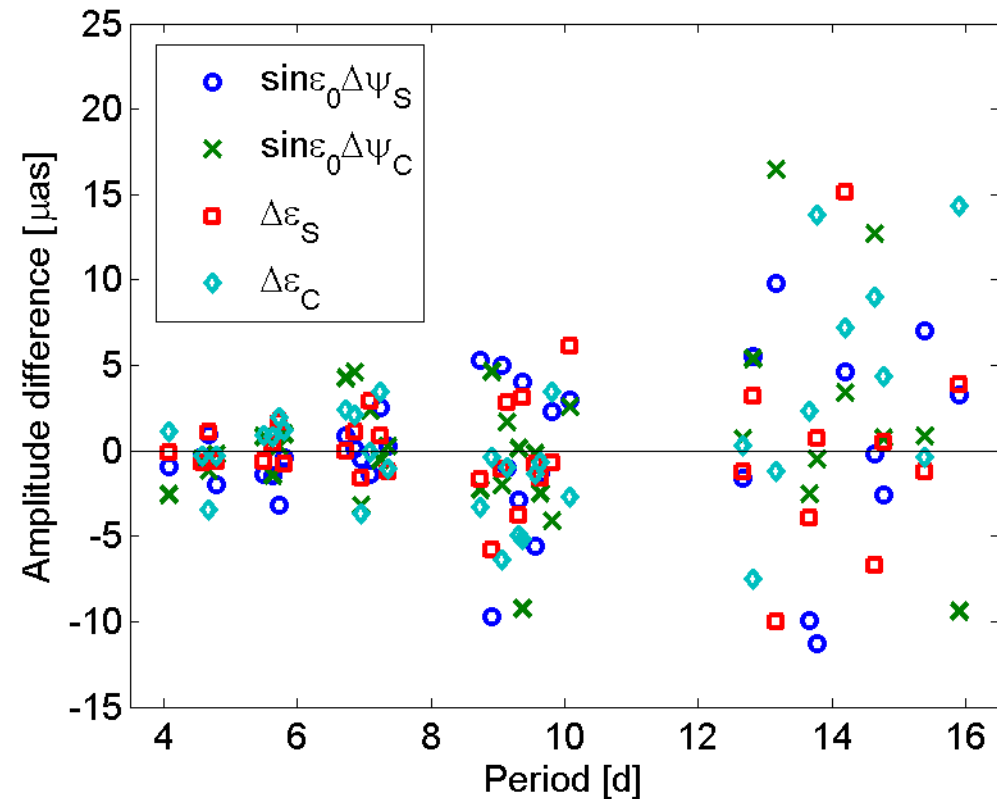
GPS vs. GPS/GLONASS

limited time interval 321/2005-366/2008

- COD: GPS+GLONASS
- CO1: GPS-only

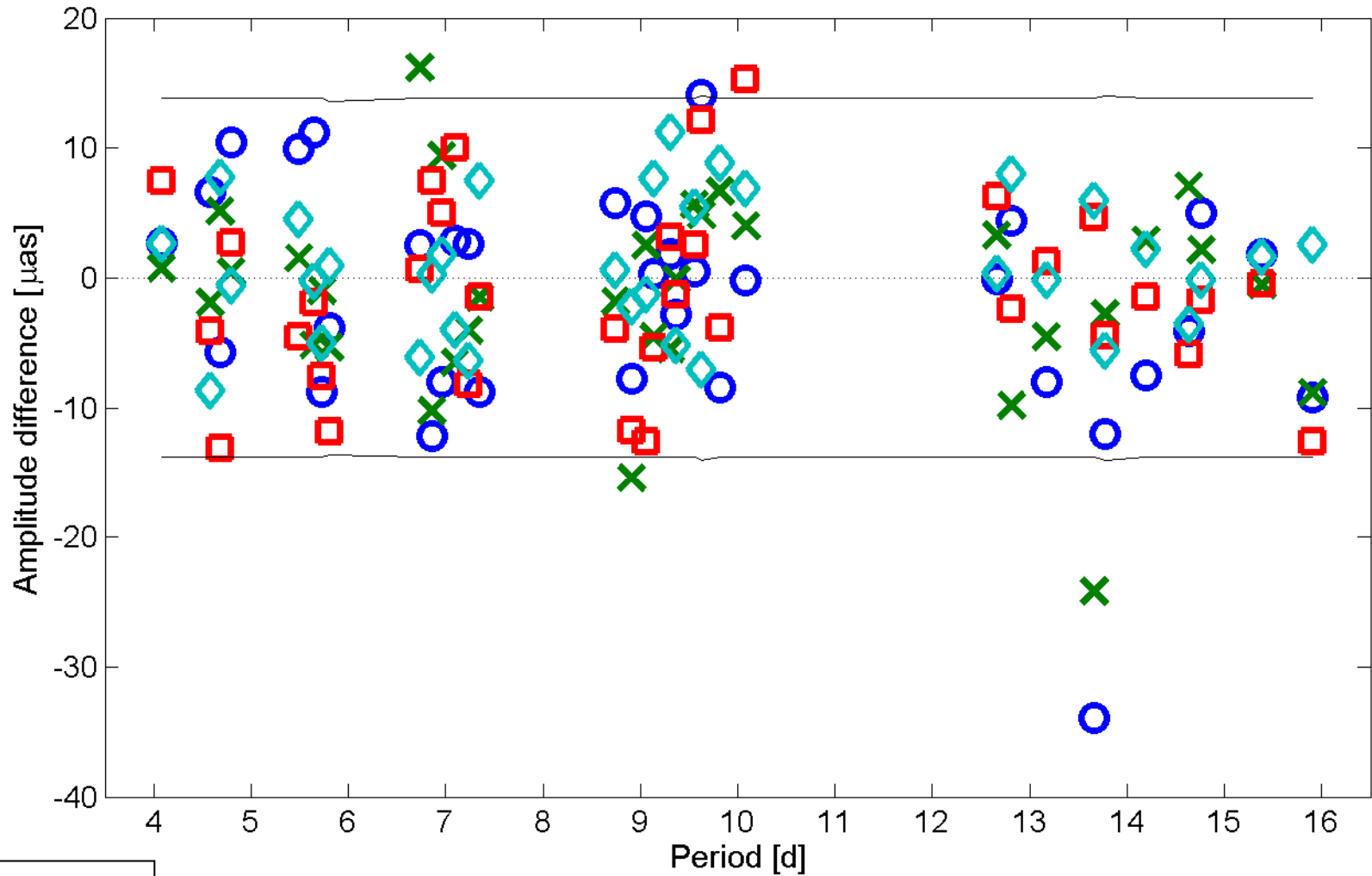


$$\overline{\Delta}_{C,S} = 4.9 \mu\text{as}$$



$$\overline{\Delta}_{C,S} = 4.6 \mu\text{as}$$

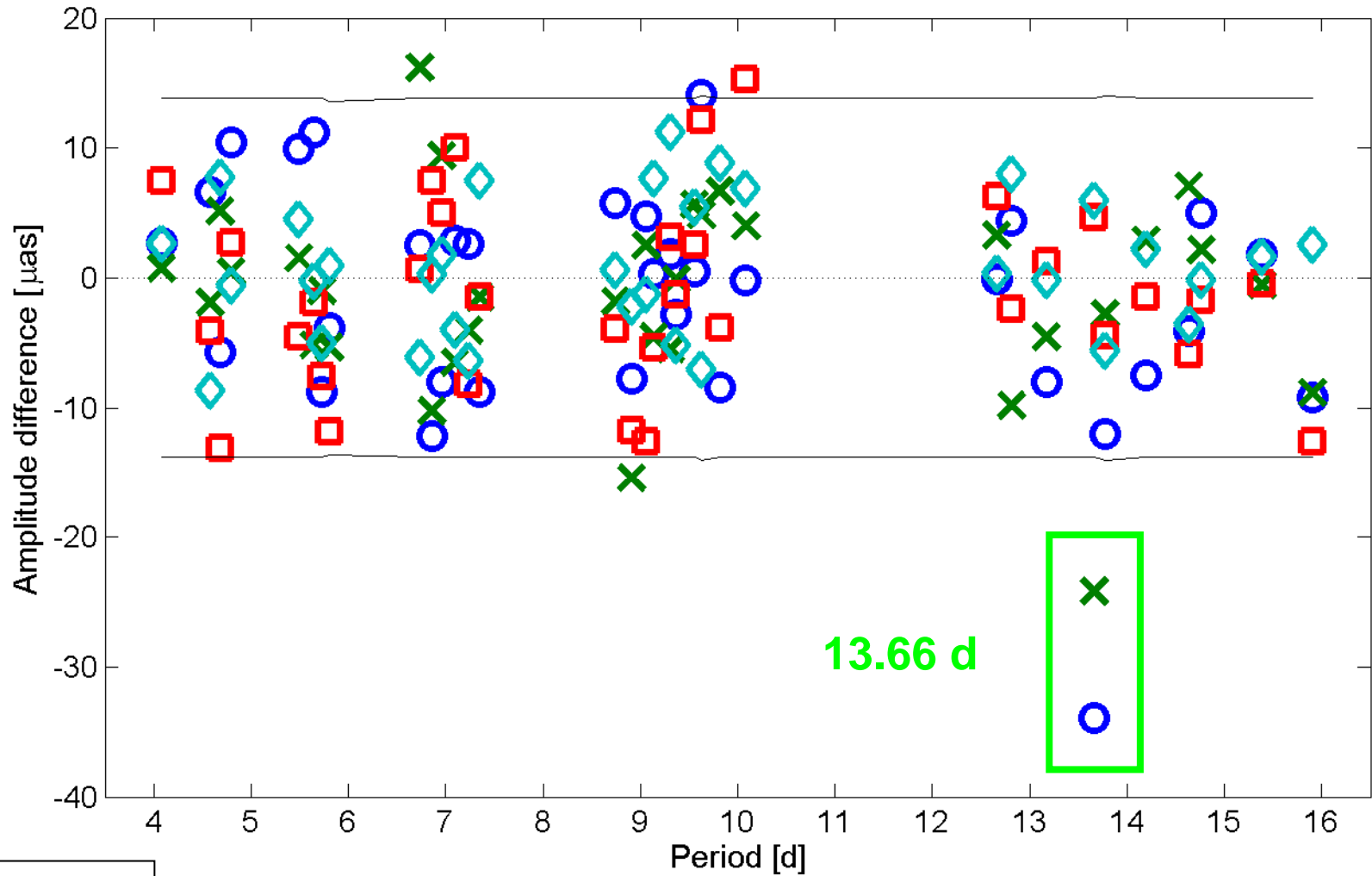
GSFC VLBI Nutation Model



Differences
to IAU2000A

○ $\sin \epsilon_0 \Delta \psi_S$
× $\sin \epsilon_0 \Delta \psi_C$
□ $\Delta \epsilon_S$
◇ $\Delta \epsilon_C$
 2 σ errors

GSFC VLBI Nutation Model



Differences
to IAU2000A



Comparison of Nutation Models

Differences between the coefficients of model 1 and model 2 in μas :

$$\overline{\Delta}_{C,S} = \sqrt{\left[\sum_{i=1}^n (\Delta\psi_1^i - \Delta\psi_2^i)^2 \cdot \sin^2 \epsilon_0 + \sum_{i=1}^n (\Delta\epsilon_1^i - \Delta\epsilon_2^i)^2 \right]} \cdot \frac{1}{4n}$$

| Model | GPS | | | VLBI | | | |
|------------|-----|------|------|------|------|------|------|
| | COD | CO1 | PD1 | DGF | GSF | GIC | IAU |
| COD | -- | 20.1 | 20.1 | 22.8 | 21.0 | 22.2 | 20.4 |
| CO1 | | -- | 4.9 | 10.0 | 9.1 | 10.3 | 7.0 |
| PD1 | | | -- | 9.1 | 8.4 | 14.2 | 6.7 |
| DGF | | | | -- | 5.9 | 3.9 | 8.2 |
| GSF | | | | | -- | 5.8 | 8.4 |
| GIC | | | | | | -- | 8.4 |

Conclusions

- **Homogeneously reprocessed** long-time series of nutation rates are a **prerequisite** for the determination of nutation with GNSS
- Still **systematic effects** from **deficiencies** of the **orbit modeling** at longer periods but **good agreement** with IAU2000A at **short periods**
- Pronounced differences between **VLBI** and **IAU2000A** for the **13.66 d period**
- GNSS and VLBI **intra-technique comparisons** agree on the level of **5 μas**
- Direct comparison of **GNSS** and **VLBI** is a **factor of two worse**
- **GNSS** and **VLBI** agree with **IAU2000A** on the level of **7 – 8 μas**
- **Improvement** of GNSS **orbit modeling** (e.g., albedo) should result in more precise and more accurate nutation rates