

*u*<sup>b</sup>

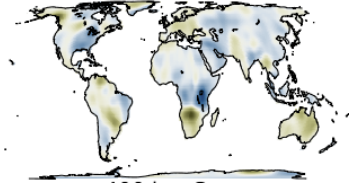
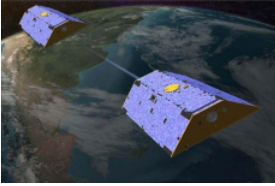
# Towards a new and consistent release of GRACE and GRACE Follow-On monthly gravity field solutions from AIUB

**Martin Lasser, Ulrich Meyer, Daniel Arnold and Adrian Jäggi**  
28<sup>th</sup> IUGG General Assembly 2023, 11 – 20 July 2023, Berlin, Germany

$u^b$

# Starting Point

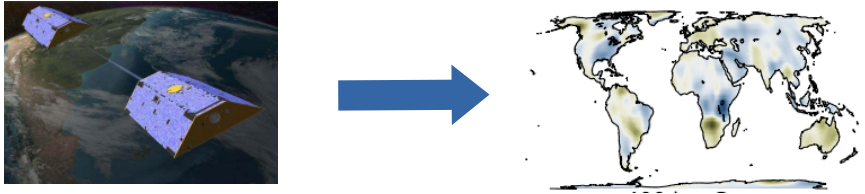
Monthly gravity fields – parametrisation



$u^b$

# Starting Point

## Monthly gravity fields – parametrisation



### Basic parametrisation

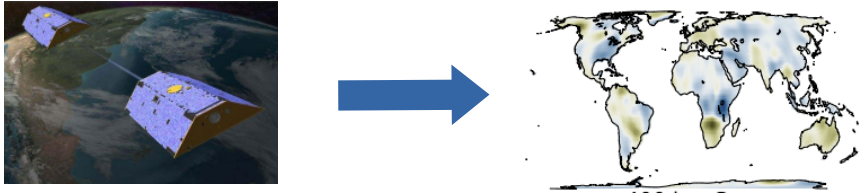
- Initial conditions  $2 \times [6]$
- Accelerometer bias  $2 \times [3] \mid [6]$
- Accelerometer scaling  $2 \times [3] \mid [9]$

Parameters per arc  $24 \mid 42$

$u^b$

# Starting Point

## Monthly gravity fields – parametrisation



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- Initial conditions  $2 \times [6]$
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Parameters per arc  $24 \mid 42$

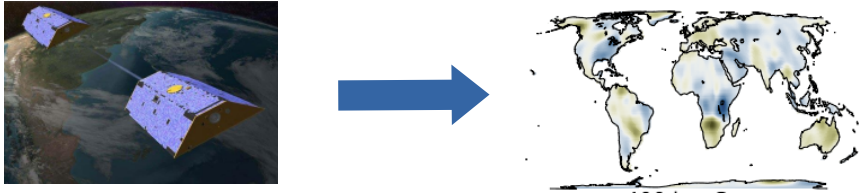
GRACE: in along-track a polynomial of order 3

GRACE: full scale matrix estimated

$u^b$ 

# Starting Point

## Monthly gravity fields – parametrisation



### Basic parametrisation

- Initial conditions  $2 \times [6]$
- Accelerometer bias  $2 \times [3] \mid [6]$
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GRACE: full scale matrix estimated

### Additional parameters

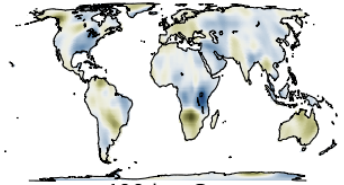
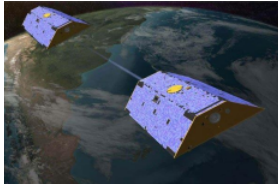
- 15 min PCA per satellite in
  - radial  $2 \times [96]$
  - along-track  $2 \times [96]$
  - cross-track  $2 \times [96]$

Parameters per arc 576

$u^b$

# Starting Point

## Monthly gravity fields – parametrisation



### Basic parametrisation

- Initial conditions 2x[6]
- Accelerometer bias 2x[3] | [6]
- Accelerometer scaling 2x[3] | [9]

Parameters per arc 24 | 42

GRACE: in along-track a polynomial of order 3

GRACE: full scale matrix estimated

### Additional parameters

- 15 min PCA per satellite in
  - radial 2x[96]
  - along-track 2x[96]
  - cross-track 2x[96]

Parameters per arc 576

in daily arcs (30 days):

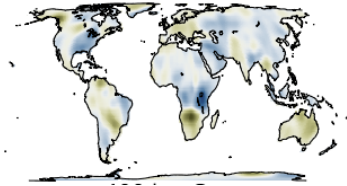
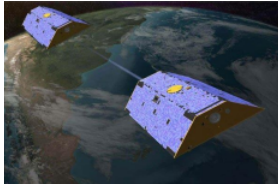
~ 18000 <orbit> parameters

+ 9405 gravity field d/o=2..96

$u^b$ 

# Starting Point

## Background force modelling



Force models

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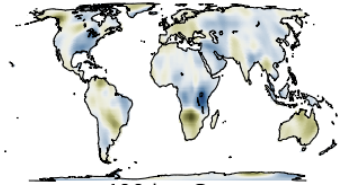
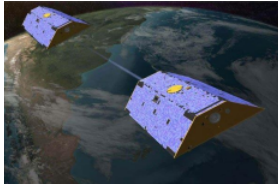
Gravity field	AIUB-GRACE03S static
Astronomic bodies	JPL DE421 (all planets)
Mean pole	Linear
Solid Earth tides	IERS2010
Solid Earth pole tides	IERS2010
Ocean tides	FES2014b (+ admittances from TUG)
Ocean pole tides	Desai
Atmospheric tides	AOD RL06
Atmospheric & oceanic dealiasing	AOD RL06
Relativistic effects	IERS2010

---

$u^b$ 

# Starting Point

## Background force modelling



Force models

Gravity field	AIUB-GRACE03S static
Astronomic bodies	JPL DE421 (all planets)
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Atmospheric & oceanic dealiasing	AOD RL06
Relativistic effects	IERS2010

??

- AOD RL07 is available:
- For now RL06 is used to keep consistency in the COST-G combination
  - RL07 is planned together with GRACE RL04 Level-1b data



$u^b$ 

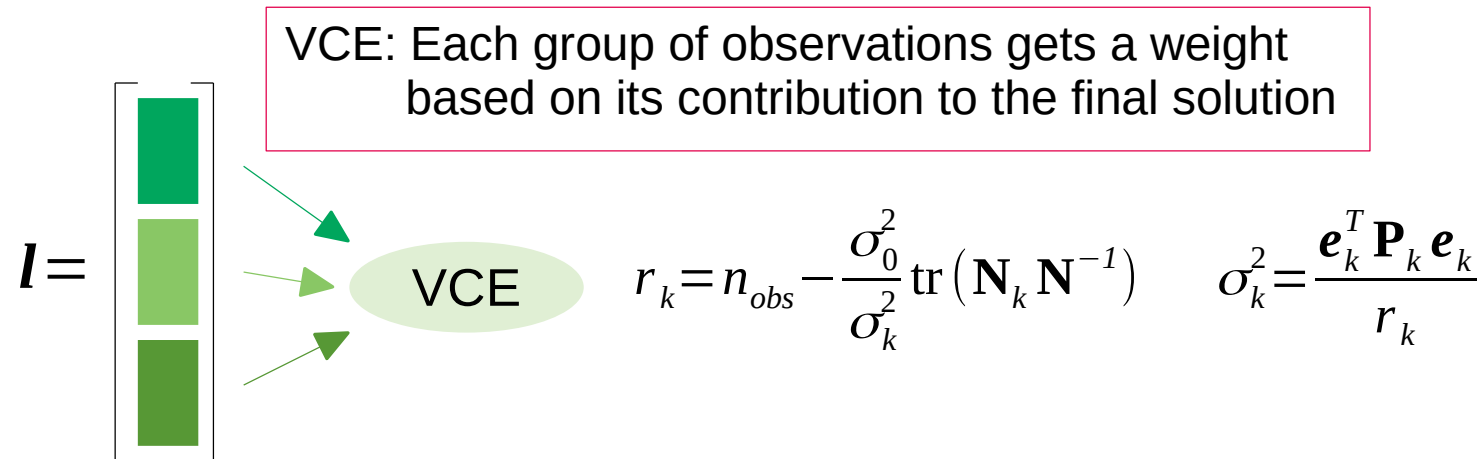
# Outlier Treatment

## Variance Component Estimation (VCE)

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A}) \quad \longrightarrow \quad \hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{b}$$

$$\mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l}$$

- The observations of each arc/block are used to set up normal equations (NEQs).
- Each arc is treated as being independent.



$u^b$ 

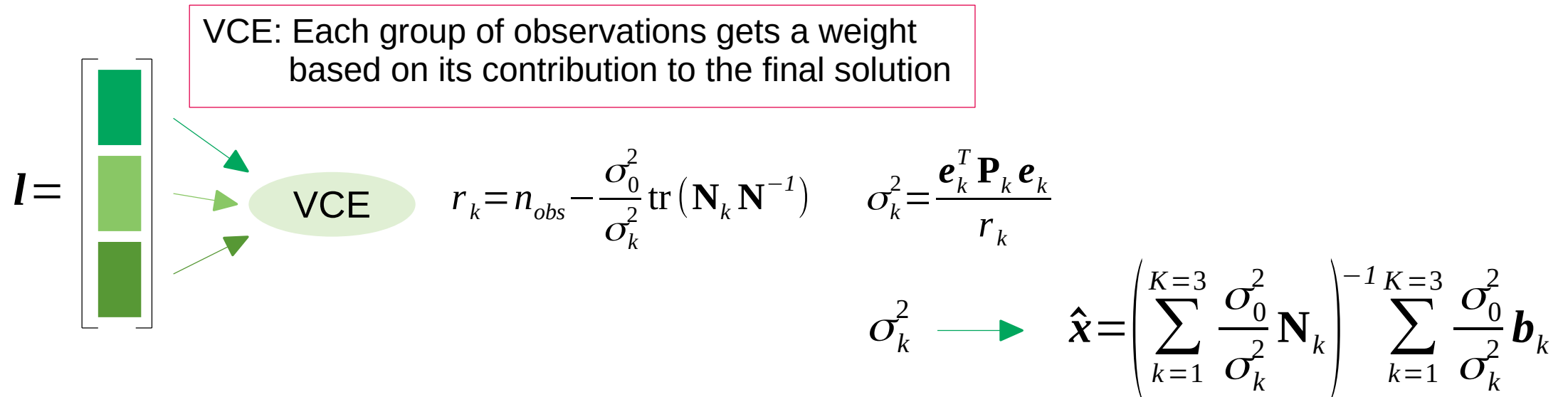
# Outlier Treatment

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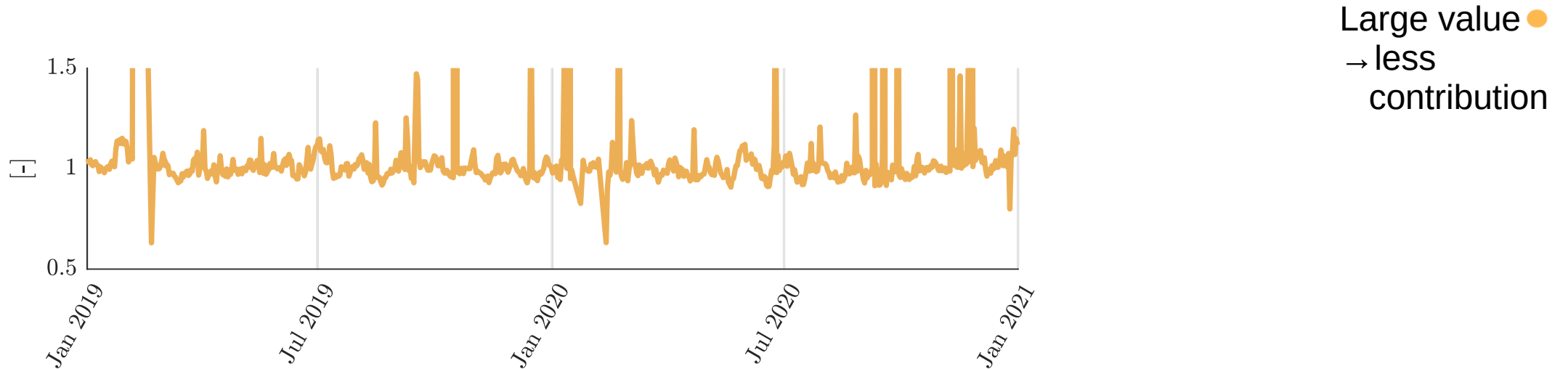
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$u^b$ 

# Outlier Treatment

## VCE – results (1)

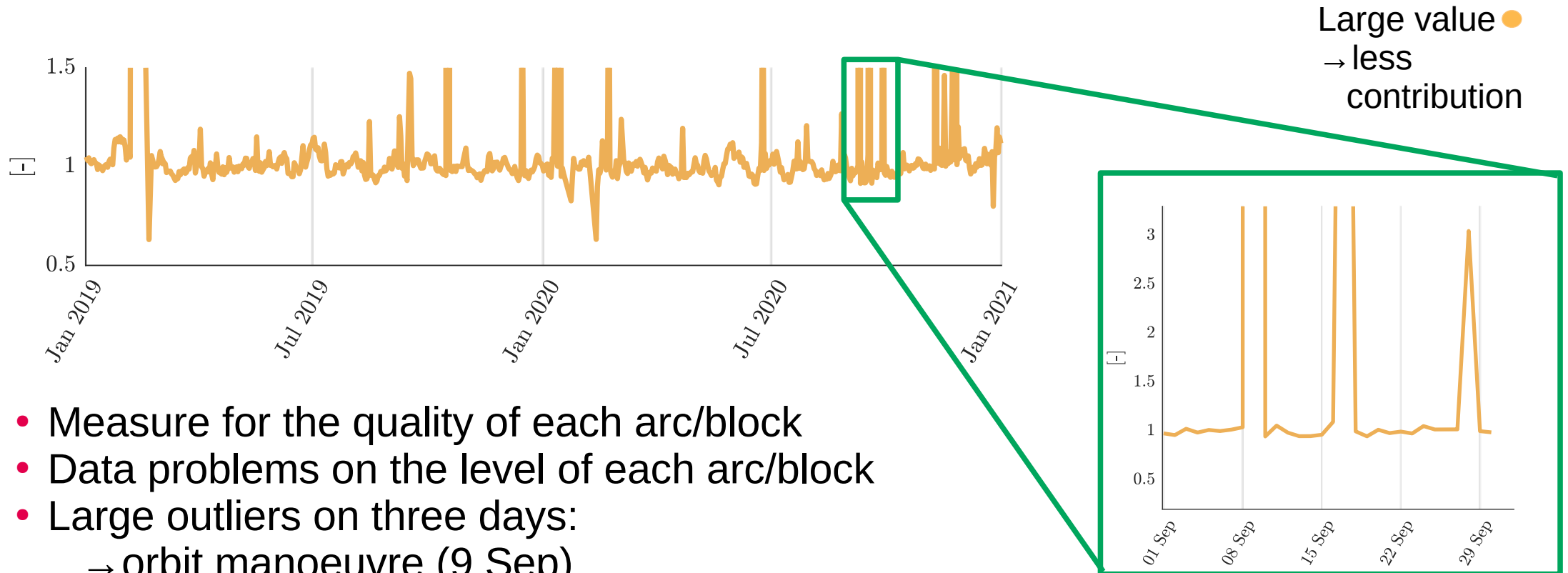


- Measure for the quality of each arc/block
- Data problems on the level of each arc/block
- Large outliers on three days:
  - orbit manoeuvre (9 Sep)
  - KBR calibration manoeuvres (17 Sep and 28 Sep)

$u^b$ 

# Outlier Treatment

## VCE – results (1)

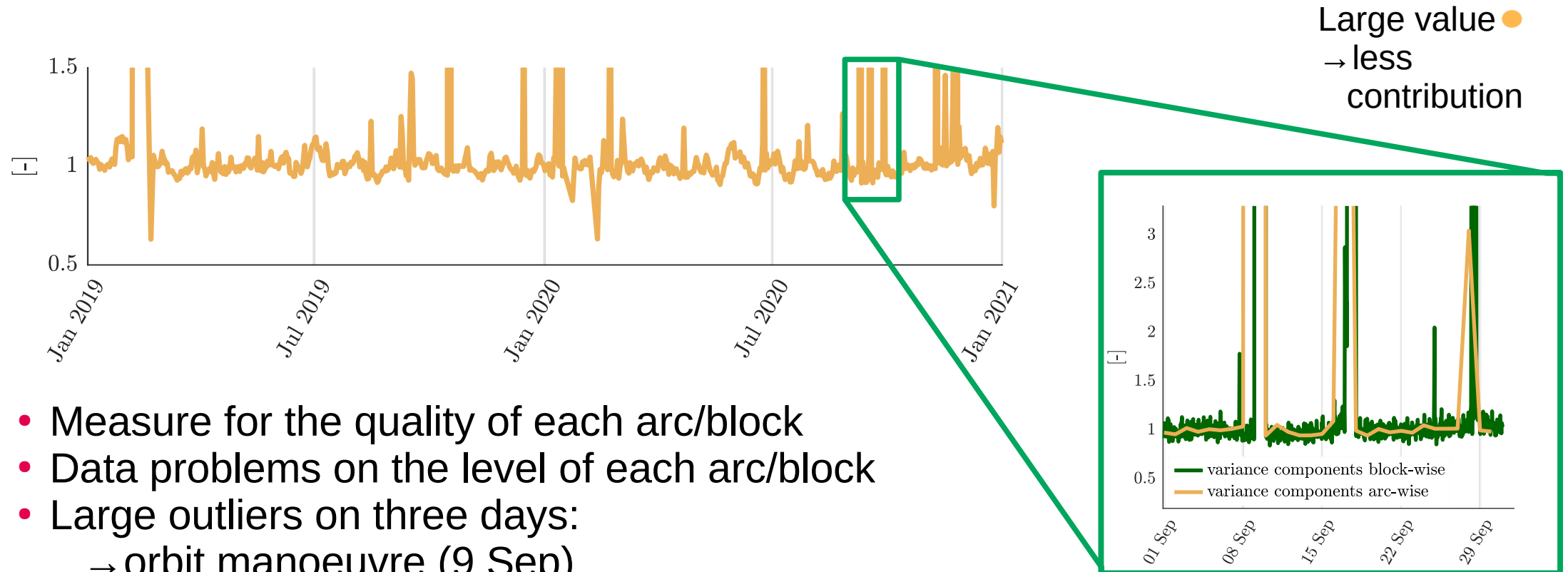


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$u^b$ 

# Outlier Treatment

## VCE – results (1)



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- Data problems on the level of each arc/block
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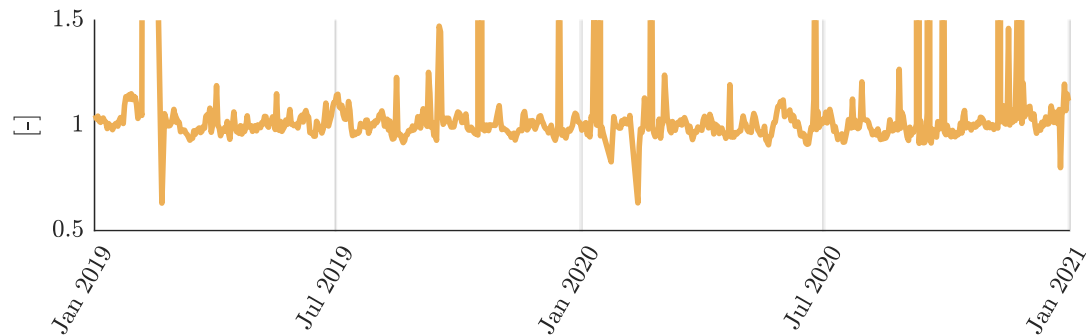
Block length of 50 min

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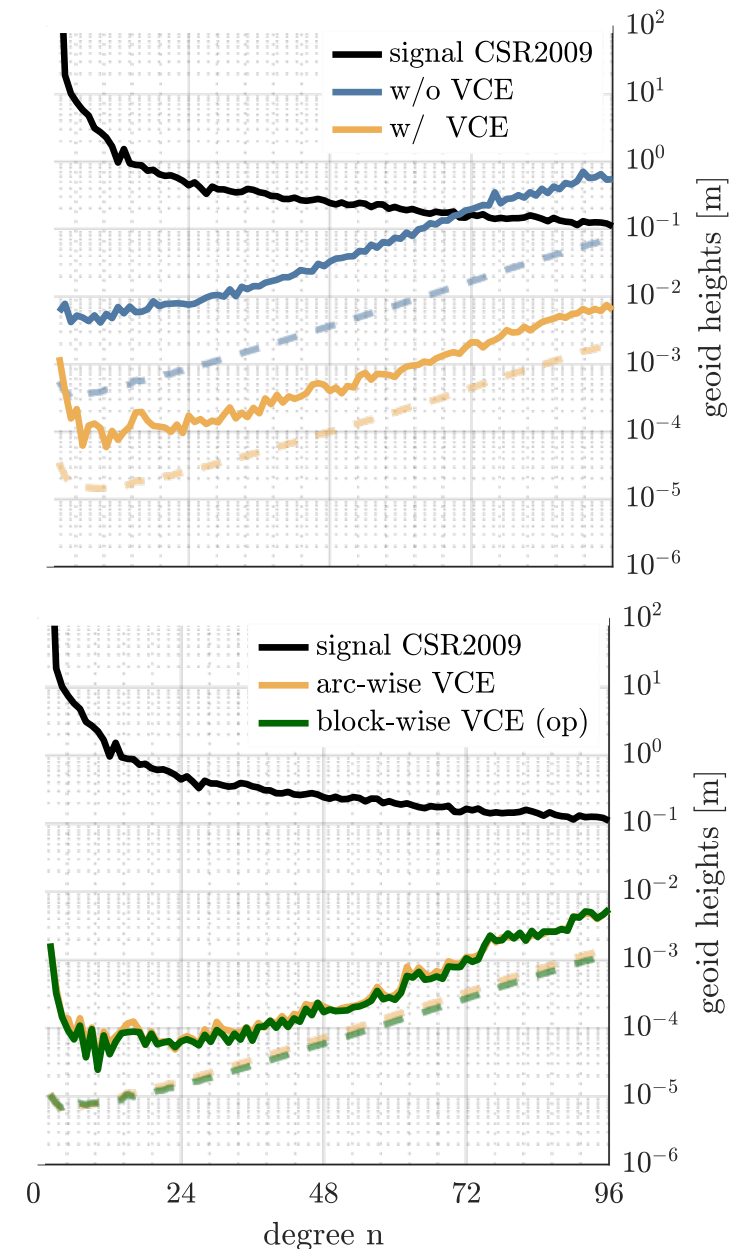
# Outlier Treatment

## VCE – results (2)

### Improvement in the gravity field solution



- Measure for the quality of each arc/block
- Data problems on the level of each arc/block
- Large outliers on three days:
  - orbit manoeuvre (9 Sep)
  - KBR calibration manoeuvres (17 Sep and 28 Sep)



$u^b$ 

# Outlier Treatment

## Robust estimators

### Weighting function

- check normalised residuals for exceeding a certain threshold
  - down-weight corresponding observation
  - Huber function, Hampel, IGG3 ...

- $m=2$ 
$$p_i = \begin{cases} \frac{p_i \cdot m}{|\hat{e}_i / \sigma_{\hat{e}_i}|} & \text{for } |\hat{e}_i / \sigma_{\hat{e}_i}| > m \\ p_i & \text{else} \end{cases}$$

$u^b$

# Outlier Treatment

## Robust estimators

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- check normalised residuals for exceeding a threshold  
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GO3p-223: Automated outlier detection with machine learning in GRACE-FO postfit residuals


Save

Info Schedule

**Abstract**

GO3p-223: Automated outlier detection with machine learning in GRACE-FO postfit residuals

J. Zbinden<sup>1</sup>, M. Lasser<sup>1</sup>, U. Meyer<sup>1</sup>, B. Panos<sup>1</sup>, D. Arnold<sup>1</sup>, A. Jäggi<sup>1</sup>.  
<sup>1</sup>University of Bern, Astronomical Institute, Bern, Switzerland.

Today 17:00 ... 

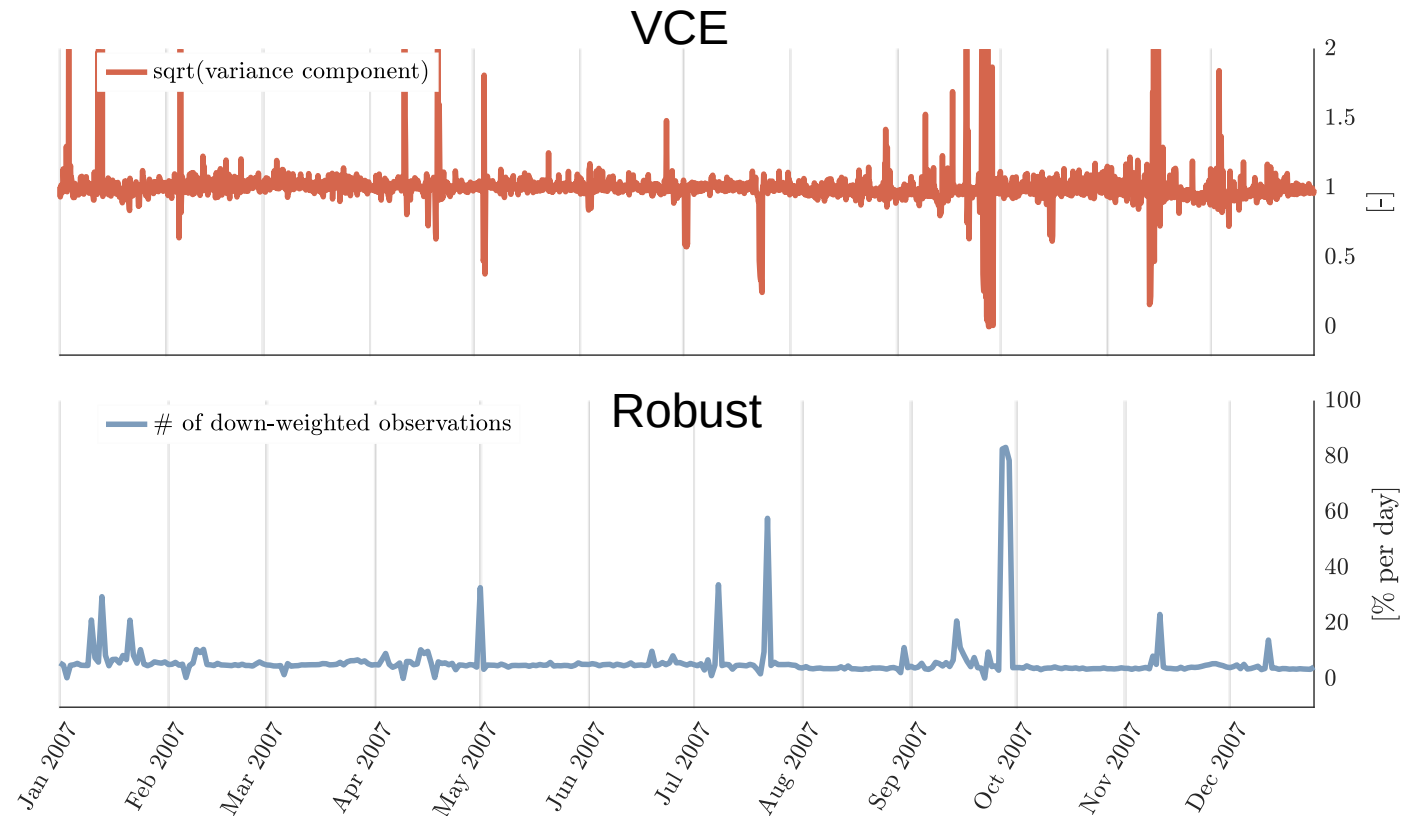
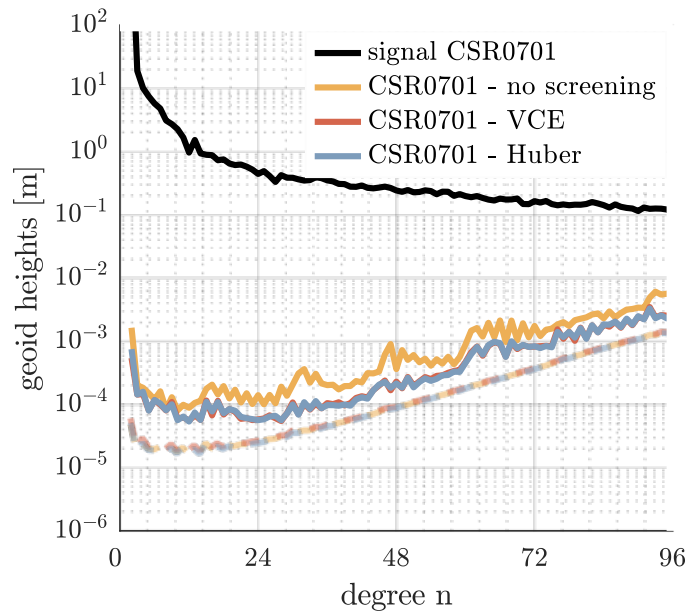


$u^b$ 

# Outlier Treatment

## Robust estimators

### Robust estimator vs. VCE



$u^b$ 

# Stochastic Noise Modelling

Empirical model from post-fit residuals

## Serial correlation of post-fit residuals

$$\hat{\mathbf{e}} = \mathbf{l} - \mathbf{A} \hat{\mathbf{x}} \quad (\text{post-fit residuals})$$

$$\text{cov}(\Delta t_k) = \frac{1}{N} \sum_{i=0}^N \hat{e}(t_i) \hat{e}(t_i + \Delta t_k)$$

- stationarity assumed
- biased estimation of auto-covariance  
→ covariance matrix nondegenerate

$u^b$ 

# Stochastic Noise Modelling

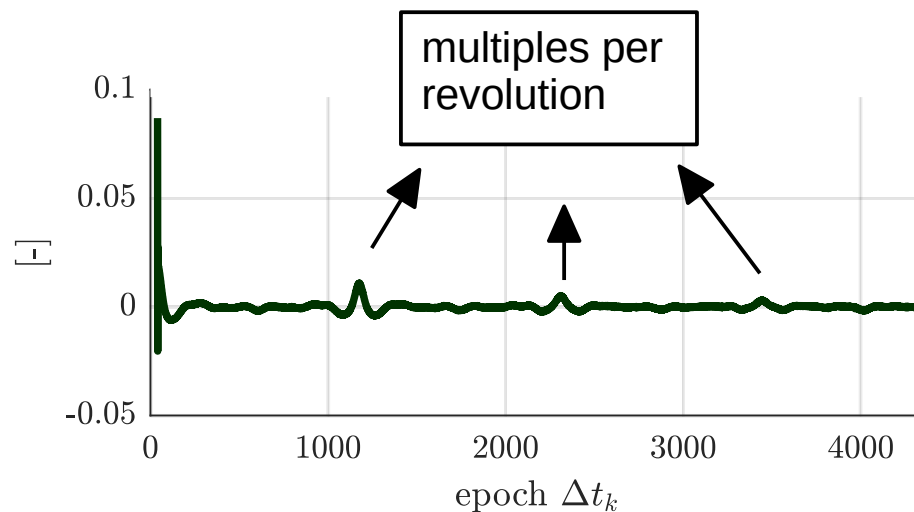
## Empirical model from post-fit residuals

### Serial correlation of post-fit residuals

$\hat{e} = l - A \hat{x}$  (post-fit residuals)

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$u^b$ 

# Stochastic Noise Modelling

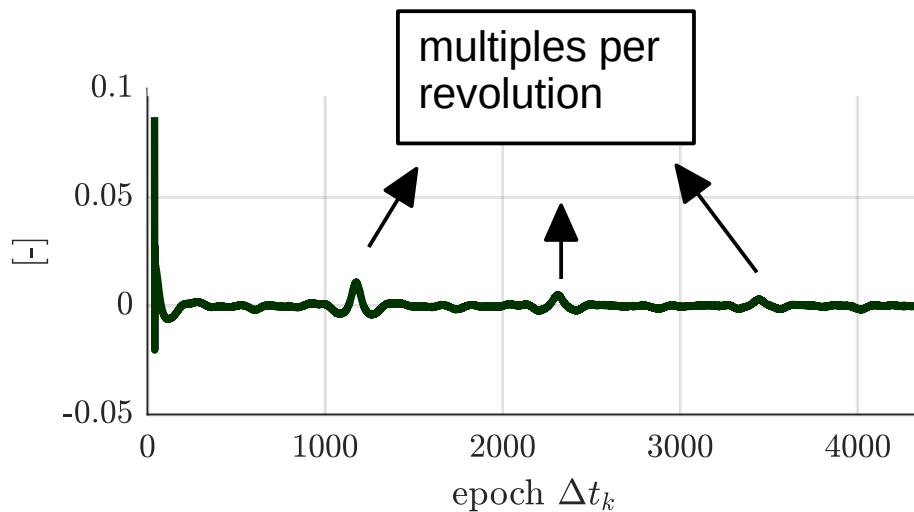
## Empirical model from post-fit residuals

### Serial correlation of post-fit residuals

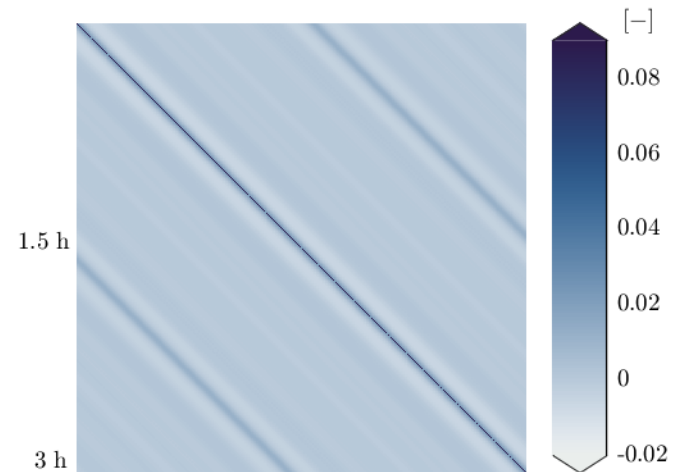
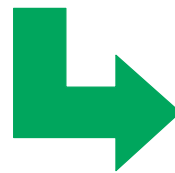
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block  
Toeplitz  
matrix

**P**

$u^b$

# Research Question

## Test scenario

Does the a priori chosen gravity field influence our monthly solutions or can we do better by co-estimating monthly solutions (up to d/o=96) together with a static component (d/o=97..160)?

$u^b$

# Research Question

## Test scenario

Does the a priori chosen gravity field influence our monthly solutions or can we do better by co-estimating monthly solutions (up to d/o=96) together with a static component (d/o=97..160)?

### Test scenario

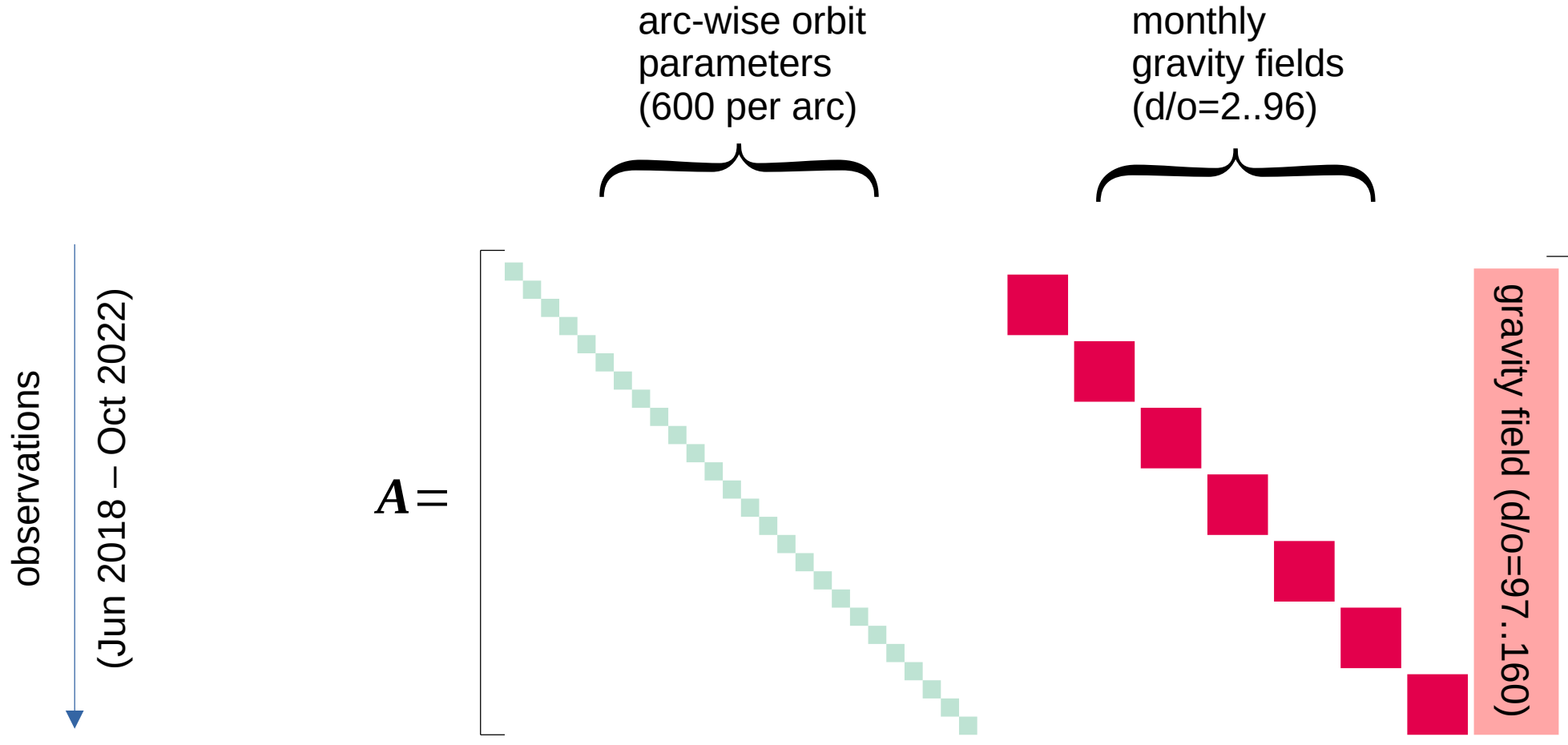
- 51 months of GRACE Follow-On (Jun 2018 – Oct 2022).
- A priori gravity field: *AIUB-GRACE-FO\_op* (d/o= 2.. 96)  
+ static *AIUB-GRACE03S* (d/o=97..160).
- With and without noise modelling from post-fit residuals.

- 1 emp monthly
- 2 stat co-est + emp monthly
- 3 stat co-est + emp full

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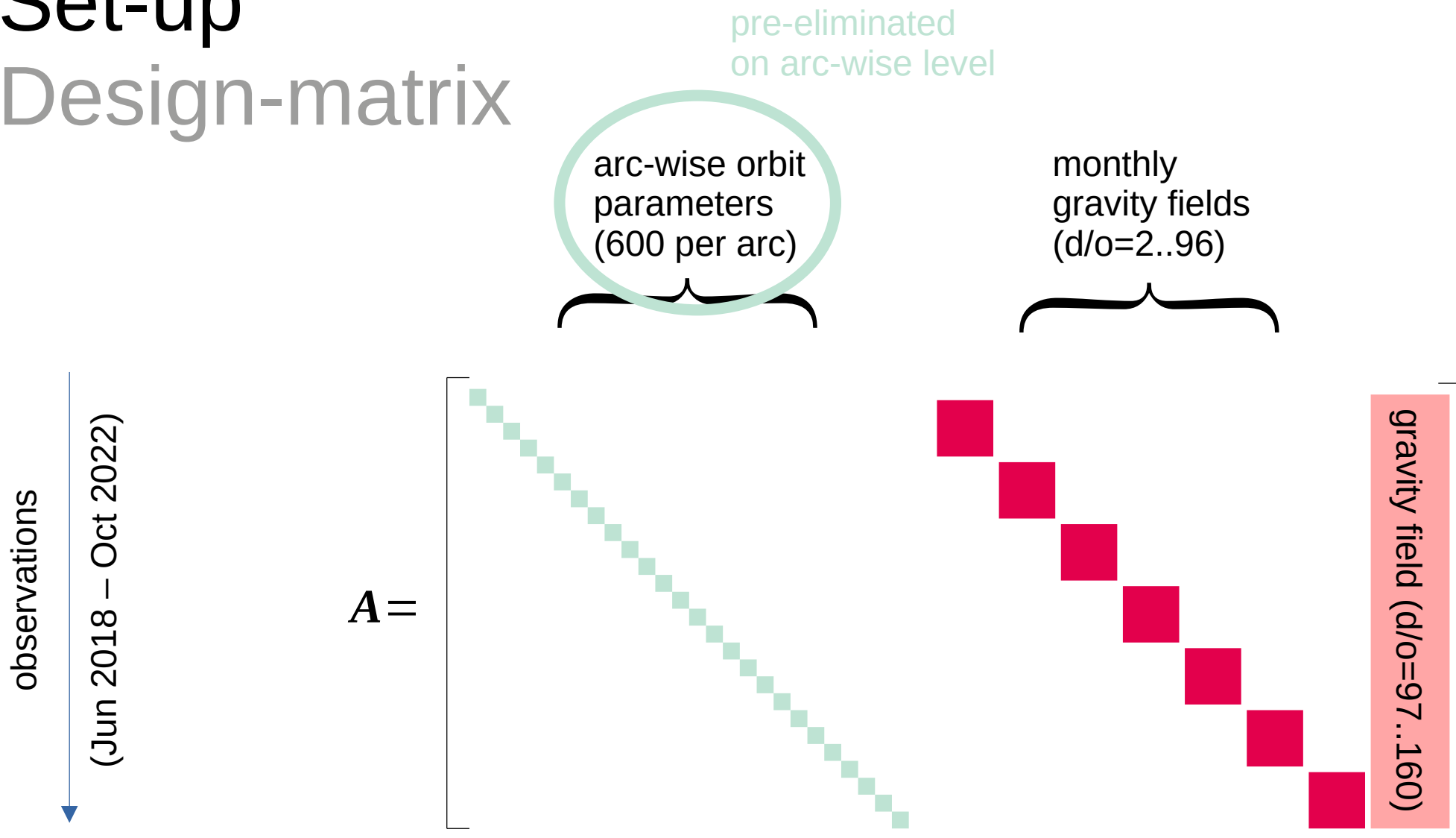
# Set-up

## Design-matrix



$u^b$

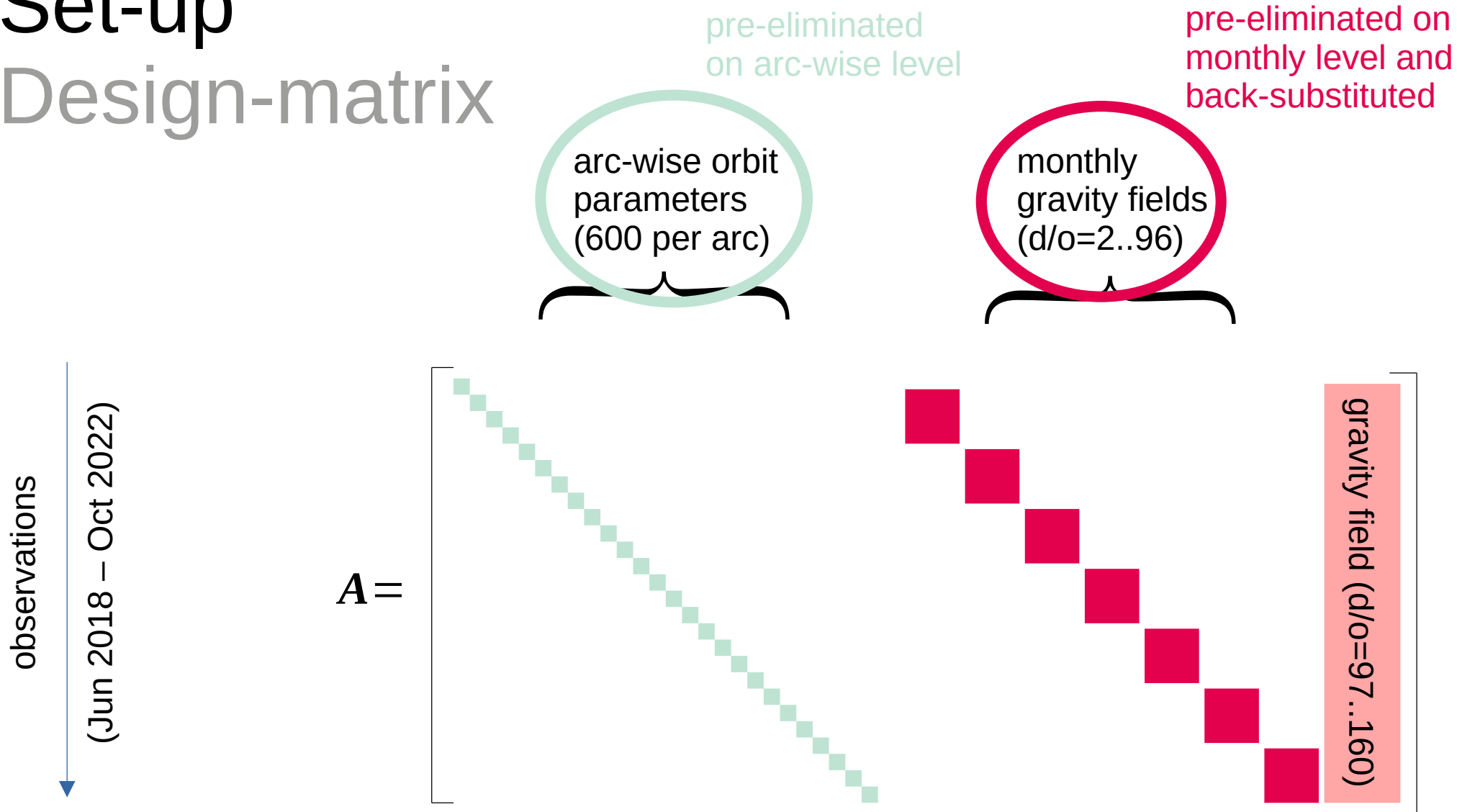
# Set-up Design-matrix





$u^b$

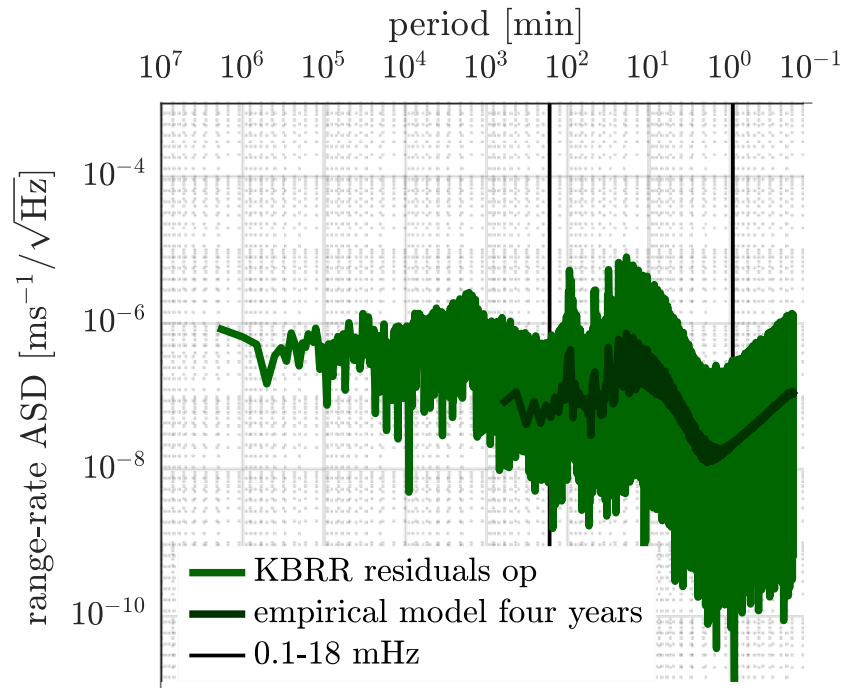
# Set-up Design-matrix



$u^b$ 

# Noise Model

Based on post-fit residuals



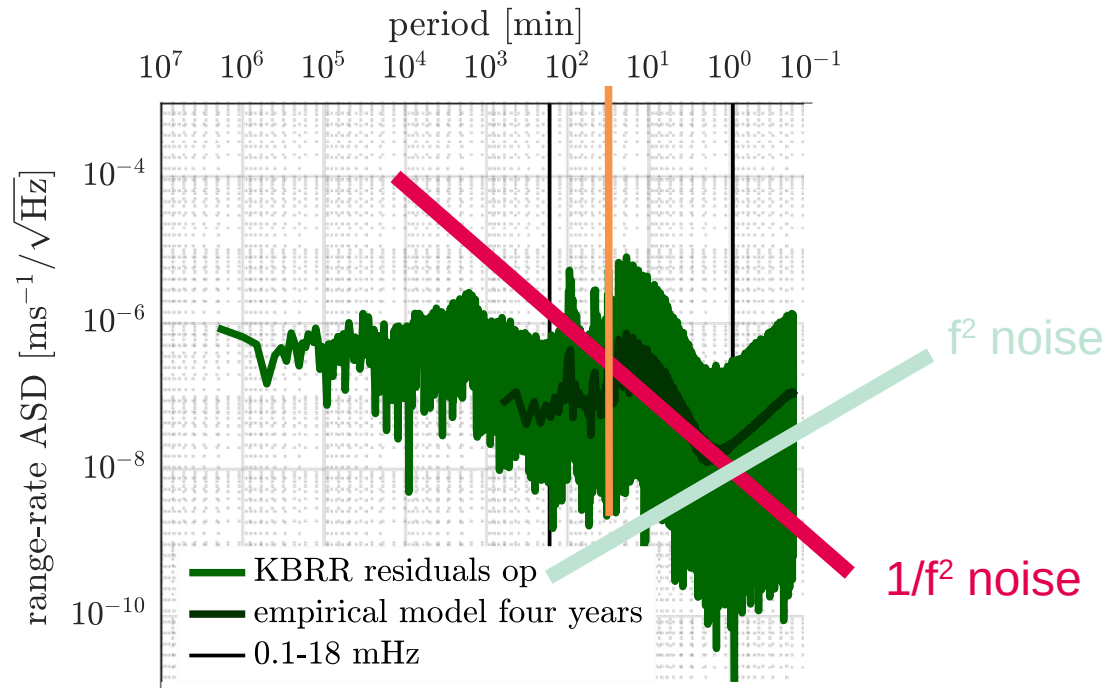
Auto covariance function → covariance matrix → weight matrix

$u^b$ 

# Noise Model

Based on post-fit residuals

15 min sampling of PCAs



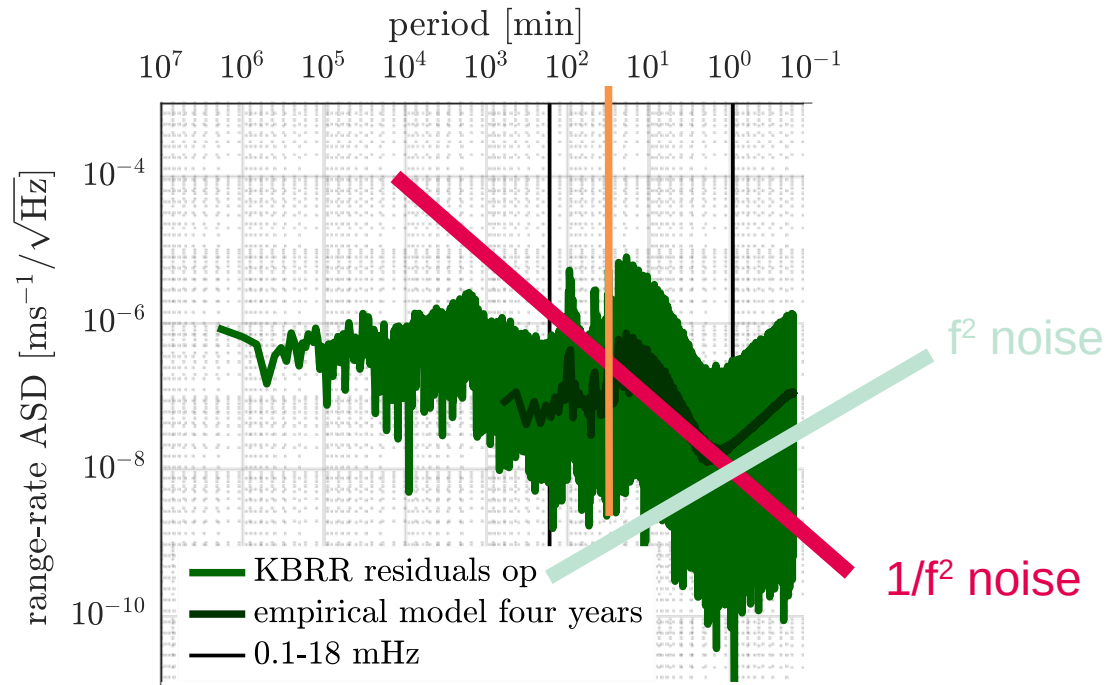
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$u^b$ 

# Noise Model

Based on post-fit residuals

15 min sampling of PCAs

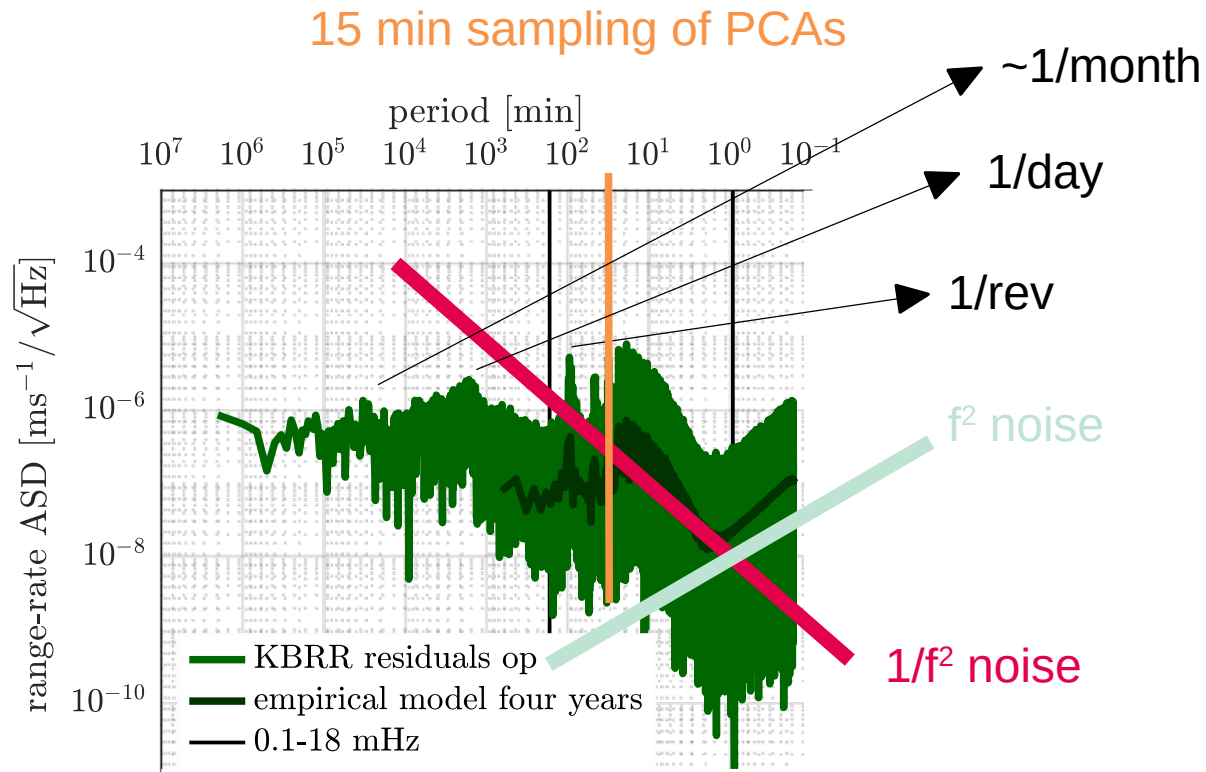


Auto covariance function → covariance matrix → weight matrix

$u^b$ 

# Noise Model

Based on post-fit residuals



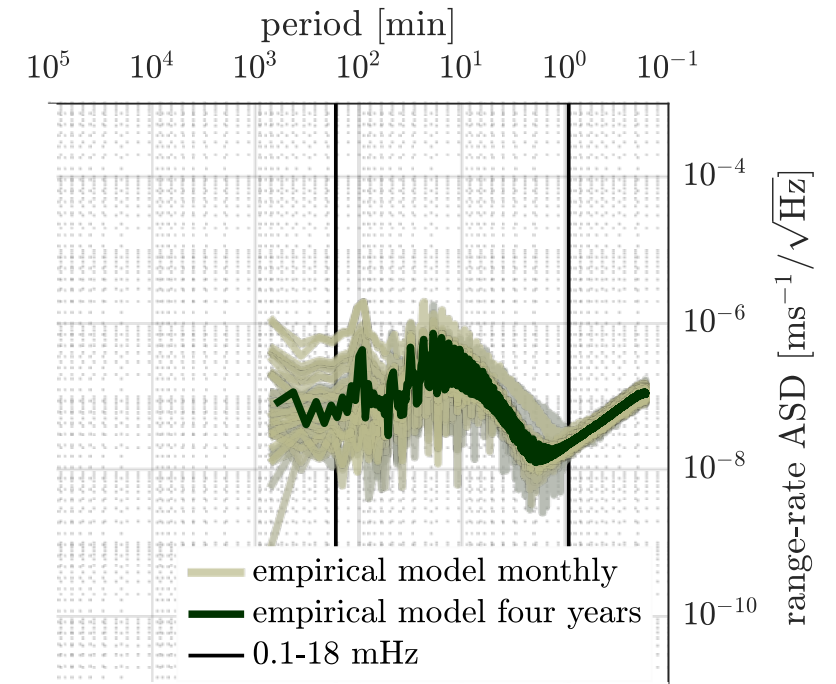
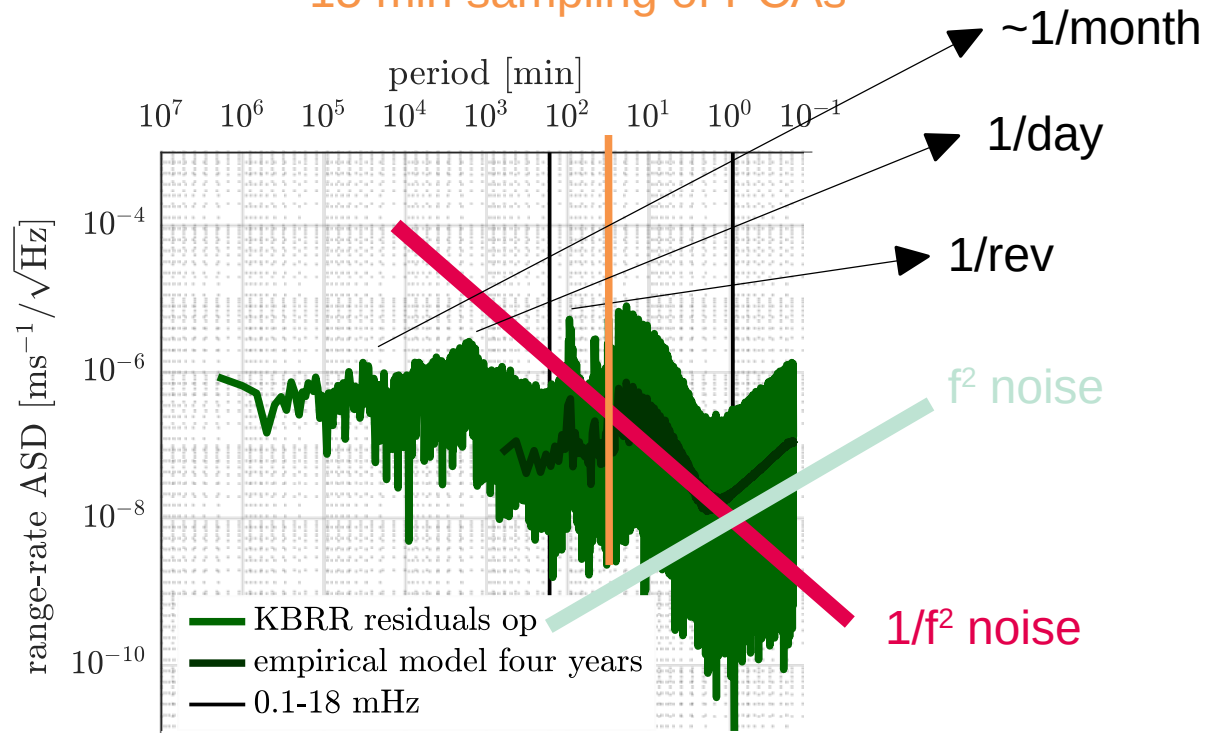
Auto covariance function → covariance matrix → weight matrix

$u^b$ 

# Noise Model

## Based on post-fit residuals

15 min sampling of PCAs



Auto covariance function  $\rightarrow$  covariance matrix  $\rightarrow$  weight matrix

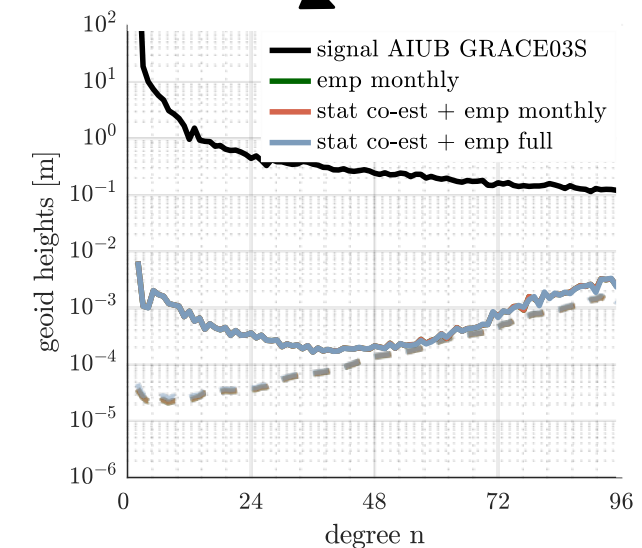
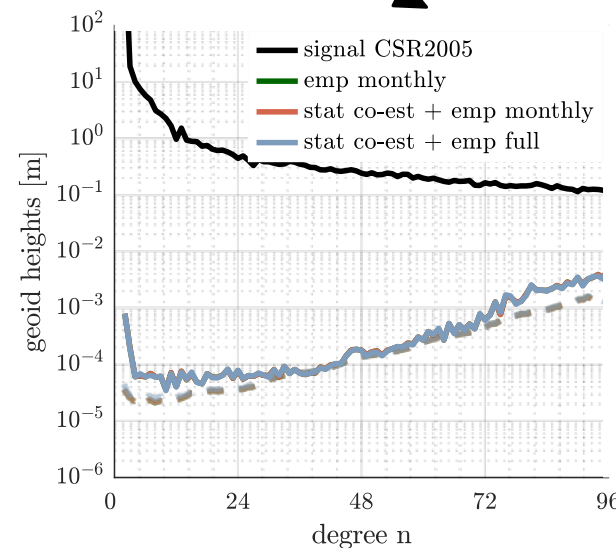
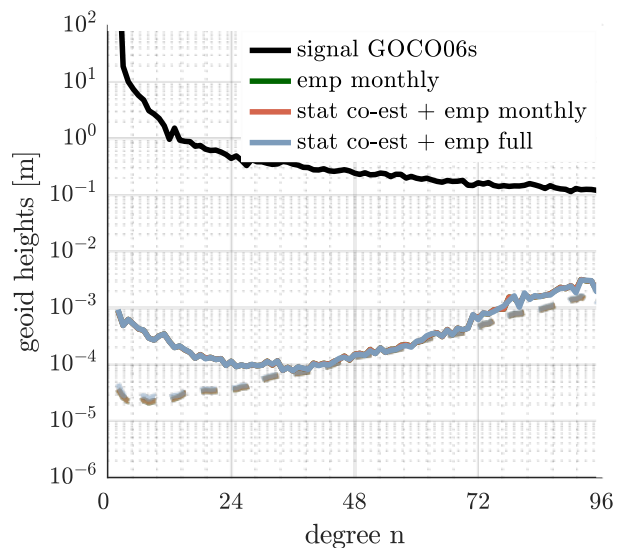
$u^b$ 

# Results

## Time-variable gravity field

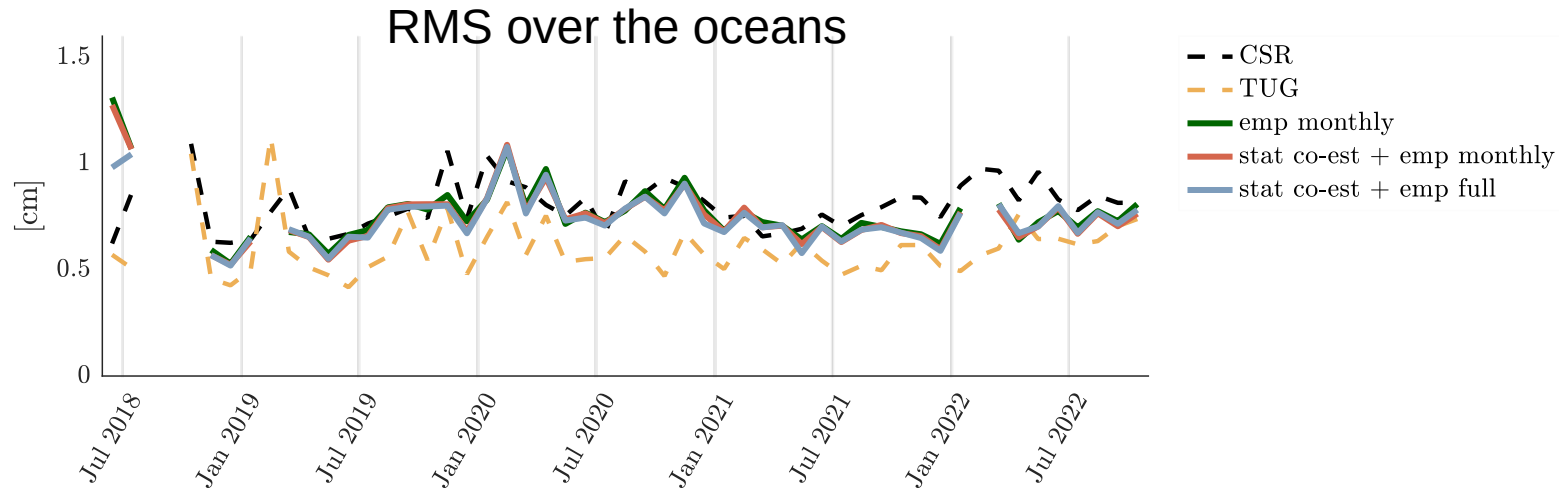
- Reference to CSR, AIUB GRACE03S and GOCO06s
- differences negligible

- 1 emp monthly
- 2 stat co-est + emp monthly
- 3 stat co-est + emp full

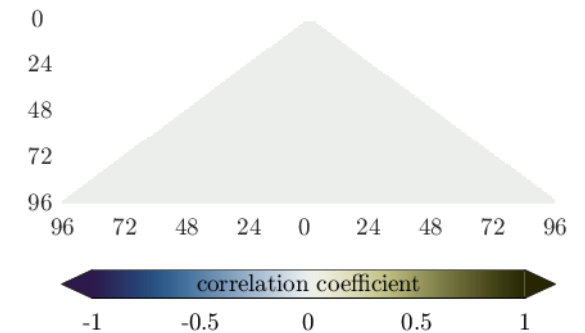


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# Results – Noise evaluation



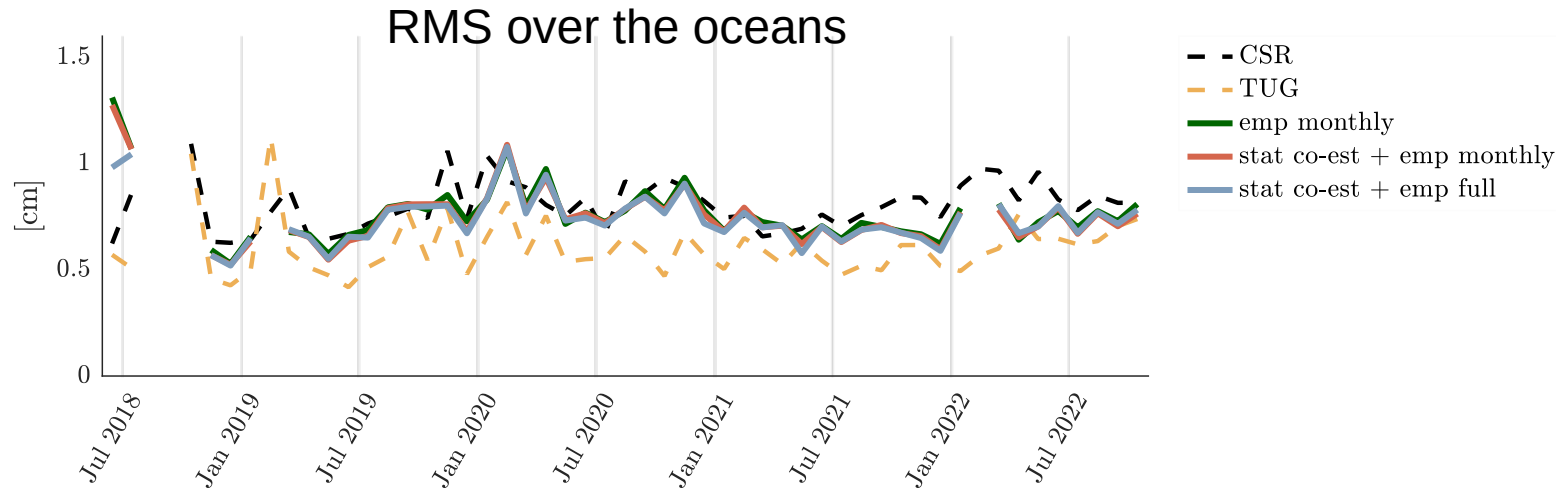
max. difference  
(w/ emp)  
= 3.51%





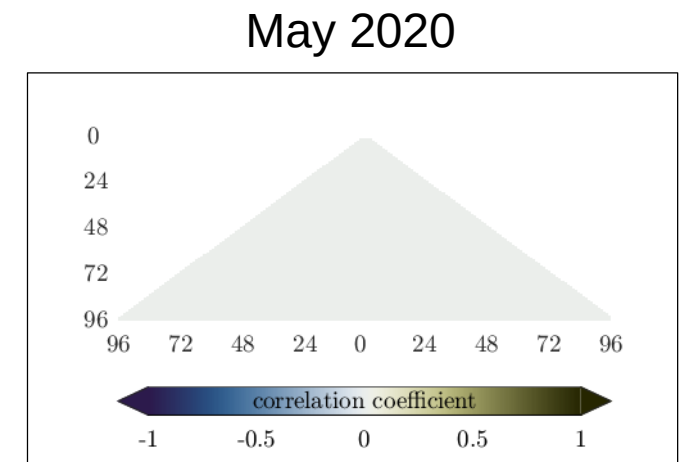
$u^b$ 

# Results – Noise evaluation



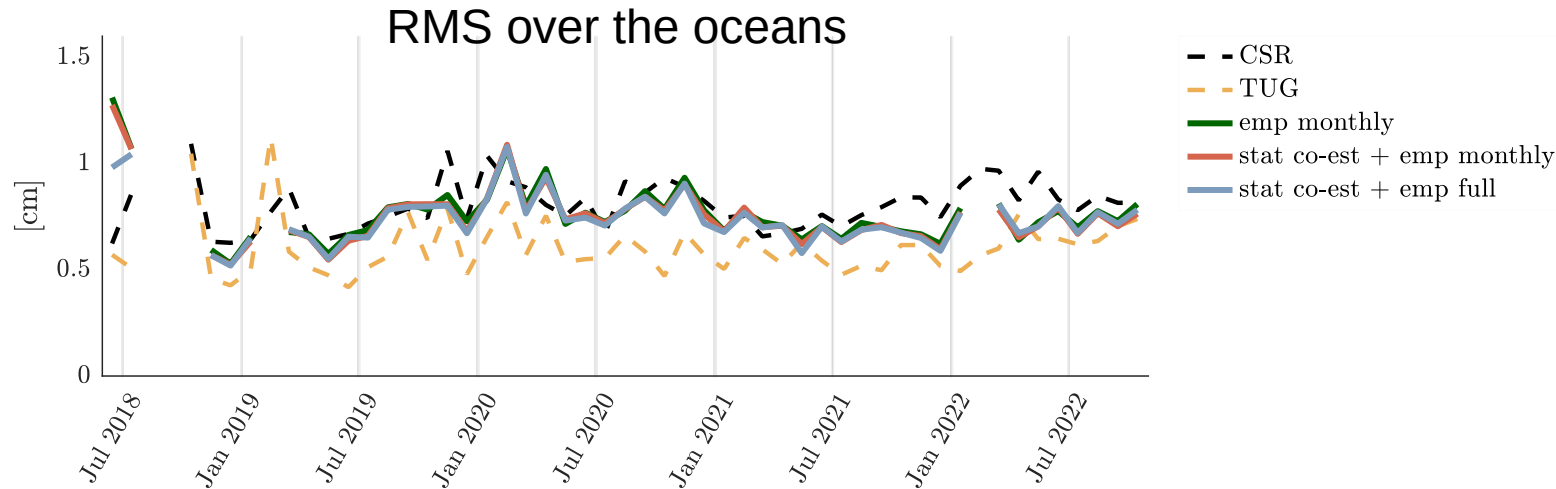
max. difference  
(w/ emp)  
= 3.51%

Formal correlation between  
<time-variable> coefficients  
and static coefficients  
(mean/minmax)



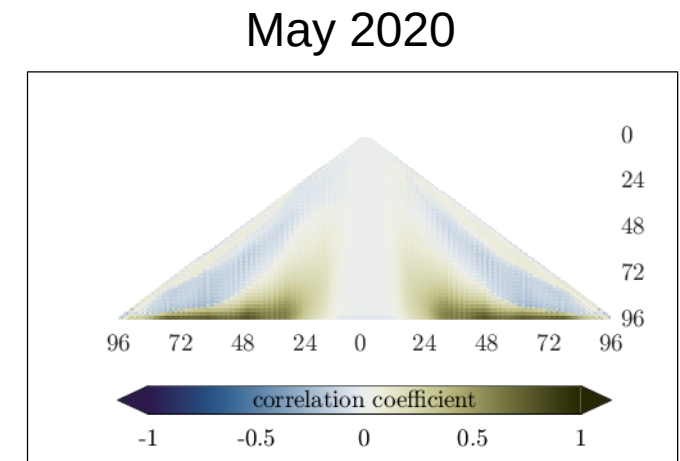
$u^b$ 

# Results – Noise evaluation



max. difference  
(w/ emp)  
= 3.51%

Formal correlation between  
<time-variable> coefficients  
and static coefficients  
(mean/minmax)



$u^b$

# Conclusions

## Co-estimation of a static gravity field

Co-estimation of a static gravity field solution from four years of GRACE Follow-On data

- Co-estimated monthly gravity field solutions
  - no significant difference to be found  
(even though covariance function varies)
  - for now not worth the time and effort
  - using monthly or averaged models reasonable  
(at the current level of precision)

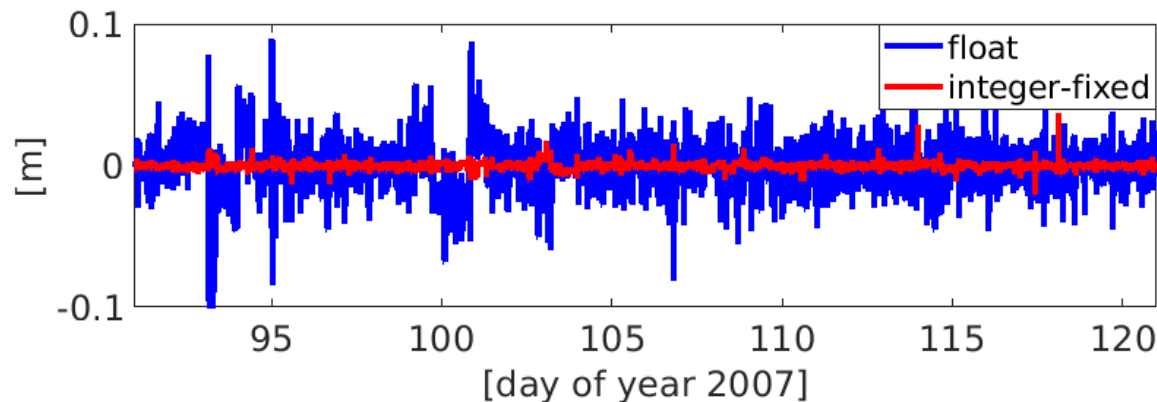
$u^b$ 

# Influence of kinematic orbits

## Ambiguity resolution

### Kinematic positions from a Precise Point Positioning (PPP)

- Provide the absolute position of the satellites in space:
  - original observation: GPS carriers phase tracking  
Attitude – antenna phase centre
  - GPS constellation
  - Ambiguity resolution either as float or fixed



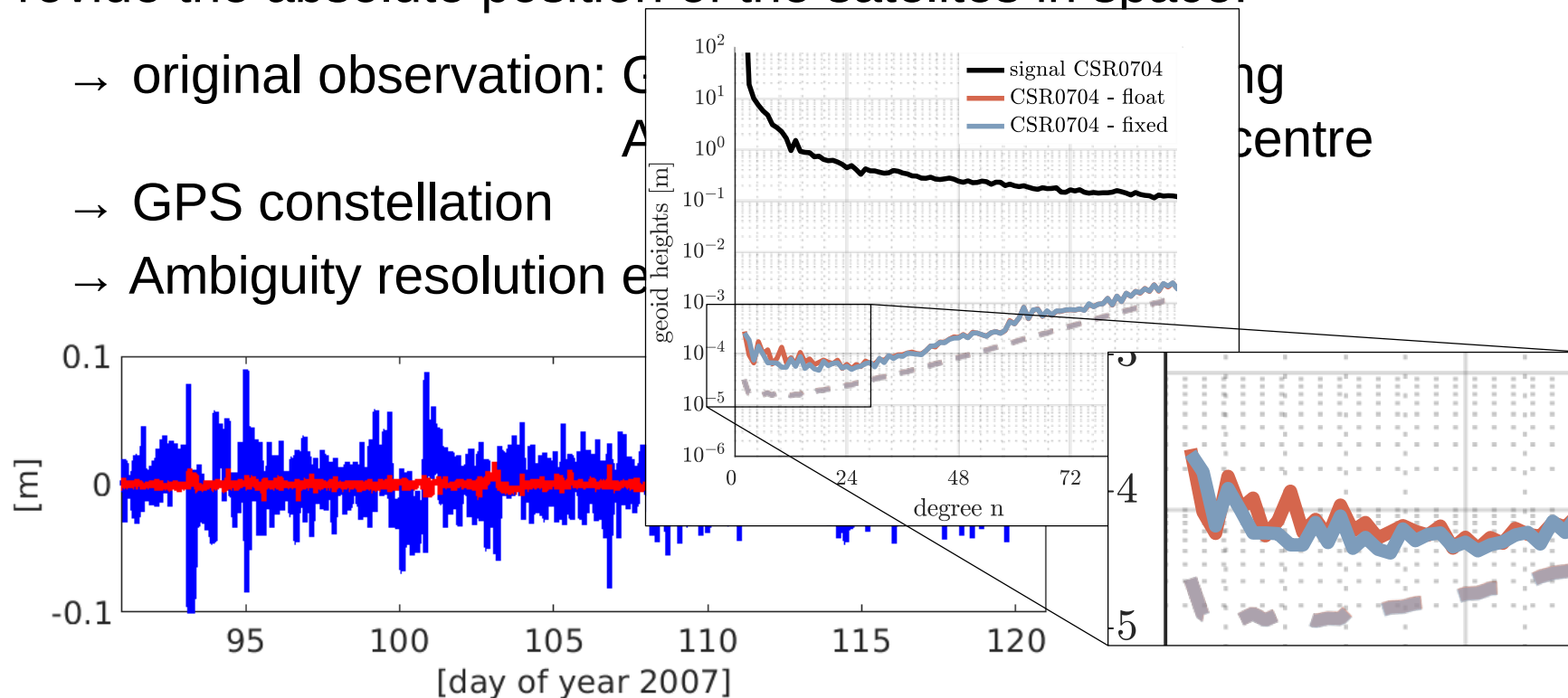
$u^b$ 

# Influence of kinematic orbits

## Ambiguity resolution

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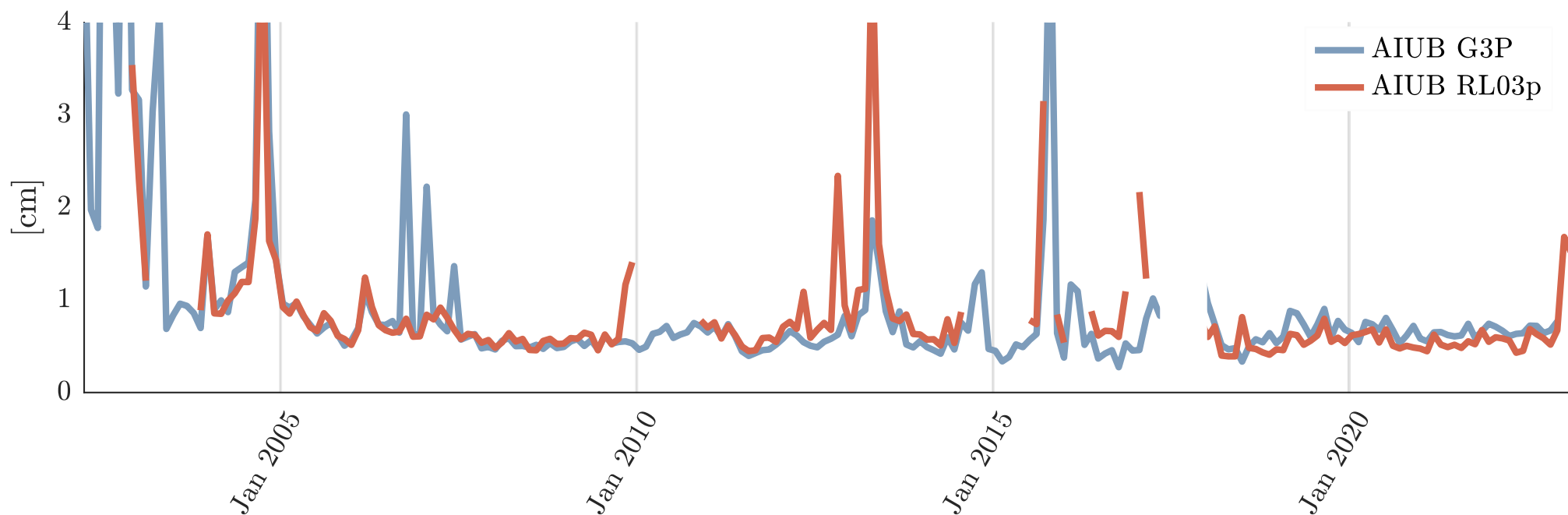
- Provide the absolute position of the satellites in space:



$u^b$

# How does it look for now...

## Noise evaluation

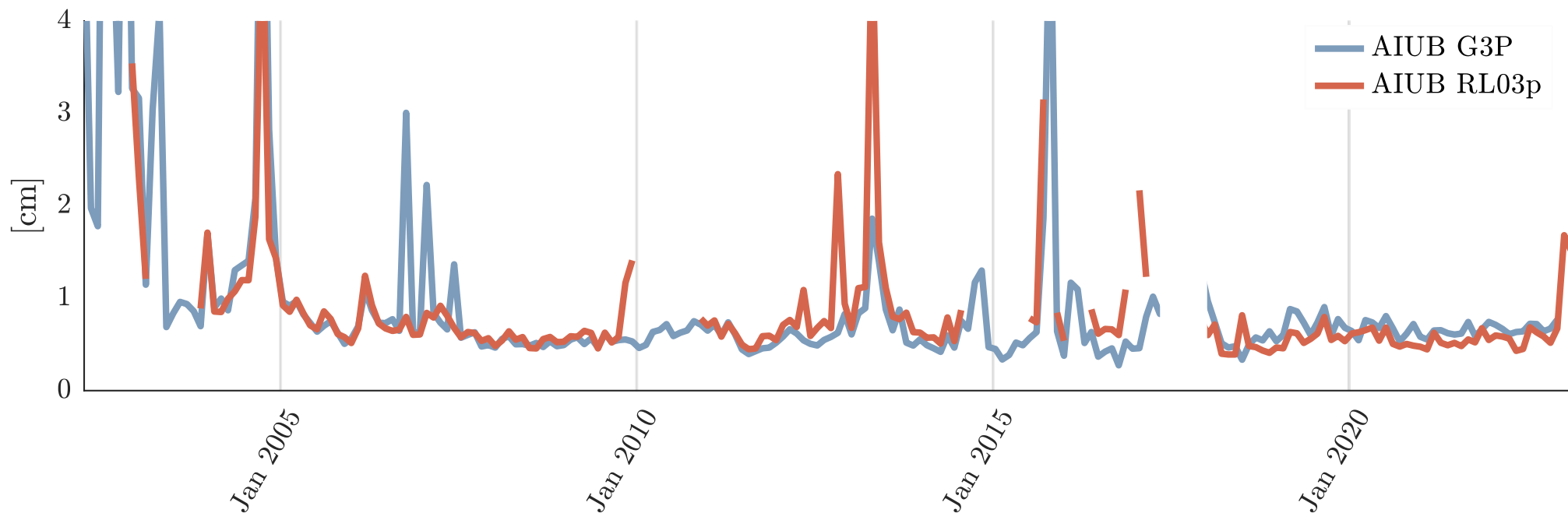


$u^b$

# How does it look for now...

## Noise evaluation

...

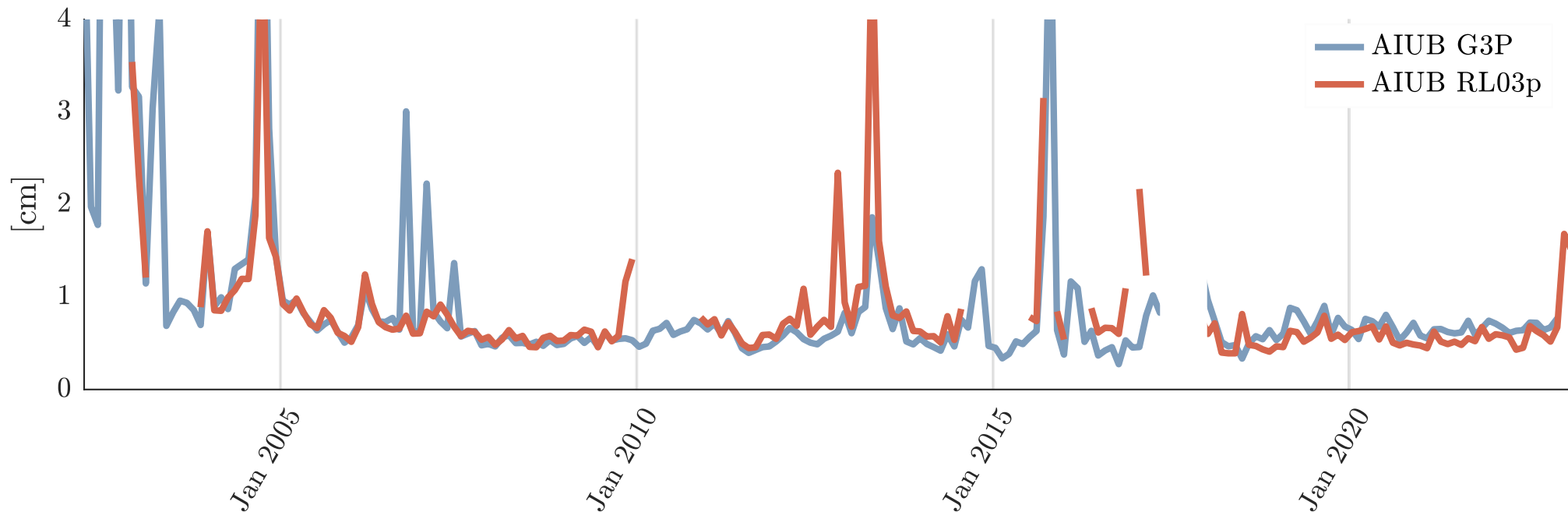


$u^b$

# How does it look for now...

## Noise evaluation

...  
→ several months missing (for now)





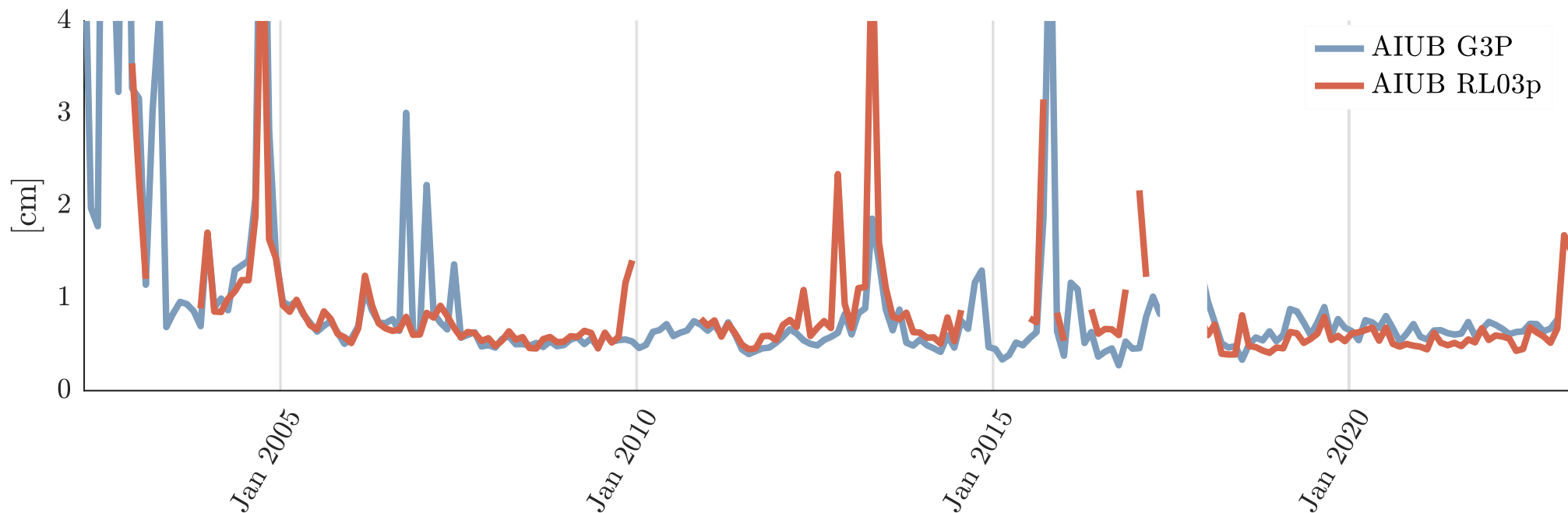
$u^b$

# How does it look for now...

## Noise evaluation

...

- several months missing (for now)
- processing efficiency to be improved



*u*<sup>b</sup>

# Thank you for your attention

## Contact

Martin Lasser

[martin.lasser@unibe.ch](mailto:martin.lasser@unibe.ch)

# References

## A

Beutler, G., Jäggi, A., Mervart, L. and Meyer, U. [2010]: The celestial mechanics approach: theoretical foundations. *Journal of Geodesy*, vol. 84(10), pp. 605-624. <https://doi.org/10.1007/s00190-010-0401-7>

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