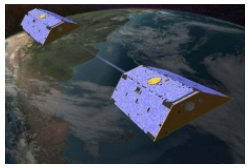
Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling

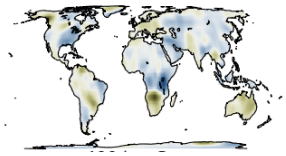
Martin Lasser, Ulrich Meyer, Daniel Arnold, Adrian Jäggi

Astronomical Institute, University of Bern, Switzerland

GRACE Follow-On Science Team Meeting 2022
18 October 2022
Potsdam, Germany

Operational processing





Basic parametrisation

▪ initial conditions	2x[6]
▪ accelerometer bias	2x[3]
▪ accelerometer scaling	2x[3]
parameters per arc	24

Additional parameters

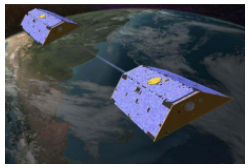
▪ 15 min PCA per satellite in	
➔ radial	2x[96]
➔ along-track	2x[96]
➔ cross-track	2x[96]
parameters per arc	576

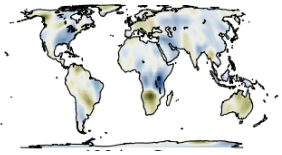
in daily arcs (30 days):

- 18000 parameters,
- 17280 for the noise model
- + gravity field

→ frequently used in the Celestial Mechanics Approach [Beutler et al., 2010]

Operational processing





Basic parametrisation

▪ initial conditions	2x[6]
▪ accelerometer bias	2x[3]
▪ accelerometer scaling	2x[3]
parameters per arc	24

Additional parameters

▪ 15 min PCA per satellite in	
➔ radial	2x[96]
➔ along-track	2x[96]
➔ cross-track	2x[96]
parameters per arc	576

in daily arcs (30 days):
- 18000 parameters,
- 17280 for the noise model
- + gravity field

Force models	
Gravity field	Internal AIUB static GRACE field
Astromomic bodies	JPL DE421 (all planets + Pluto)
Mean pole	Linear
Solid Earth tides	IERS2010
Solid Earth pole tides	IERS2010
Ocean tides	FES2014b (+ admittances from TUG)
Ocean pole tides	Desai
Atmospheric tides	AOD RL06
Atmospheric & oeanic dealiasing	AOD RL06
Relativistic effects	IERS2010

Non-conservative forces:
ACT from TUG

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

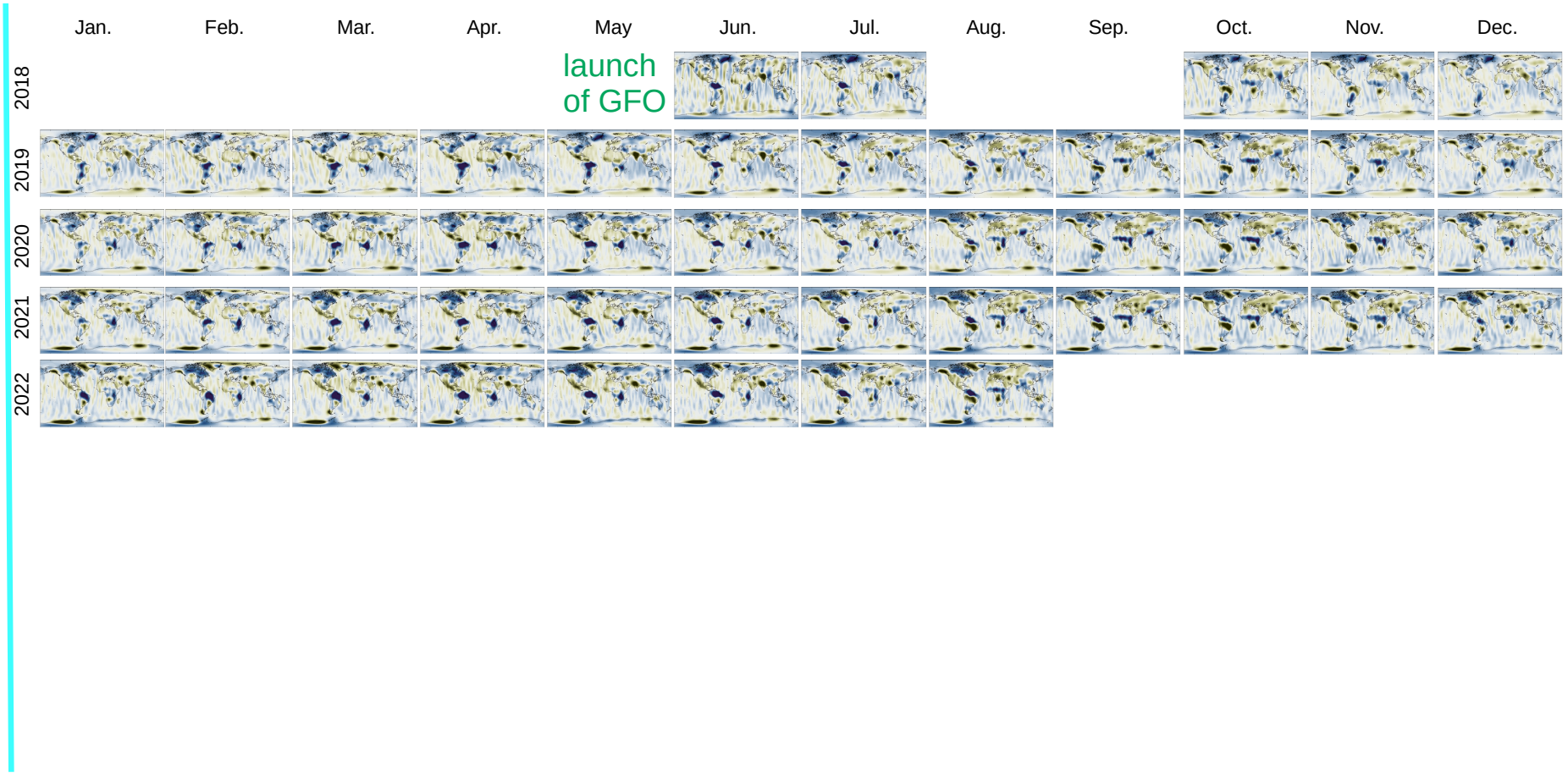
AIUB

Processing – operational solution

$$\mathbf{N} = \left(\mathbf{A}^T \mathbf{P}\, \mathbf{A}\right)$$

$$\boldsymbol{b} = \mathbf{A}^T \mathbf{P}\, \boldsymbol{l}$$

$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1}\, \boldsymbol{b}$$



M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

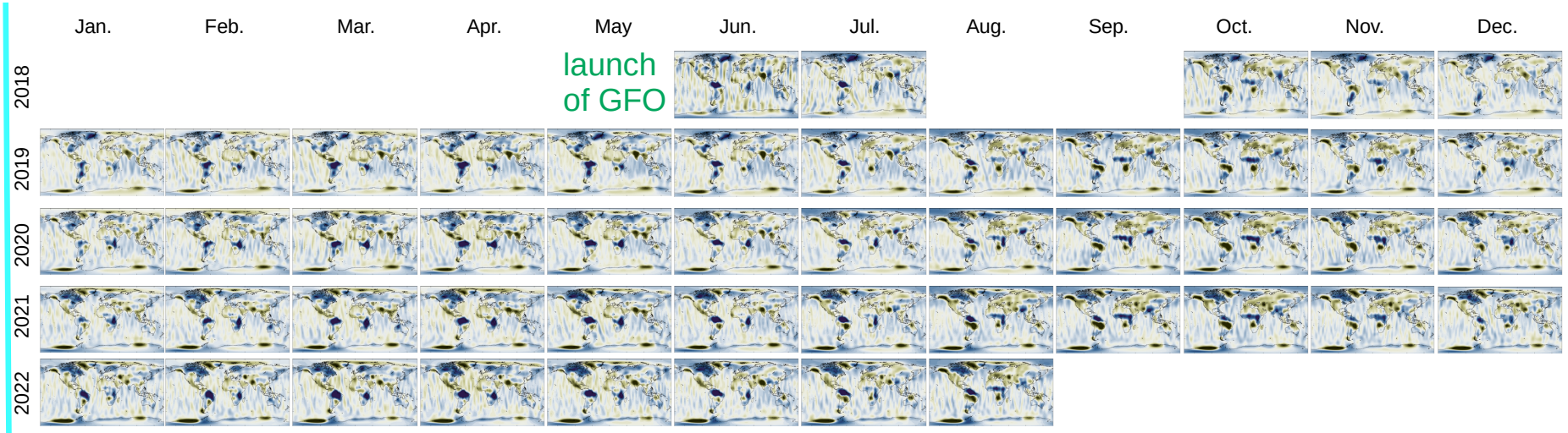
AIUB

Processing – operational solution

$$\mathbf{N} = \left(\mathbf{A}^T \mathbf{P}\, \mathbf{A}\right)$$

$$\boldsymbol{b} = \mathbf{A}^T \mathbf{P}\, \boldsymbol{l}$$

$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1}\, \boldsymbol{b}$$



Facts

- operational since Sepember 2020
- available at ICGEM as AIUB-GRACE-FO_operational
- continuation of AIUB-RL02
- current status: 49 months from June 2018 until August 2022

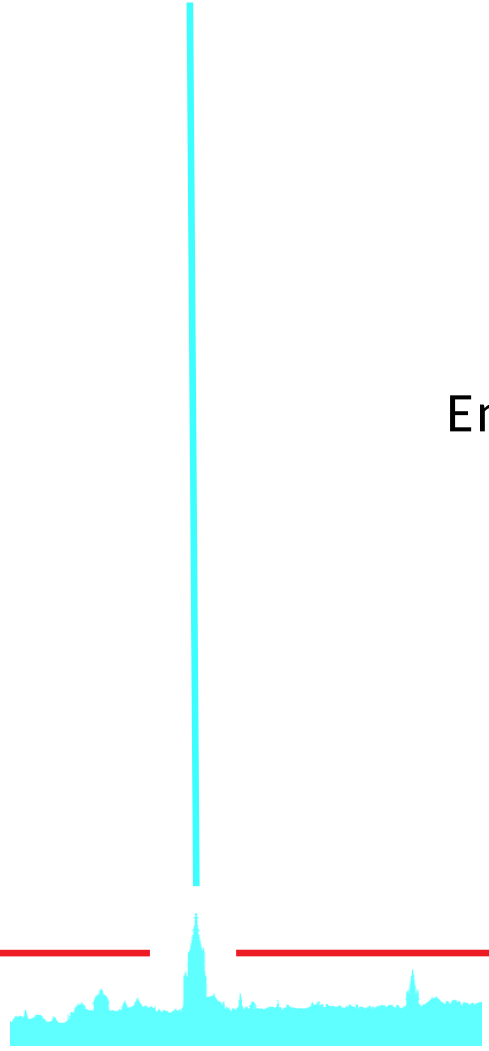
Features

- updated background models
- data screening with variance component estimation
- use of alternative GF2 Level 1B ACT product from TUG

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Empirical Noise Modelling



AIUB

Empirical noise modelling based on post-fit residuals

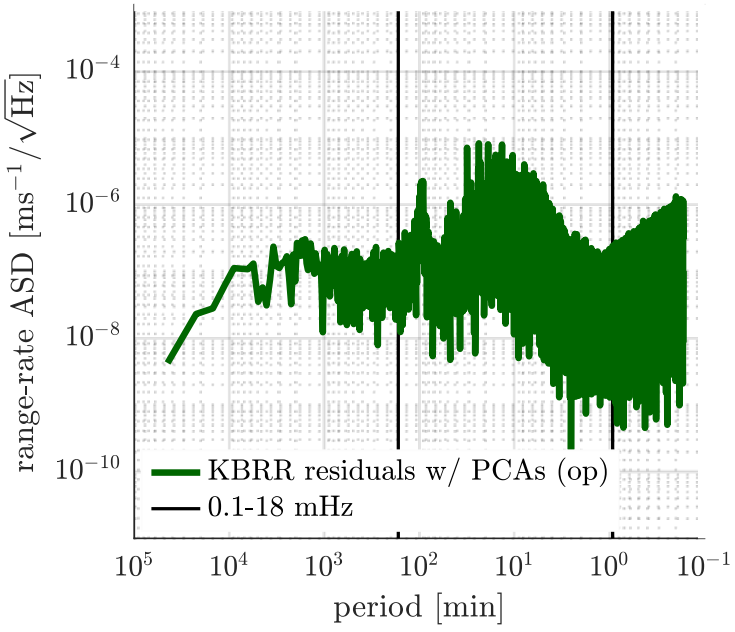
Least-squares

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A})$$

$$\boldsymbol{b} = \mathbf{A}^T \mathbf{P} \boldsymbol{l}$$

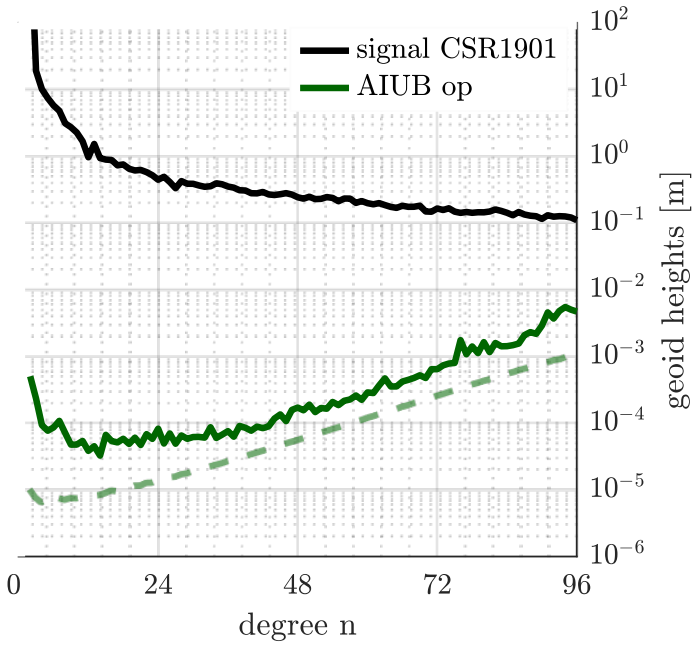
$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$$

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\hat{\boldsymbol{x}}$$



- information in the residuals (noise?)

$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$$



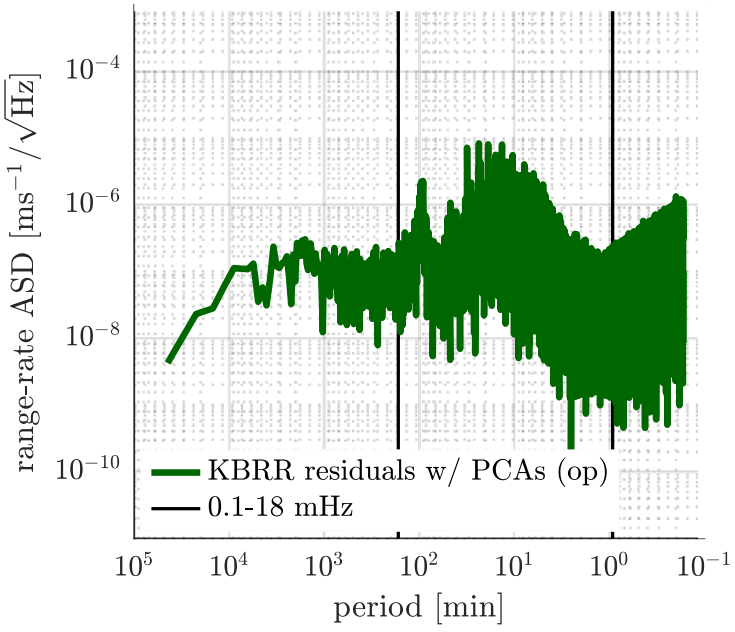
- information in the parameters (signal?)

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

What do we expect from residuals?

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



The estimator is BLUE
(best – linear – unbiased) if

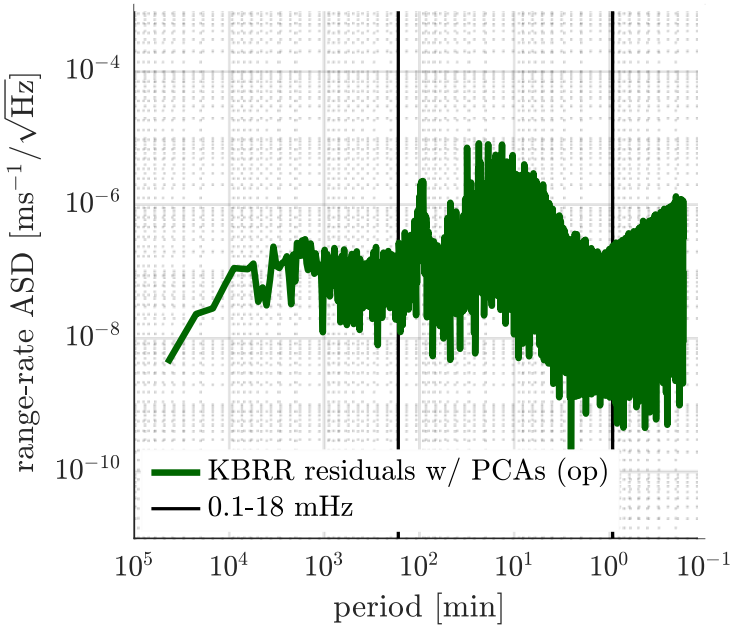
- $\mathrm{E}(\hat{\boldsymbol{x}}) = \boldsymbol{x}$
- $\mathrm{E}(\boldsymbol{e}|\boldsymbol{x}) = \mathrm{E}(\hat{\boldsymbol{e}}) = \mathbf{0}$
- $\mathrm{D}(\boldsymbol{e}|\boldsymbol{x}, \sigma_0^2) = \mathrm{D}(\boldsymbol{l}|\sigma_0^2) = \sigma_0^2 \mathbf{P}^{-1}$
  $= \sigma_0^2 \mathbf{Q}_{\boldsymbol{ee}} = \sigma_0^2 \mathbf{Q}_{\boldsymbol{ll}}$

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

What do we expect from residuals?

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



The estimator is BLUE
(best – linear – unbiased) if

- $\mathrm{E}(\hat{\boldsymbol{x}}) = \boldsymbol{x}$
- $\mathrm{E}(\boldsymbol{e}|\boldsymbol{x}) = \mathrm{E}(\hat{\boldsymbol{e}}) = \mathbf{0}$
- $\mathrm{D}(\boldsymbol{e}|\boldsymbol{x}, \sigma_0^2) = \mathrm{D}(\boldsymbol{l}|\sigma_0^2) = \sigma_0^2 \mathbf{P}^{-1}$
  $= \sigma_0^2 \mathbf{Q}_{\boldsymbol{ee}} = \sigma_0^2 \mathbf{Q}_{\boldsymbol{ll}}$

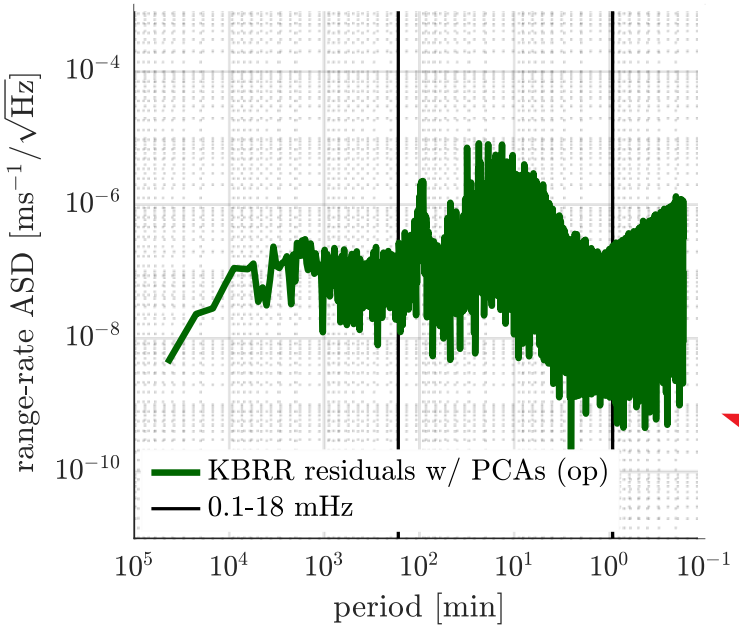
$$\mathbf{P} = \frac{1}{\sigma_0^2}\mathbf{I}$$

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

What do we expect from residuals?

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



The estimator is BLUE
(best – linear – unbiased) if

- $\mathrm{E}(\hat{\boldsymbol{x}}) = \boldsymbol{x}$
- $\mathrm{E}(\boldsymbol{e}|\boldsymbol{x}) = \mathrm{E}(\hat{\boldsymbol{e}}) = \mathbf{0}$
- $\mathrm{D}(\boldsymbol{e}|\boldsymbol{x}, \sigma_0^2) = \mathrm{D}(\boldsymbol{l}|\sigma_0^2) = \sigma_0^2 \mathbf{P}^{-1}$
  $= \sigma_0^2 \mathbf{Q}_{\boldsymbol{ee}} = \sigma_0^2 \mathbf{Q}_{\boldsymbol{ll}}$

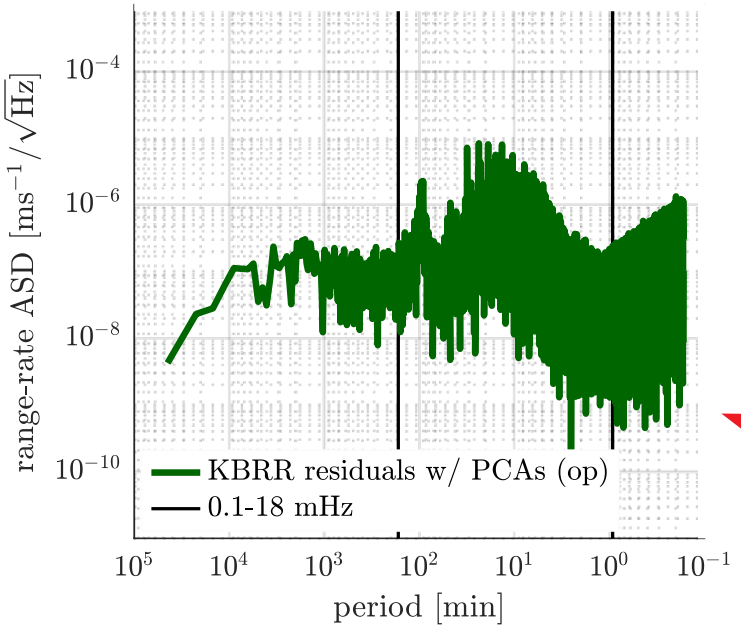
$$\mathbf{P} = \frac{1}{\sigma_0^2}\mathbf{I}$$

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

What do we expect from residuals?

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



The estimator is BLUE
(best – linear – unbiased) if

- $\mathrm{E}(\hat{\boldsymbol{x}}) = \boldsymbol{x}$
- $\mathrm{E}(\boldsymbol{e}|\boldsymbol{x}) = \mathrm{E}(\hat{\boldsymbol{e}}) = \mathbf{0}$
- $\mathrm{D}(\boldsymbol{e}|\boldsymbol{x}, \sigma_0^2) = \mathrm{D}(\boldsymbol{l}|\sigma_0^2) = \sigma_0^2 \mathbf{P}^{-1}$
  $= \sigma_0^2 \mathbf{Q}_{\boldsymbol{ee}} = \sigma_0^2 \mathbf{Q}_{\boldsymbol{ll}}$

$$\mathbf{P} = \frac{1}{\sigma_0^2}\mathbf{I}$$

$\hat{\boldsymbol{x}} = \mathbf{N}^{-1}\boldsymbol{b}$ unbiased

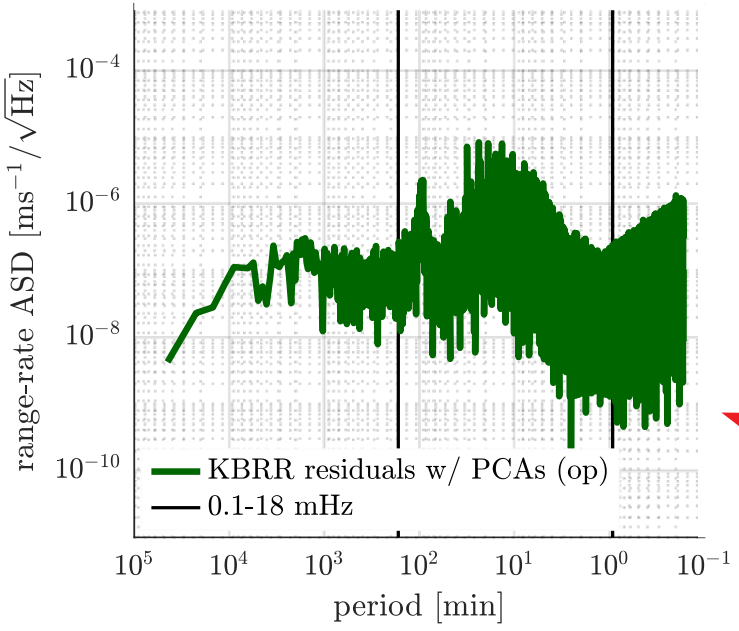
$\mathbf{C}_{\hat{\boldsymbol{x}}\hat{\boldsymbol{x}}} = \sigma_0^2 (\mathbf{A}^T \mathbf{P}\,\mathbf{A})^{-1}$ biased

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

What do we expect from residuals?

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$

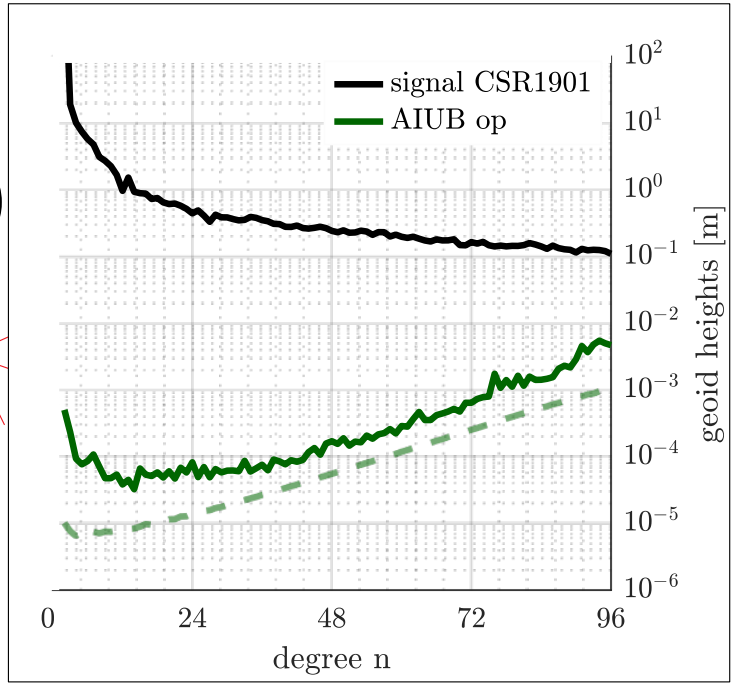


$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1}\boldsymbol{b} \qquad \text{unbiased}$$

$$\mathbf{C}_{\hat{\boldsymbol{x}}\hat{\boldsymbol{x}}} = \sigma_0^2\left(\mathbf{A}^T\mathbf{P}\,\mathbf{A}\right)^{-1} \qquad \text{biased}$$

The estimator is BLUE
(best – linear – unbiased) if

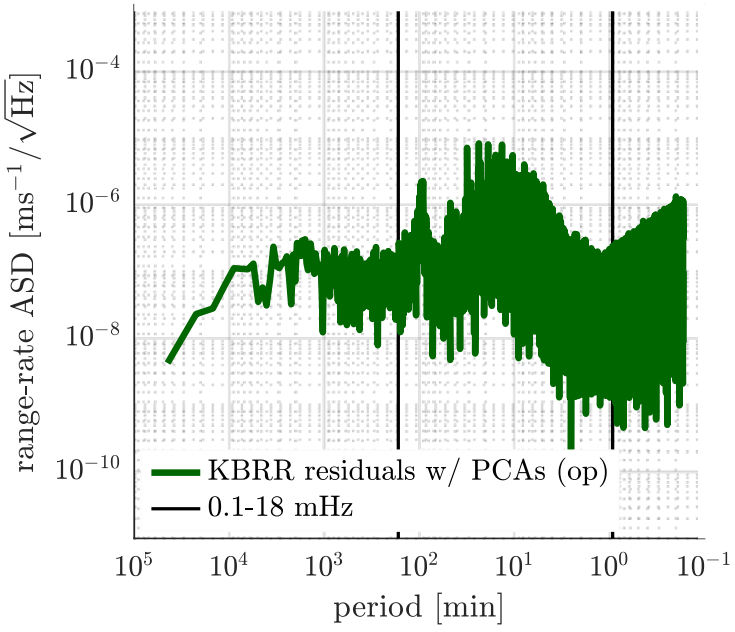
- $\mathrm{E}\left(\hat{\boldsymbol{x}}\right) = \boldsymbol{x}$
- $\mathrm{E}\left(\boldsymbol{e}|\boldsymbol{x}\right) = \mathrm{E}$
- $\mathrm{D}\left(\boldsymbol{e}|\boldsymbol{x}, \sigma_0^2\right)$



M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Auto-covariance function



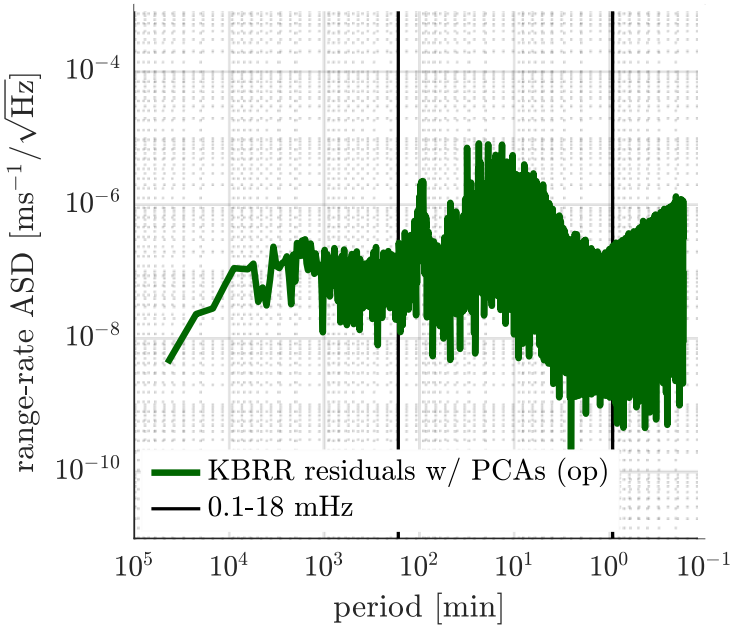
$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}} \qquad \text{(post-fit residuals)}$$

$$\operatorname{cov}(\Delta t_k) = \frac{1}{N}\sum_{i=0}^{N} \hat{e}(t_i)\,\hat{e}(t_i + \Delta t_k)$$

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Auto-covariance function



$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}} \qquad \text{(post-fit residuals)}$$

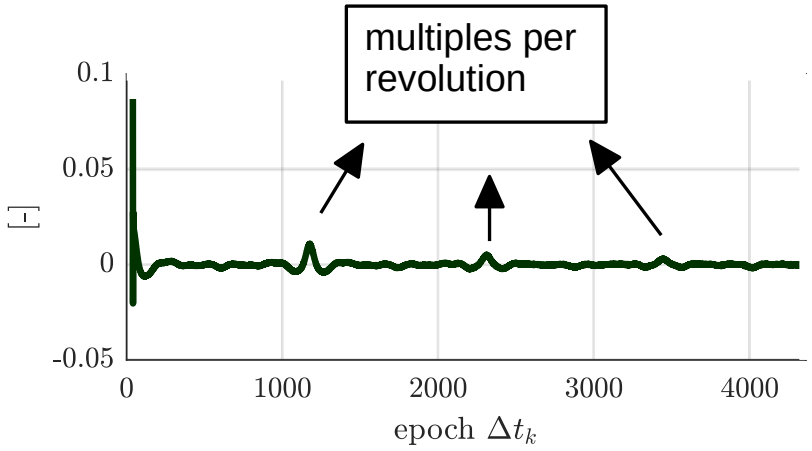
$$\operatorname{cov}(\Delta t_k) = \frac{1}{N}\sum_{i=0}^{N} \hat{e}(t_i)\,\hat{e}(t_i + \Delta t_k)$$

- stationarity assumed
- biased estimation of auto-covariance
  → covariance matrix nondegenerate

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Auto-covariance function



$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}} \qquad \text{(post-fit residuals)}$$

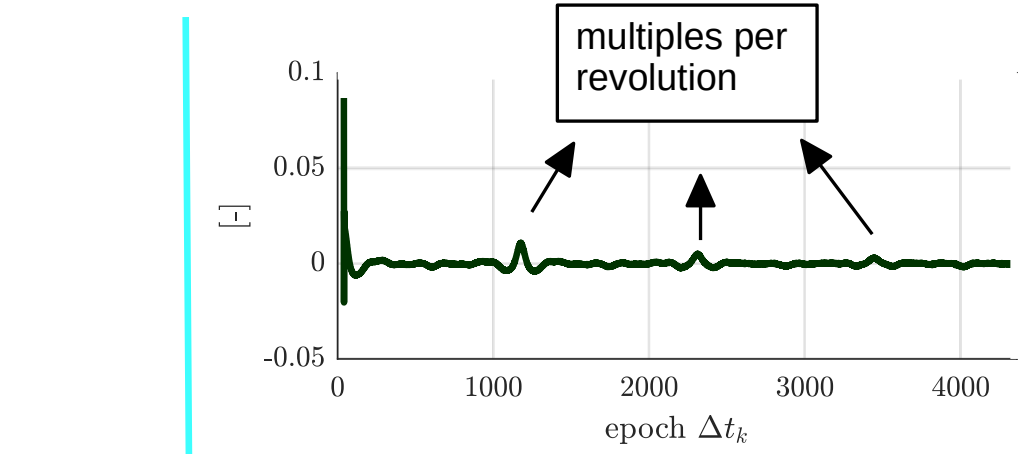
$$\mathrm{cov}\left(\Delta t_k\right) = \frac{1}{N}\sum_{i=0}^{N} \hat{e}\left(t_i\right)\hat{e}\left(t_i + \Delta t_k\right)$$

- stationarity assumed
- biased estimation of auto-covariance
  → covariance matrix nondegenerate

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Auto-covariance function and weight matrix

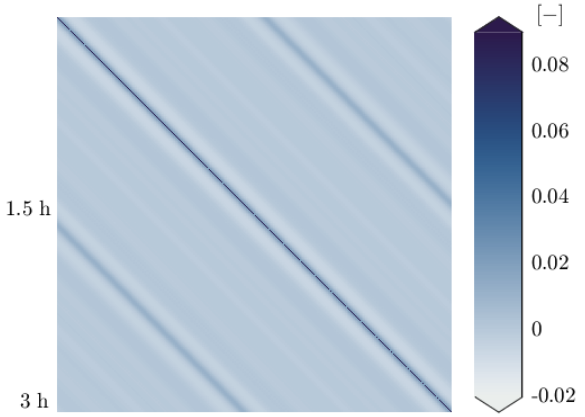


$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}} \qquad \text{(post-fit residuals)}$$

$$\operatorname{cov}(\Delta t_k) = \frac{1}{N}\sum_{i=0}^{N} \hat{e}(t_i)\,\hat{e}(t_i + \Delta t_k)$$

- stationarity assumed
- biased estimation of auto-covariance
  → covariance matrix nondegenerate

block Toeplitz matrix



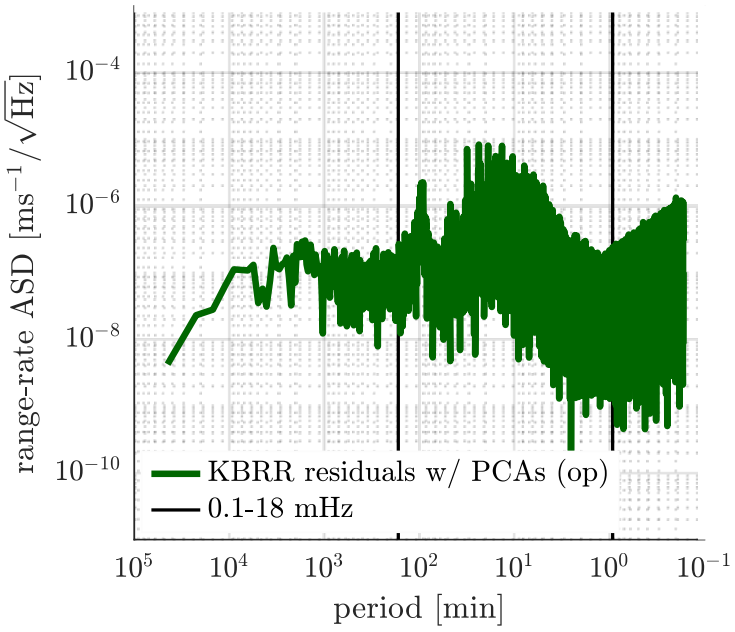
P

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Auto-covariance function – examples

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



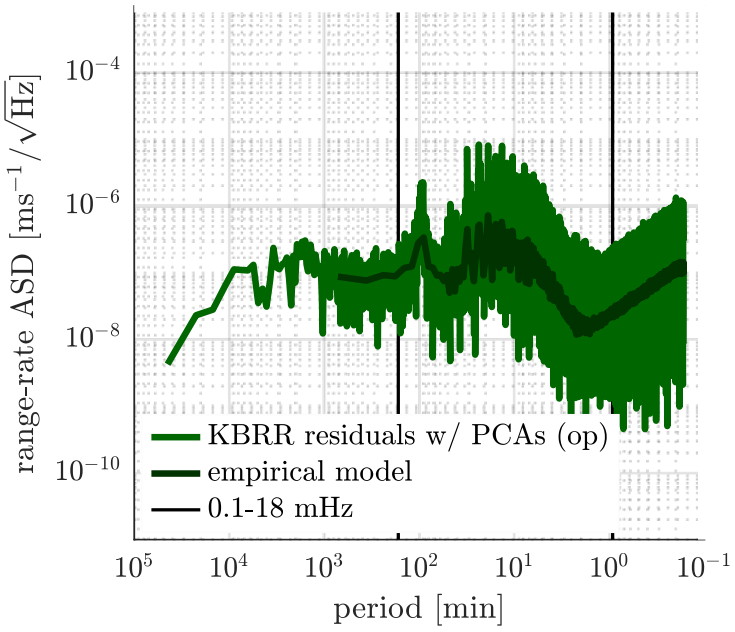
M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Auto-covariance function – examples

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$

$$\mathrm{cov}(\Delta t_k) = \frac{1}{N} \cdot \sum_{i=0}^{N} \hat{e}(t_i)\,\hat{e}(t_i + \Delta t_k)$$



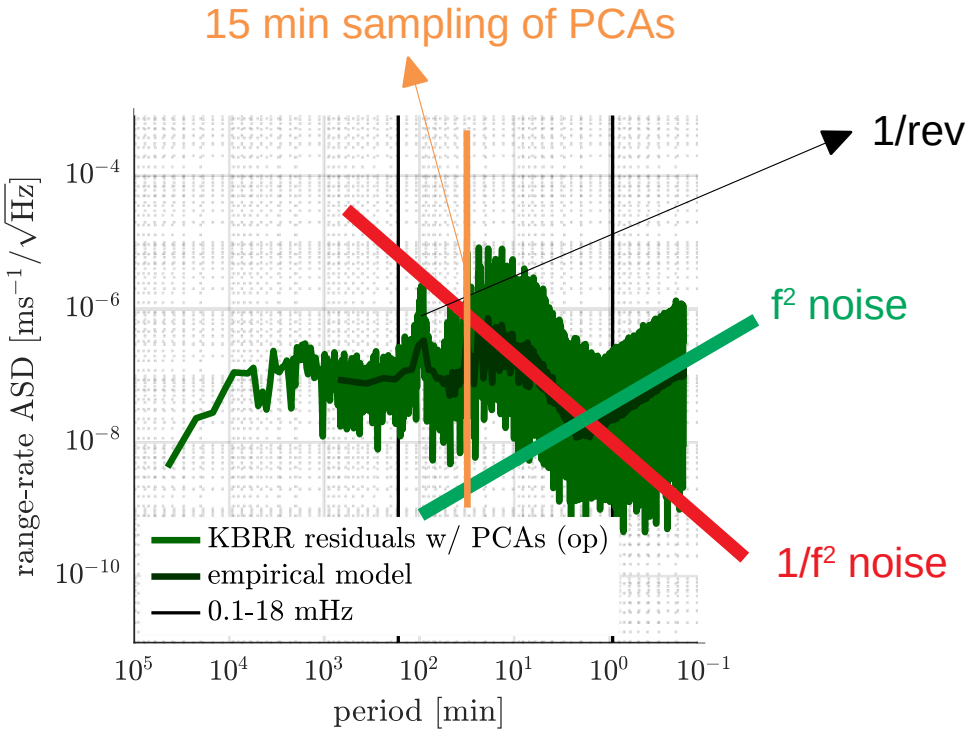
M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Auto-covariance function – examples

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$

$$\mathrm{cov}(\Delta t_k) = \frac{1}{N} \cdot \sum_{i=0}^{N} \hat{e}(t_i)\,\hat{e}(t_i + \Delta t_k)$$



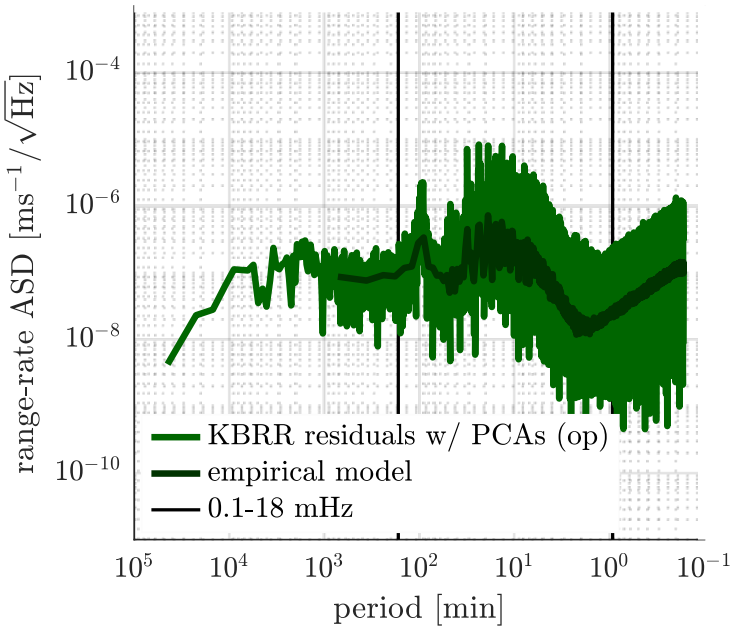
M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

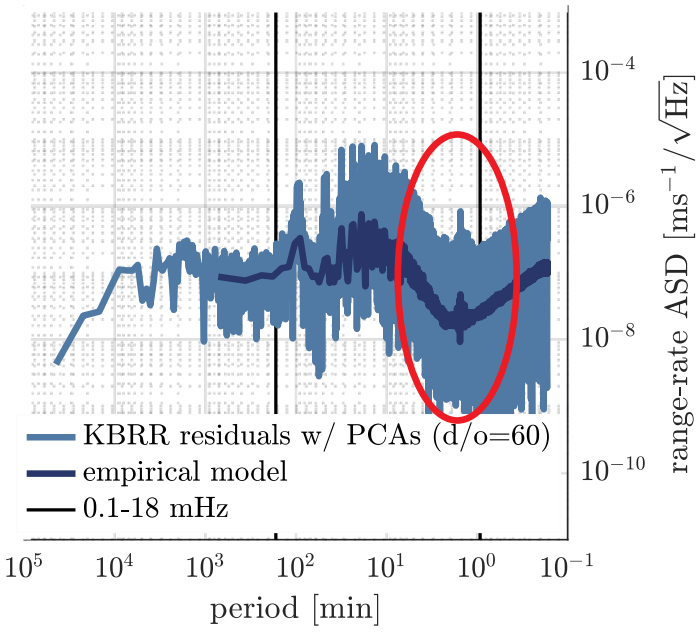
Auto-covariance function – examples

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$

$$\mathrm{cov}(\Delta t_k) = \frac{1}{N} \cdot \sum_{i=0}^{N} \hat{e}(t_i)\,\hat{e}(t_i + \Delta t_k)$$



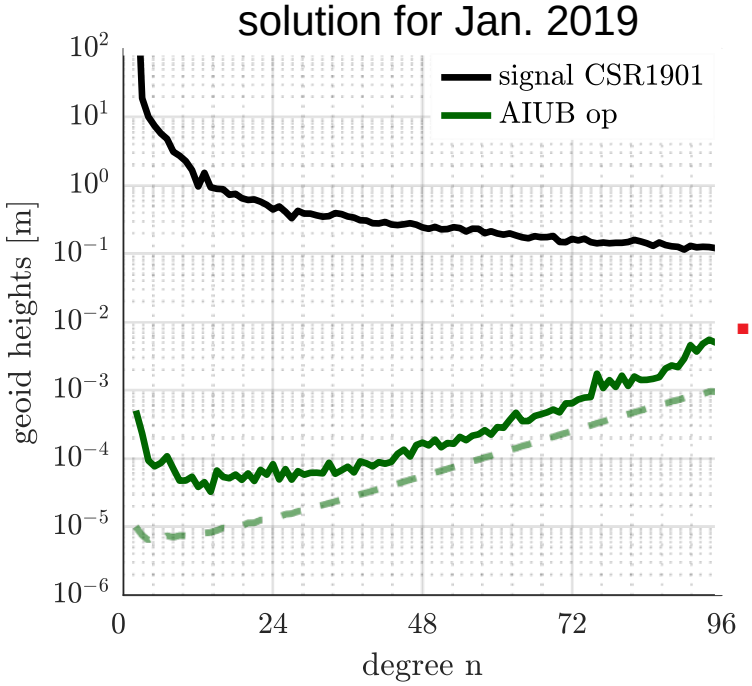
truncation at d/o=60 (artefact → bias) is reflected



M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Results of empirical modelling – gravity field

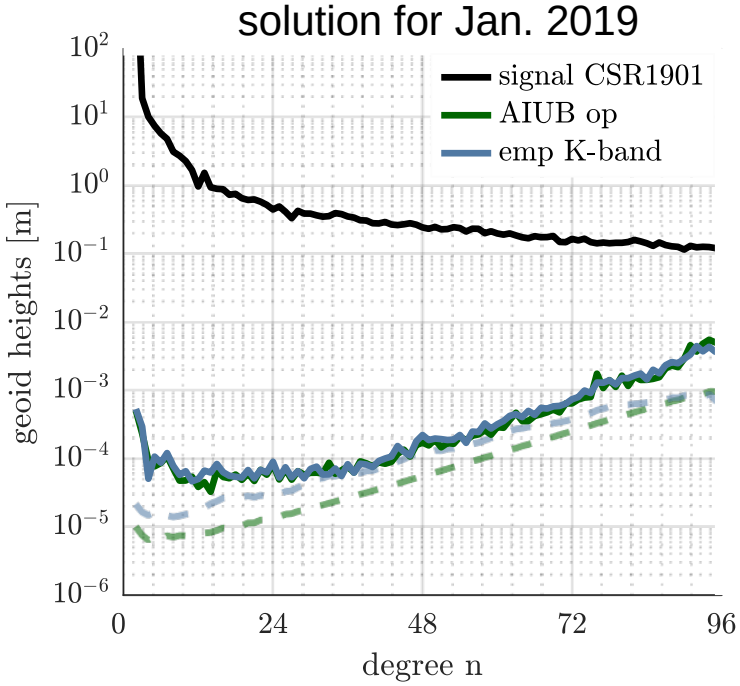


- formal errors too optimistic compared to the assessed noise

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Results of empirical modelling (K-band) – gravity field

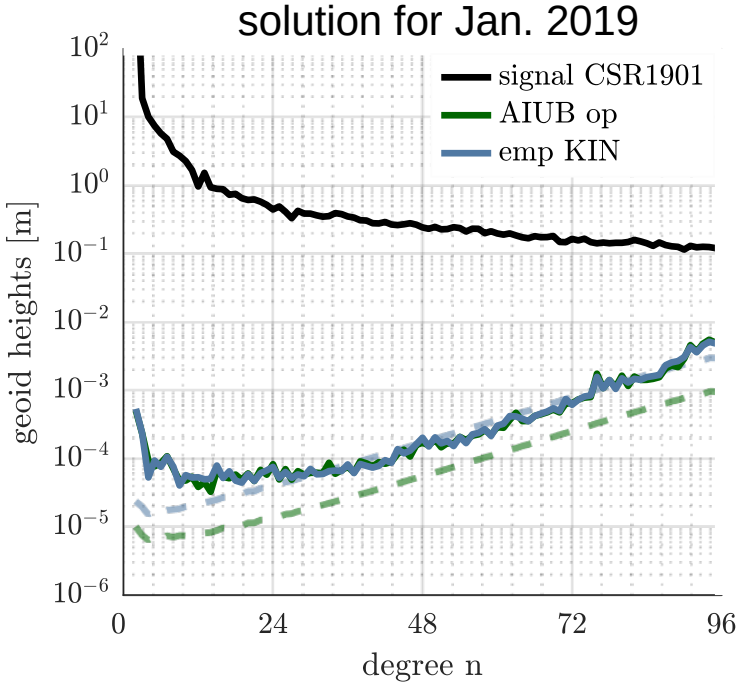


- formal errors reflect assessed noise very well
- including features of resonance orders

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Results of empirical modelling (KIN) – gravity field

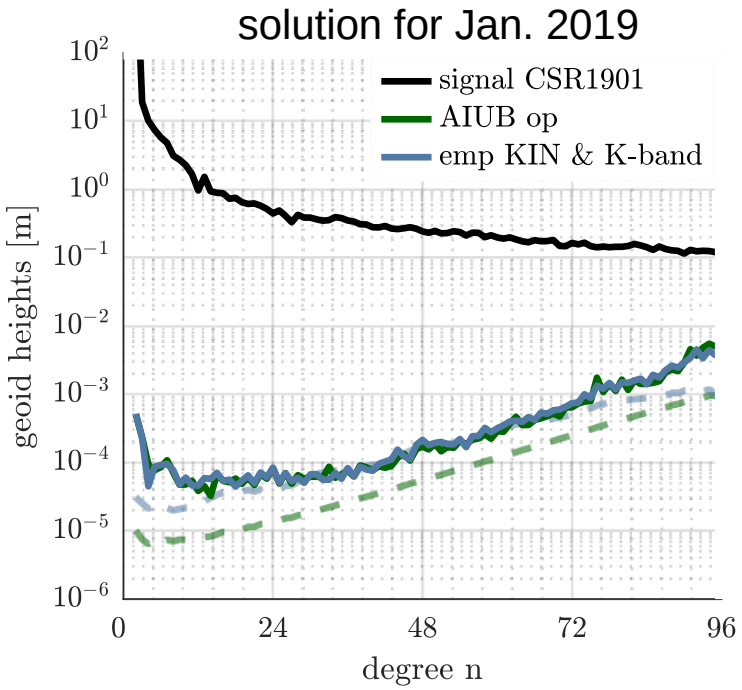


- formal errors reflect assessed noise very well
- basically the formal error curve is shifted

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Results of empirical modelling (KIN & K-band) – gravity field

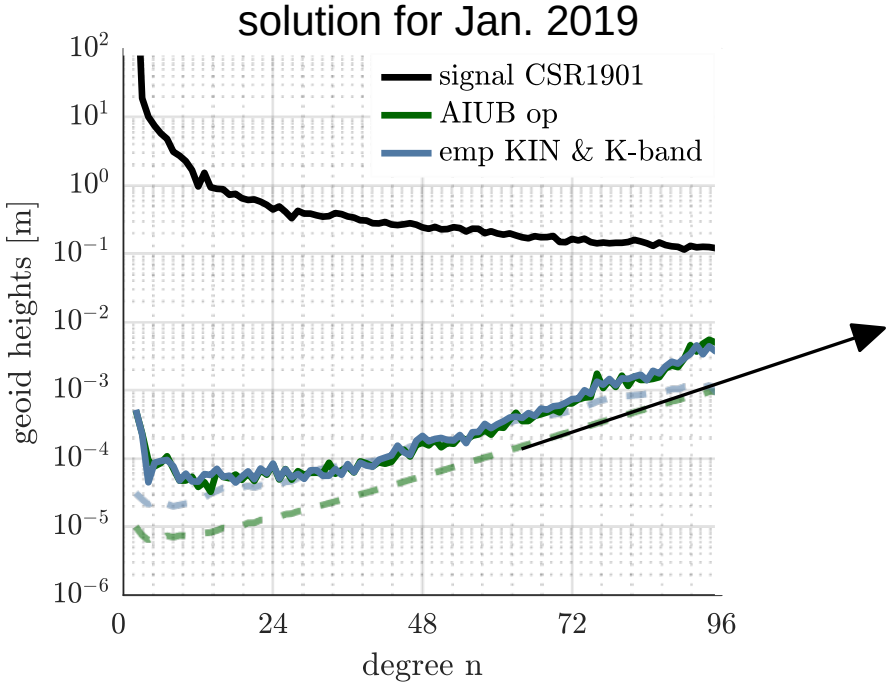


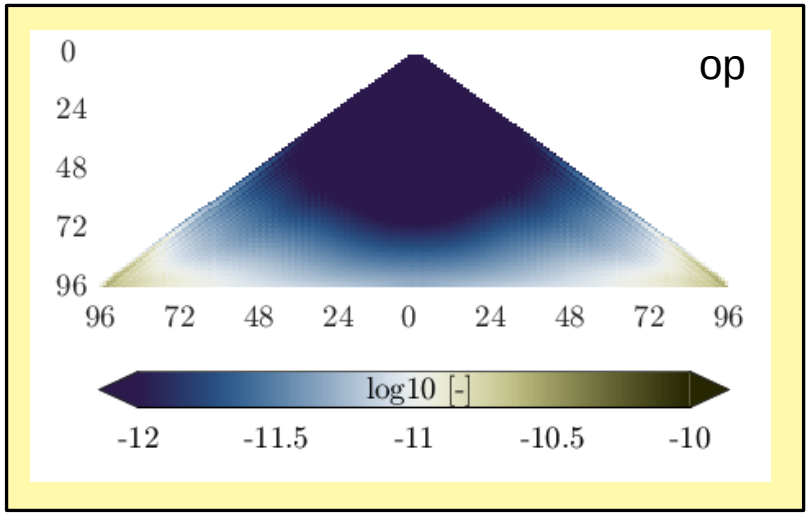
- formal errors reflect assessed noise very well
- including features of resonance orders

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Results of empirical modelling – formal errors

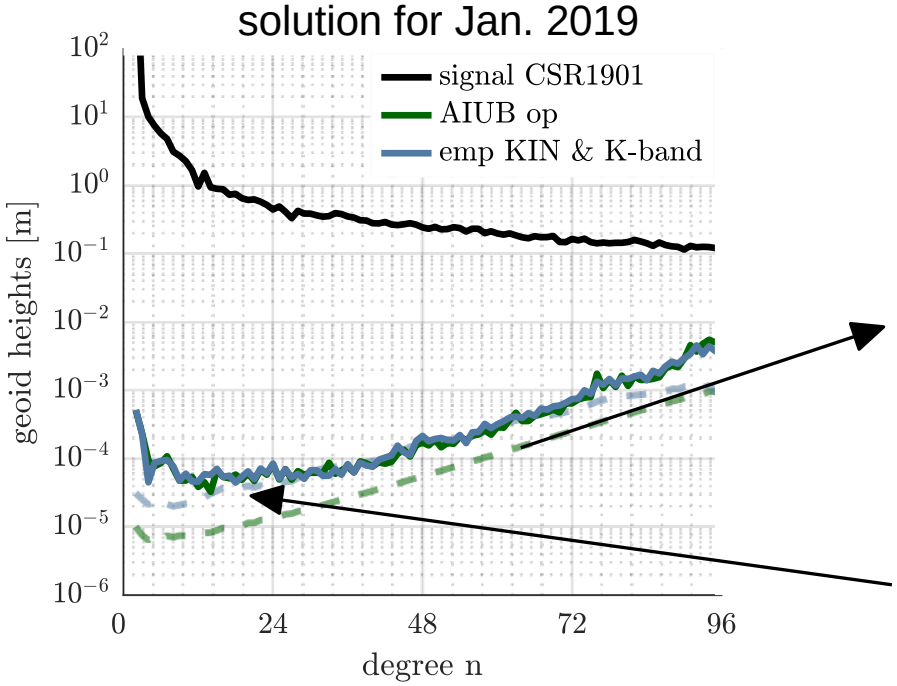


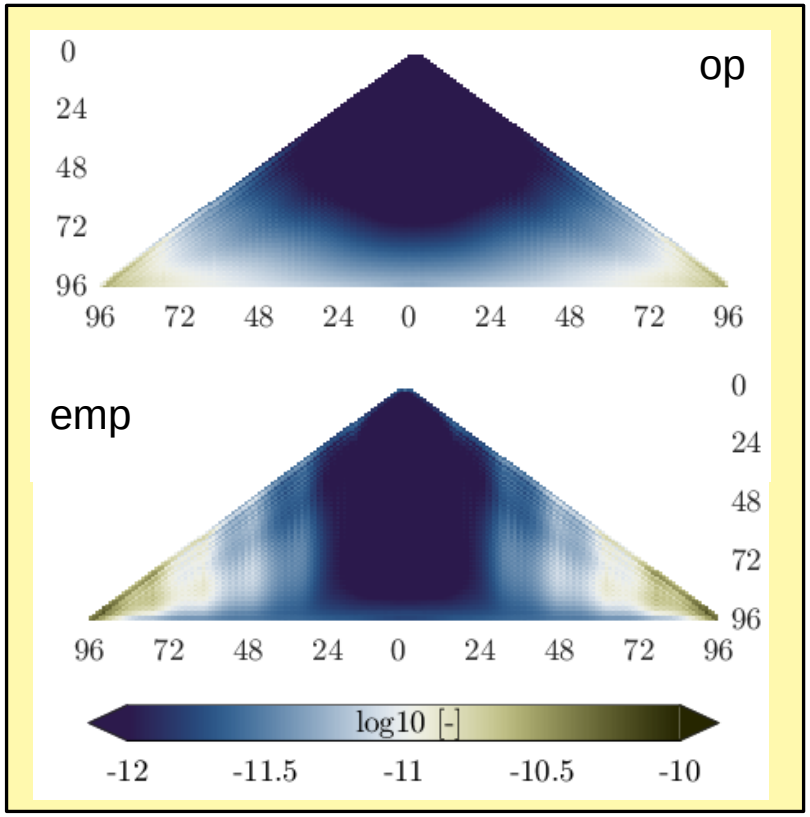


M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Results of empirical modelling – formal errors

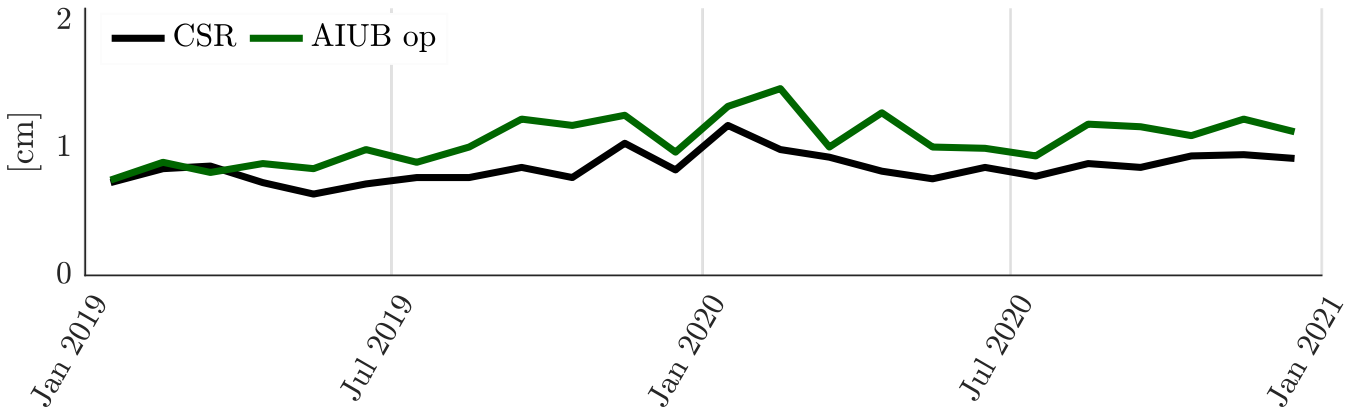




M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

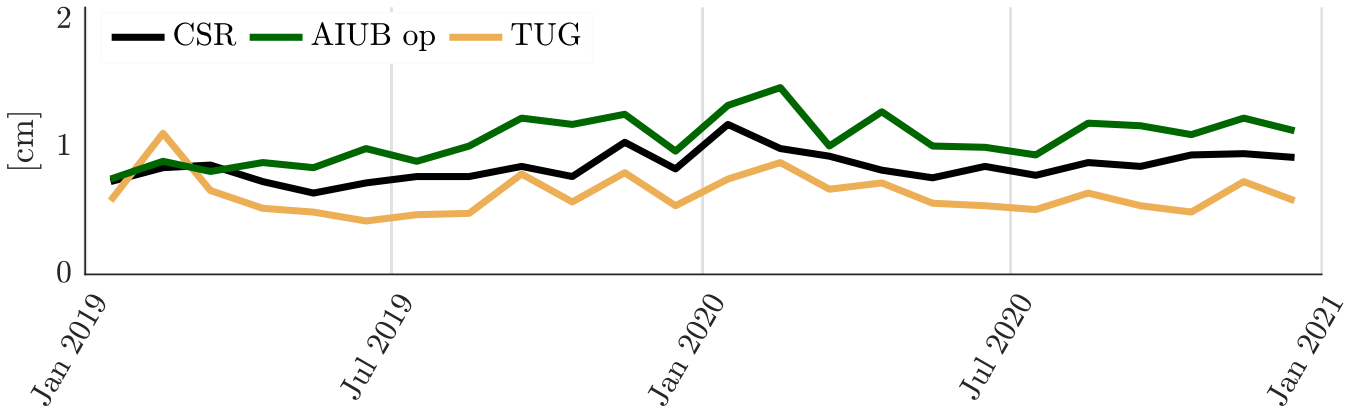
Results of empirical modelling – RMS over the ocean



M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

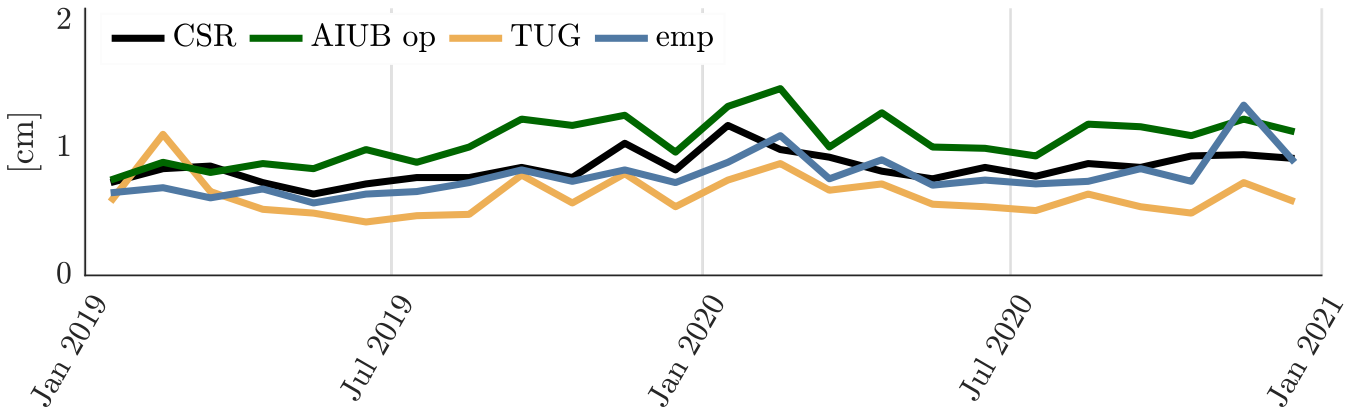
AIUB

Results of empirical modelling – RMS over the ocean



M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

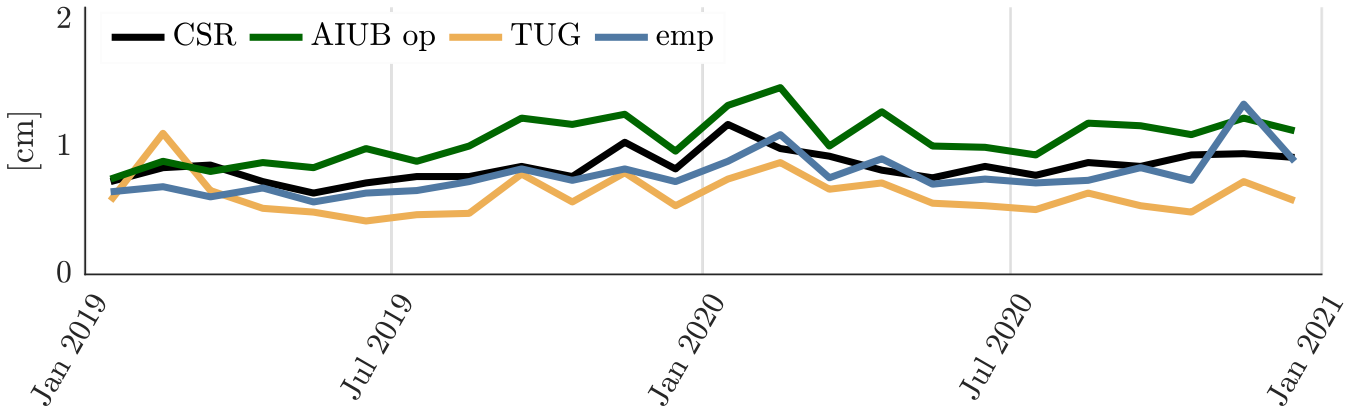
Results of empirical modelling – RMS over the ocean



M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Results of empirical modelling – summary



- possible on any (stationary) residuals time series
- additional parameters can be reduced as stationary behaviour can be absorbed
- formal errors much more realistic and show resonance orders (if correlation length > 3 h)
- no constraints needed
- no/few a priori knowledge needed

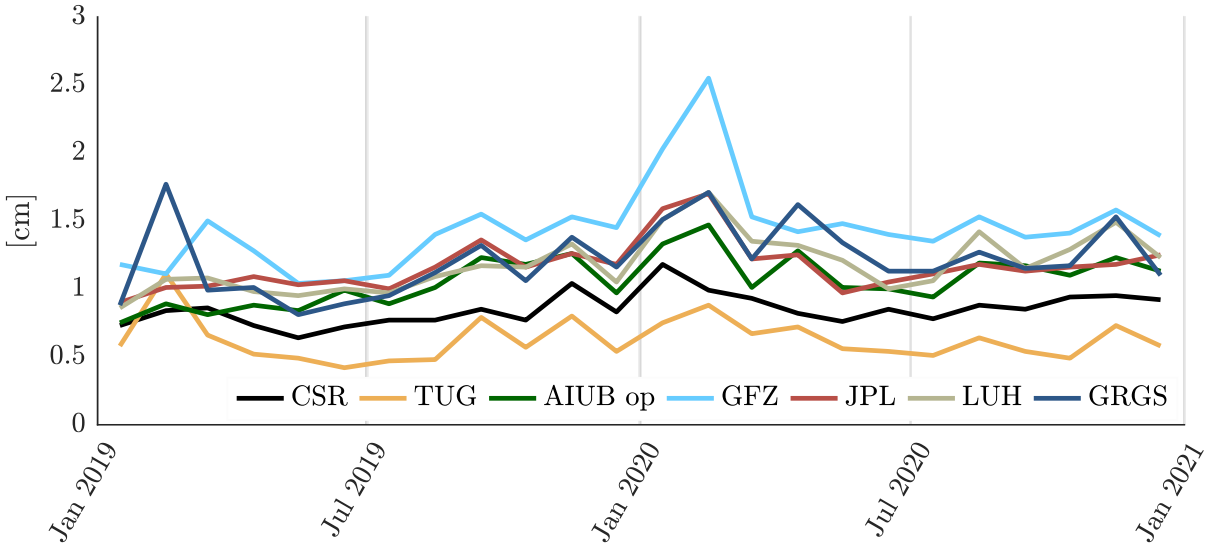
- iterations required (might be time consuming)
- memory consumption and inversion time dependent on length of auto-covariance function

M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

AIUB

Performance in COST-G – RMS over the ocean

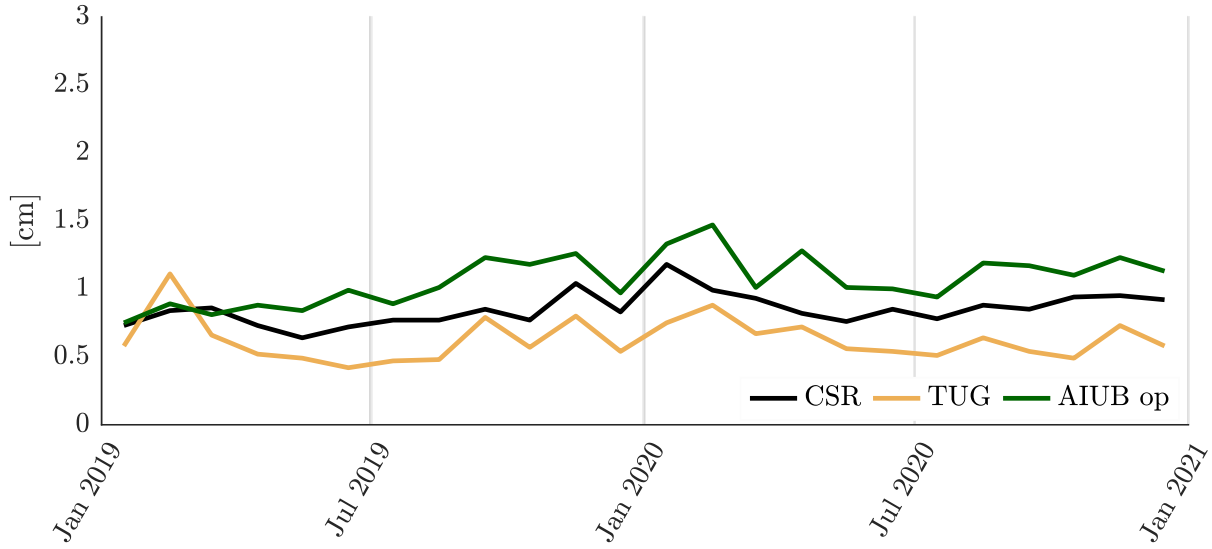
Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability



M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

Performance in COST-G – RMS over the ocean

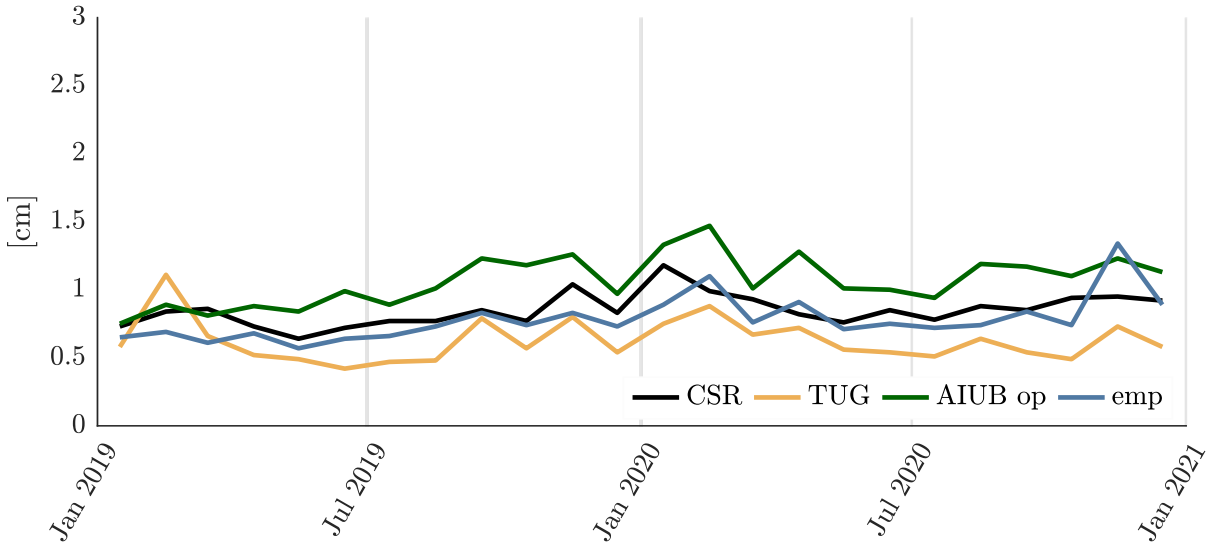
Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability



M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

Performance in COST-G – RMS over the ocean

Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability

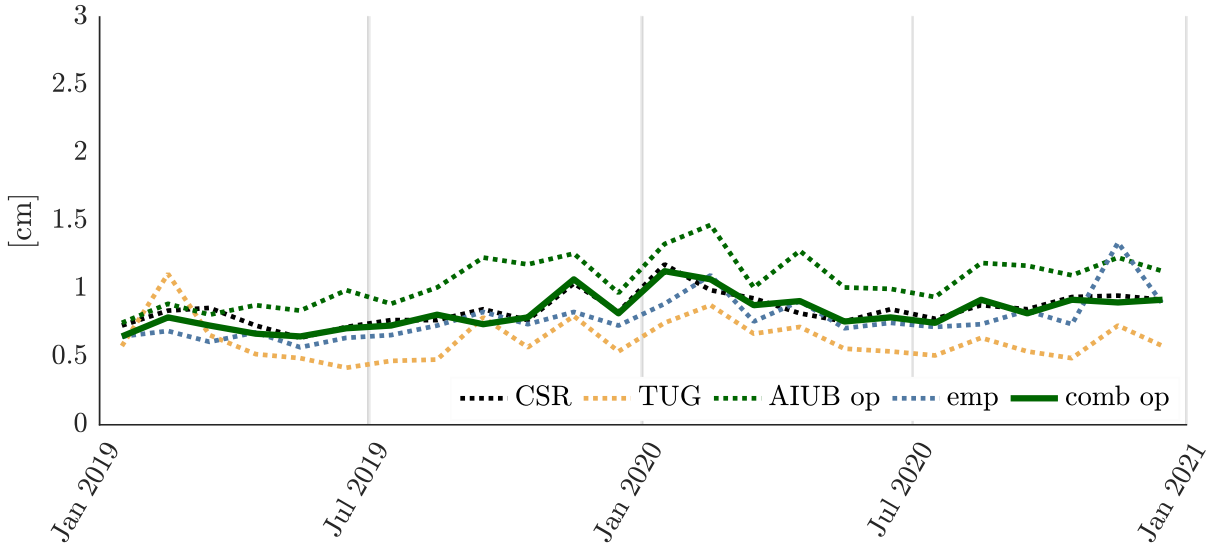


M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

Performance in COST-G – RMS over the ocean

Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability

- Combination not at the level of the best individual solutions

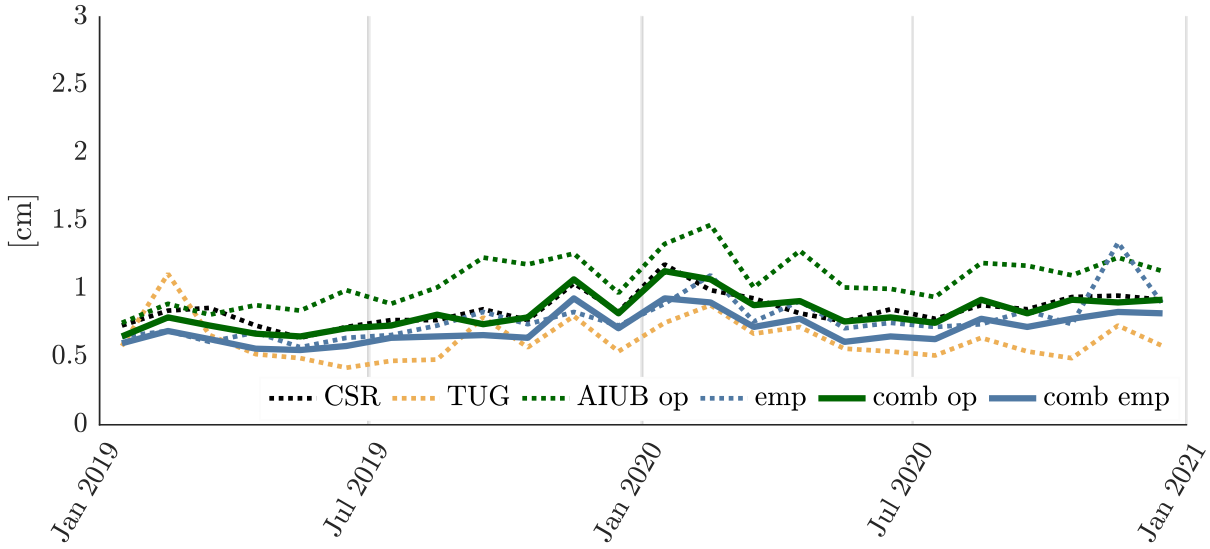


M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

Performance in COST-G

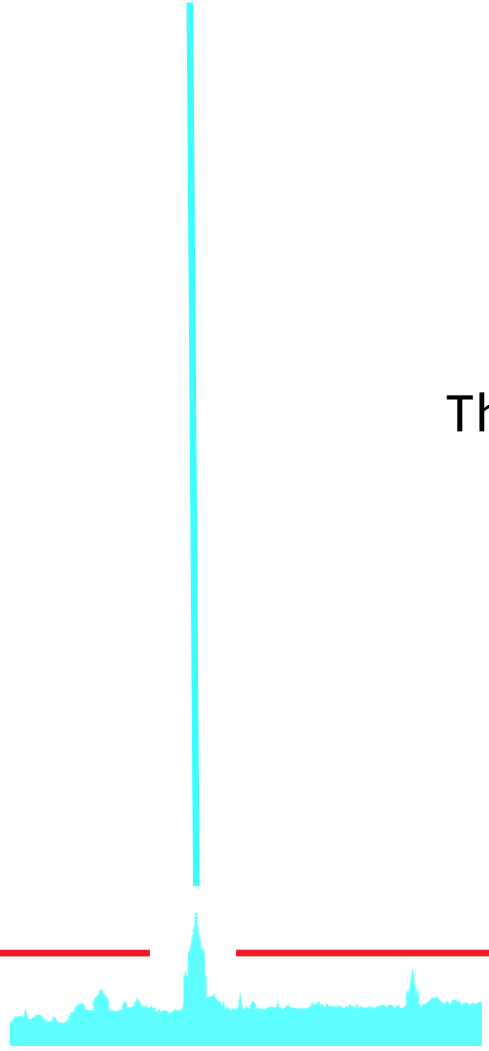
Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability

- (Combination not at the level of the best individual solutions)
- Combination significantly improved by the new processing scheme of one analysis centre



M. Lasser et al.: Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling, GRACE Follow-On Science Team Meeting,, 18 October 2022

Thank you for your attention



AIUB

References

Beutler, G., Jäggi, A., Mervart, L. and Meyer, U. [2010]: The celestial mechanics approach: theoretical foundations. Journal of Geodesy, vol. 84(10), pp. 605-624. https://doi.org/10.1007/s00190-010-0401-7

Ellmer, M. [2018]: Contributions to GRACE Gravity Field Recovery: Improvements in Dynamic Orbit Integration Stochastic Modelling of the Antenna Offset Correction, and Co-Estimation of Satellite Orientations. PhD thesis, Graz University of Technology, In Monographic Series TU Graz, number 1, Verlag der Technischen Universität Graz, Graz, Austria. https://doi.org/10.3217/978-3-85125-646-8

Jäggi, A., Meyer, U., Lasser, M., Jenny, B., Lopez, T., Flechtner, F., Dahle, C., Förste, C., Mayer-Gürr, T., Kvas, A., Lemoine, J.-M., Bourgogne, S., Weigelt, M. and Groh, A. [2020]: International Combination Service for Time-Variable Gravity Fields (COST-G) – Start of Operational Phase and Future Perspectives. In J. Freymueller, editor, International Association of Geodesy Symposia, pages 1–9, Springer Berlin-Heidelberg, Germany. https://doi.org/10.1007/1345_2020_109

Lasser, M., Meyer, U., Arnold, D. and Jäggi, A. [2020]: AIUB-GRACE-FO-operational - Operational GRACE Follow-On monthly gravity field solutions. https://doi.org/10.5880/ICGEM.2020.001

Kvas, A., Behzadpour, S., Ellmer, M., Klinger, B., Strasser, S., Zehentner, N. and Mayer-Gürr, T. [2019]: Overview and evaluation of a new GRACE-only gravity field time series. Journal of Geophysical Research: Solid Earth. ISSN 2169-9313. https://doi.org/10.1029/2019JB017415

NASA Jet Propulsion Laboratory (JPL) [2019]: GRACE-FO Monthly Geopotential Spherical Harmonics CSR Release 6.0. https://doi.org/10.5067/GFL20-MC060