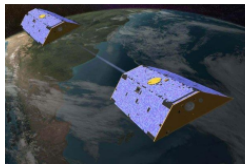
Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery

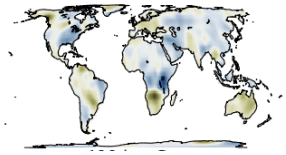
Martin Lasser, Ulrich Meyer, Daniel Arnold, Adrian Jäggi

Astronomical Institute, University of Bern, Switzerland

Gravity, Geoid, and Height Systems 2022 Symposium
12 September 2022
Austin, TX, USA

Operational processing





Basic parametrisation

▪ initial conditions	2x[6]
▪ accelerometer bias	2x[3]
▪ accelerometer scaling	2x[3]
parameters per arc	24

Additional parameters

▪ 15 min PCA per satellite in	
➔ radial	2x[96]
➔ along-track	2x[96]
➔ cross-track	2x[96]
parameters per arc	576

in daily arcs (30 days):

- 18000 parameters,
- 17280 for the noise model
- + gravity field

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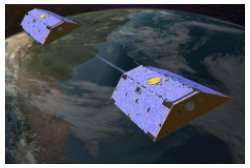

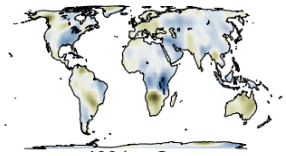

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Force models	
Gravity field	Internal AIUB static GRACE field
Astromomic bodies	JPL DE421 (all planets + Pluto)
Mean pole	Linear
Solid Earth tides	IERS2010
Solid Earth pole tides	IERS2010
Ocean tides	FES2014b (+ admittances from TUG)
Ocean pole tides	Desai
Atmospheric tides	AOD RL06
Atmospheric & oeanic dealiasing	AOD RL06
Relativistic effects	IERS2010

Non-conservative forces:
ACT from TUG

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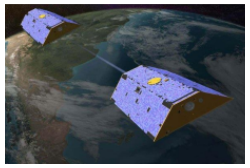
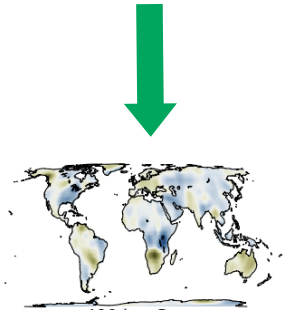
VCE – constraints





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- Perturbation theory [Kim, 2000]:
  Errors in background models will (mostly) sum up in 1/rev

→ frequently used in the Celestial Mechanics Approach
[Beutler et al., 2010]

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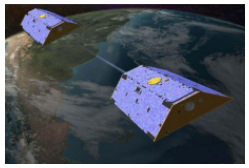

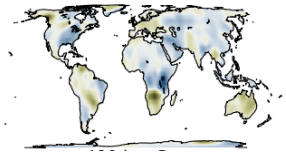

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How to constrain their impact
to the correct magnitude?

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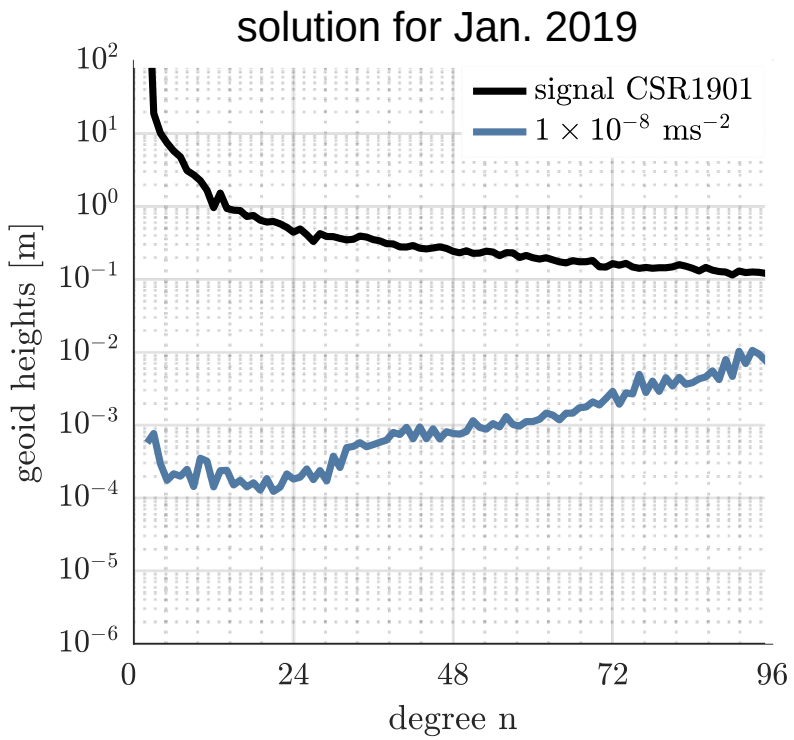
Impact of different constraints

loose PCAs may become large



1×10^{-8} ms^{-2}

«loose» constraint

(gravity field signal absorbed in PCAs)

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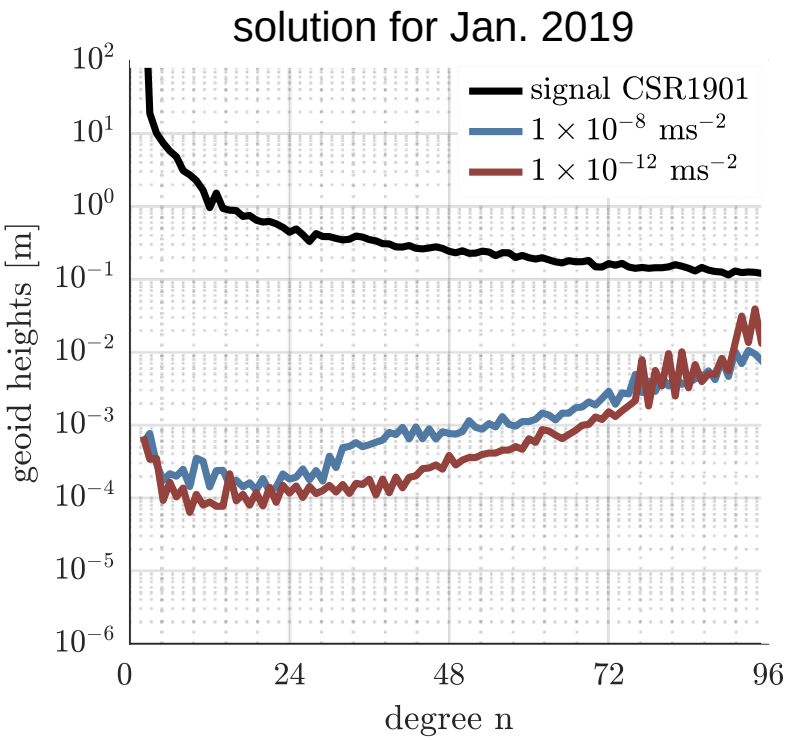
Impact of different constraints

loose
PCAs
may
become
large

tight
PCAs
strongly
confined



1×10^{-12} ms^{-2}

«tight» constraint

(not enough to absorb mis-modellings)

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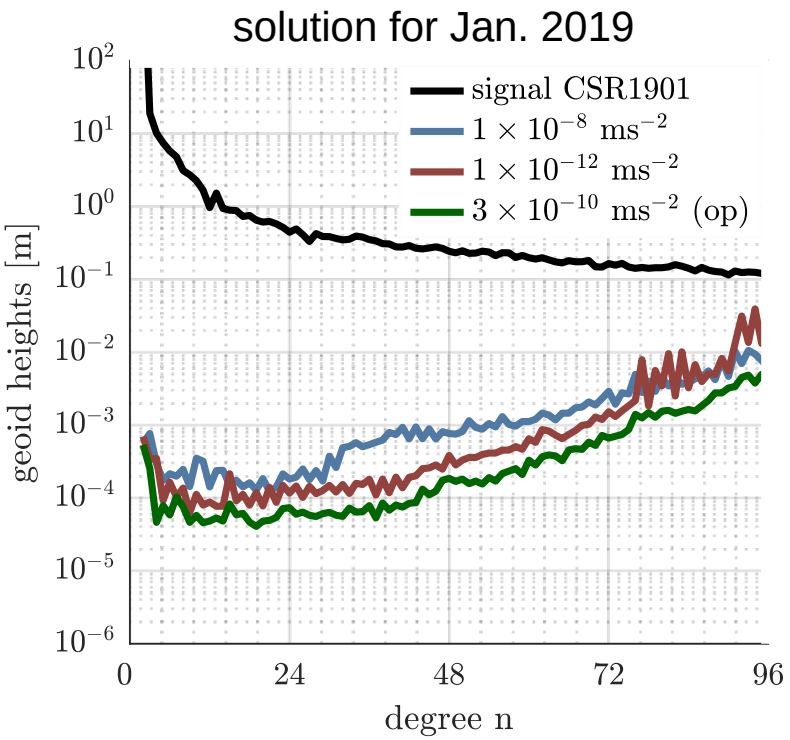

3×10^{-10} ms^{-2}

reasonable balance

(applied in the operational solutions)

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Constraining

A design matrix

P weight matrix

l obser-vations

$$\mathbf{N} = \left(\mathbf{A}^T \mathbf{P} \mathbf{A} \right) \quad \text{and} \quad \boldsymbol{b} = \mathbf{A}^T \mathbf{P} \boldsymbol{l} \longrightarrow \hat{\boldsymbol{x}} = \mathbf{N}^{-1} \boldsymbol{b}$$

$$\mathbf{N} = \left(\mathbf{A}^T \mathbf{P} \mathbf{A} + \mathbf{W} \right)$$

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$$\boldsymbol{N} = \begin{bmatrix} \blacksquare & & \blacksquare & \blacksquare & & \blacksquare \\ & \blacksquare & & \blacksquare & \blacksquare & \\ \blacksquare & & \blacksquare & & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & & \blacksquare & & \blacksquare \\ & \blacksquare & \blacksquare & & \blacksquare & \\ \blacksquare & & \blacksquare & \blacksquare & & \blacksquare \end{bmatrix} + \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \blacksquare & \blacksquare \end{bmatrix}$$

$$\frac{\sigma_0^2}{\sigma_{PCA}^2},$$

$$\sigma_{PCA}^2 = \text{e.g., } 3 \times 10^{-10} \text{ ms}^{-2}$$

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VCE and constraints

$\mathbf{A}$ design matrix

$\mathbf{P}$ weight matrix

$\boldsymbol{l}$ observations

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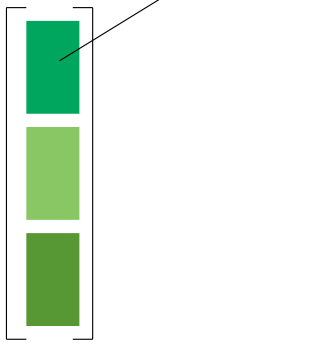
- The observations of each arc are used to set up the normal equations (NEQs)

$\boldsymbol{l}=$ 

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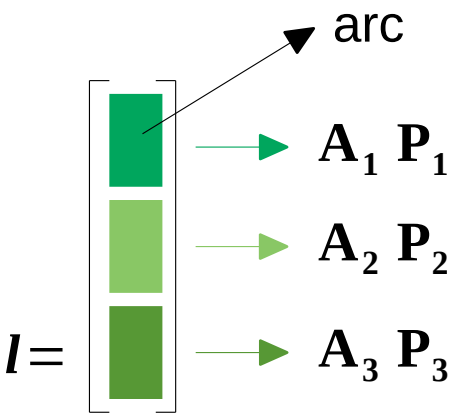
- Each arc is treated as being independent



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$$\boldsymbol{l} = \begin{bmatrix} \\ \\ \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \longrightarrow \mathbf{A}_1 \, \mathbf{P}_1 \\ \longrightarrow \mathbf{A}_2 \, \mathbf{P}_2 \\ \longrightarrow \mathbf{A}_3 \, \mathbf{P}_3 \end{matrix}$$

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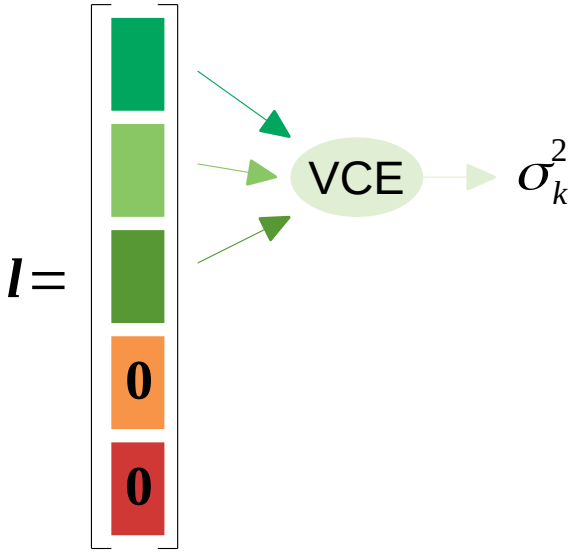
VCE: Each group of observations gets a weight based on its contribution to the final solution



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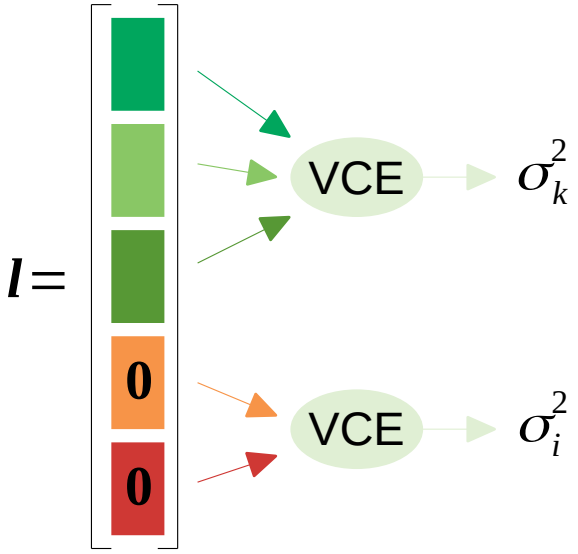

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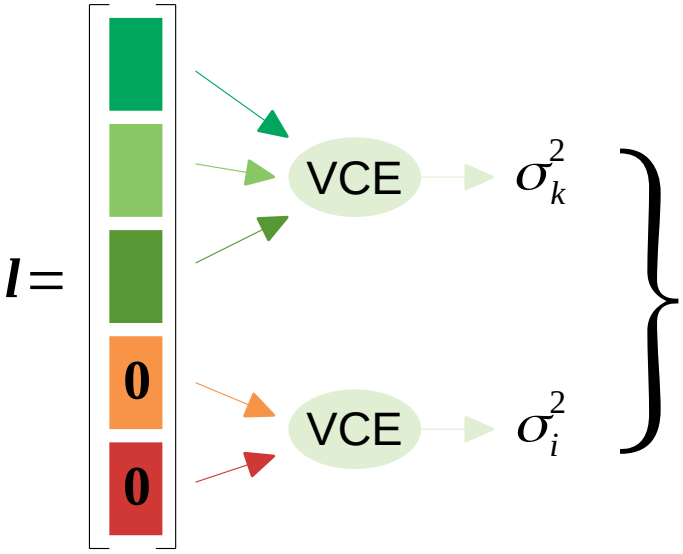

$$\hat{\boldsymbol{x}} = \left(\sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} \mathbf{N}_k + \sum_{i=1}^{I=2} \frac{\sigma_0^2}{\sigma_i^2} \mathbf{W}_i \right)^{-1} \sum_{k=1}^{K=3} \frac{\sigma_0^2}{\sigma_k^2} \boldsymbol{b}_k$$

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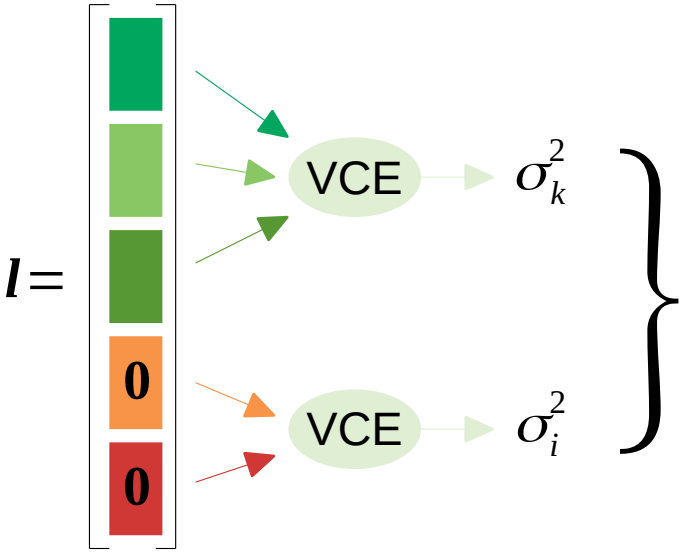

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information about observations introduced via σ_0^2

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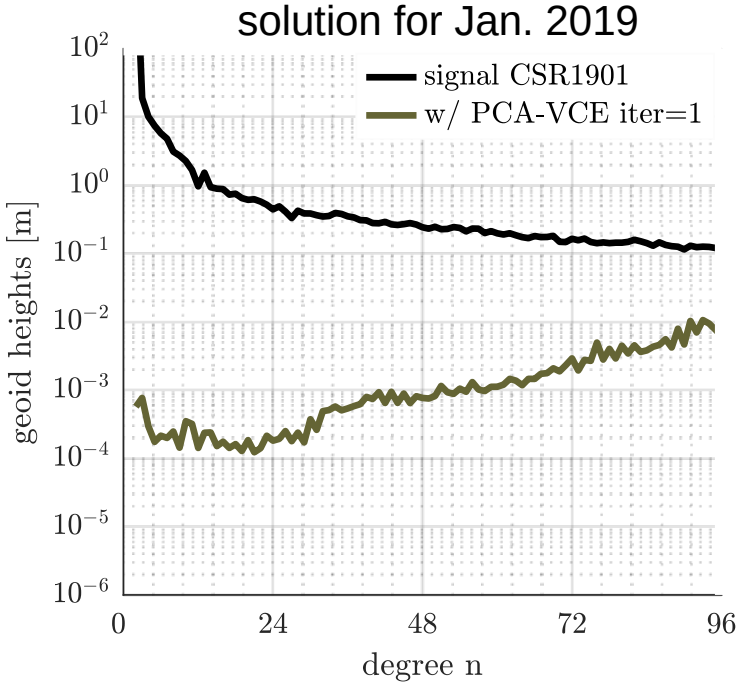
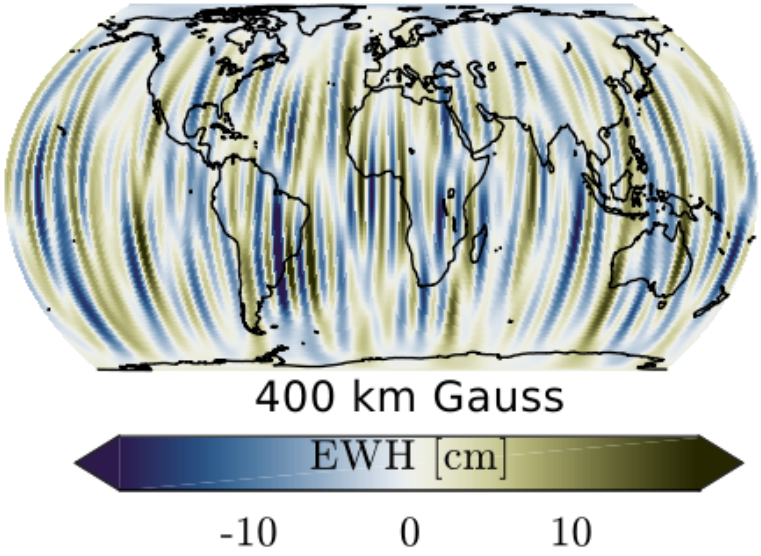
Results for VCE on constraints





radial 1×10^{-8} ms^{-2} along-track 1×10^{-8} ms^{-2} cross-track 1×10^{-8} ms^{-2}

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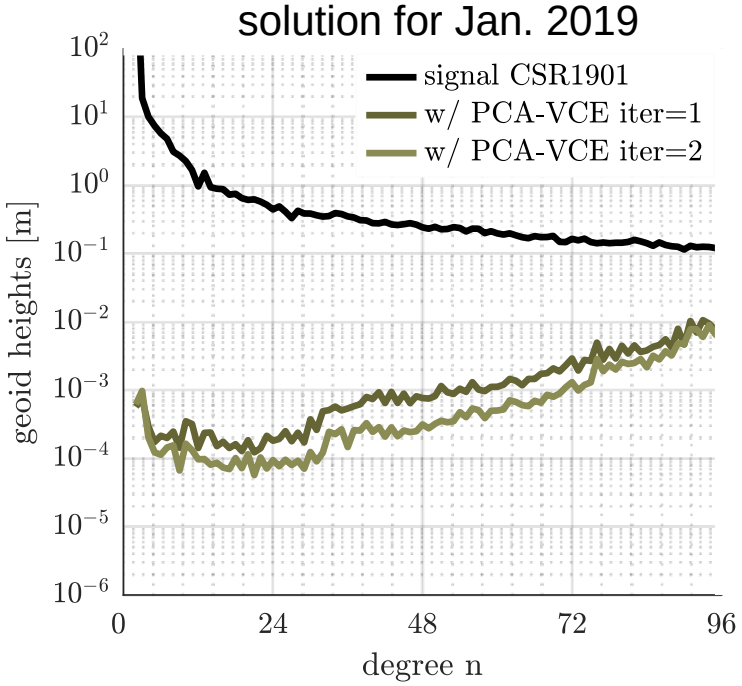

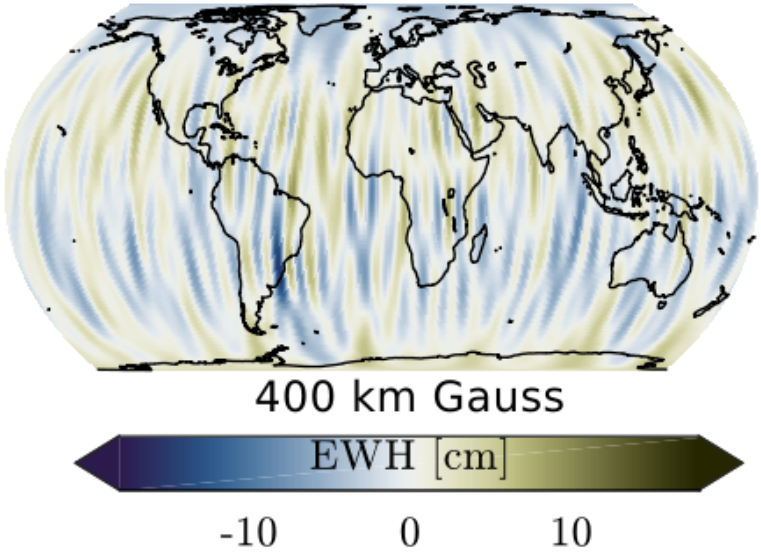

radial 1.9×10^{-8} ms^{-2} along-track 9.8×10^{-9} ms^{-2} cross-track 8.6×10^{-9} ms^{-2}

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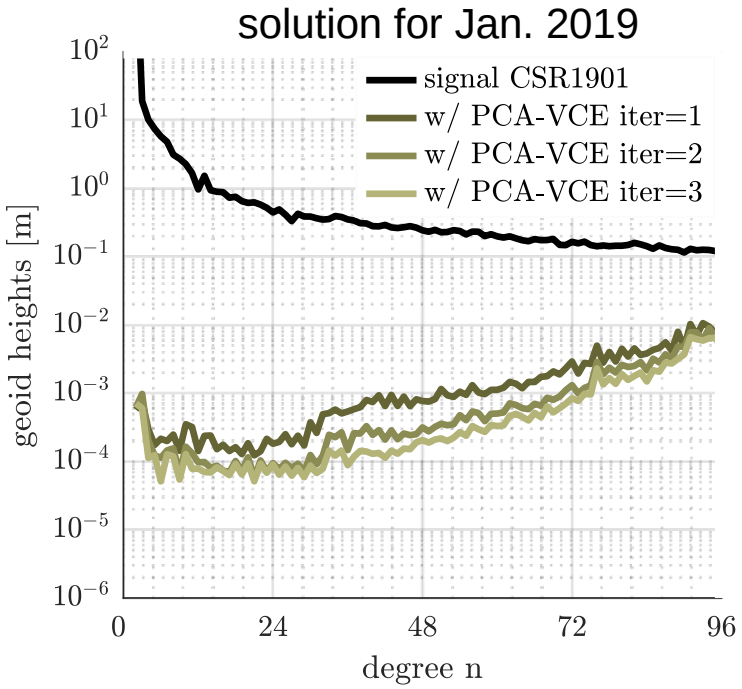
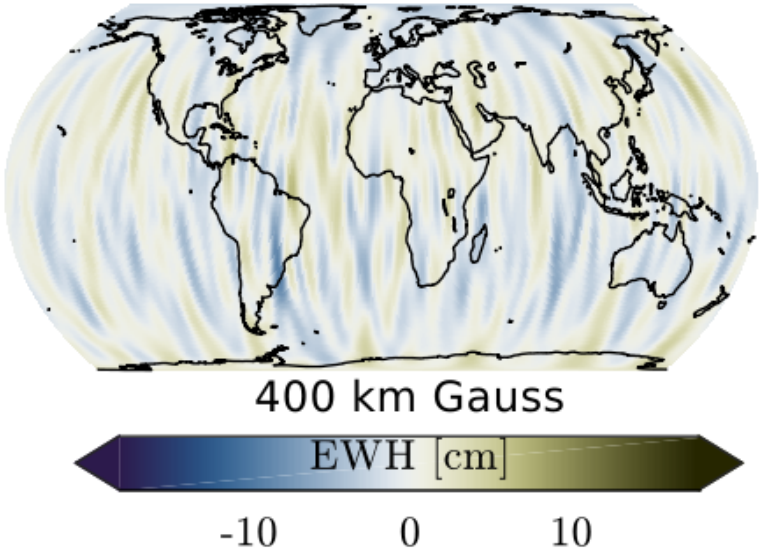
Results for VCE on constraints





radial 2.9×10^{-9} ms^{-2} along-track 1.5×10^{-9} ms^{-2} cross-track 1.3×10^{-9} ms^{-2}

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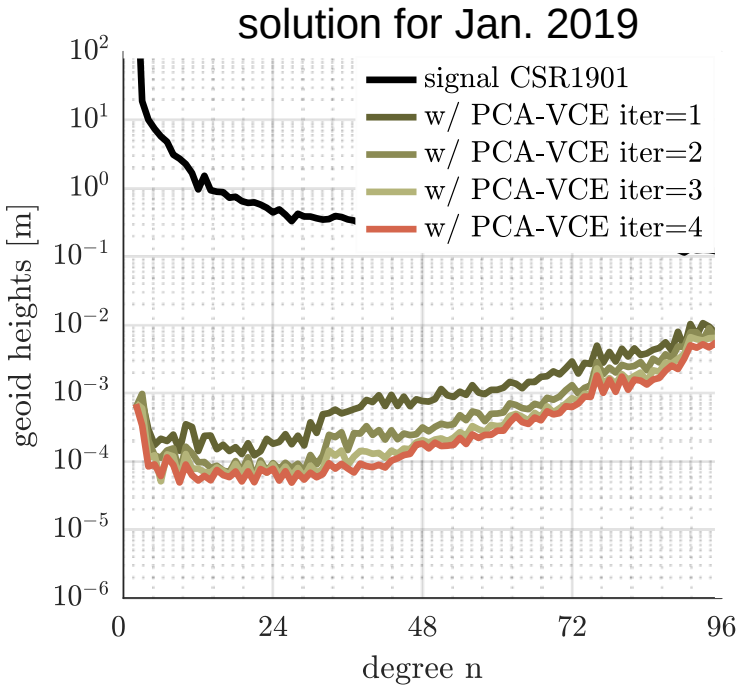

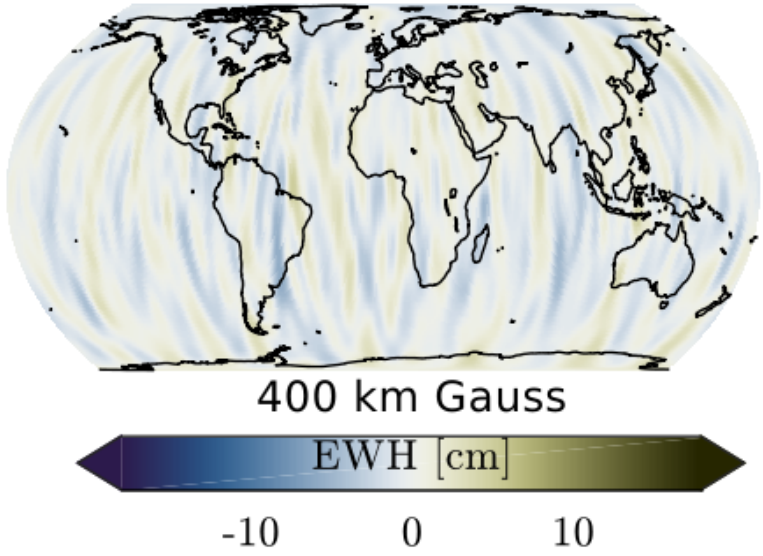

radial 1.2×10^{-9} ms^{-2} along-track 6.2×10^{-10} ms^{-2} cross-track 6.9×10^{-10} ms^{-2}

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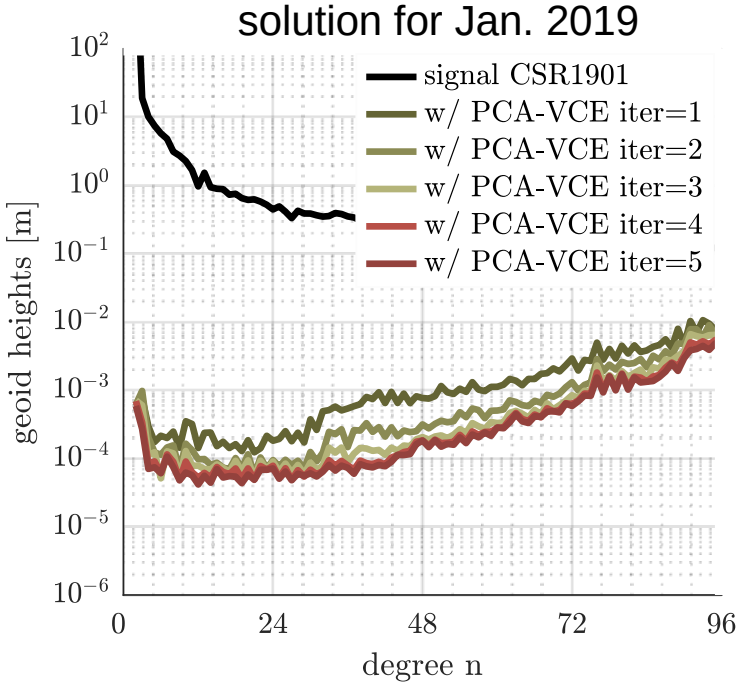

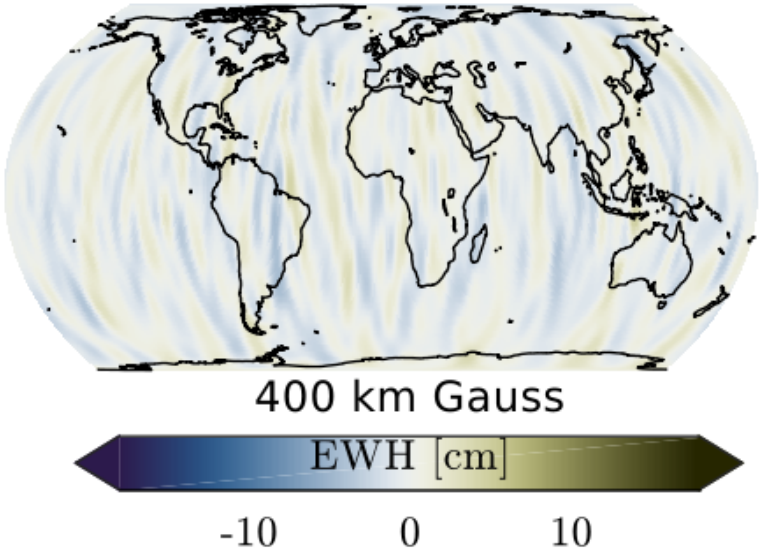

radial $5.6\times10^{-10}\,\mathrm{ms}^{-2}$ along-track $2.6\times10^{-10}\,\mathrm{ms}^{-2}$ cross-track $6.1\times10^{-10}\,\mathrm{ms}^{-2}$

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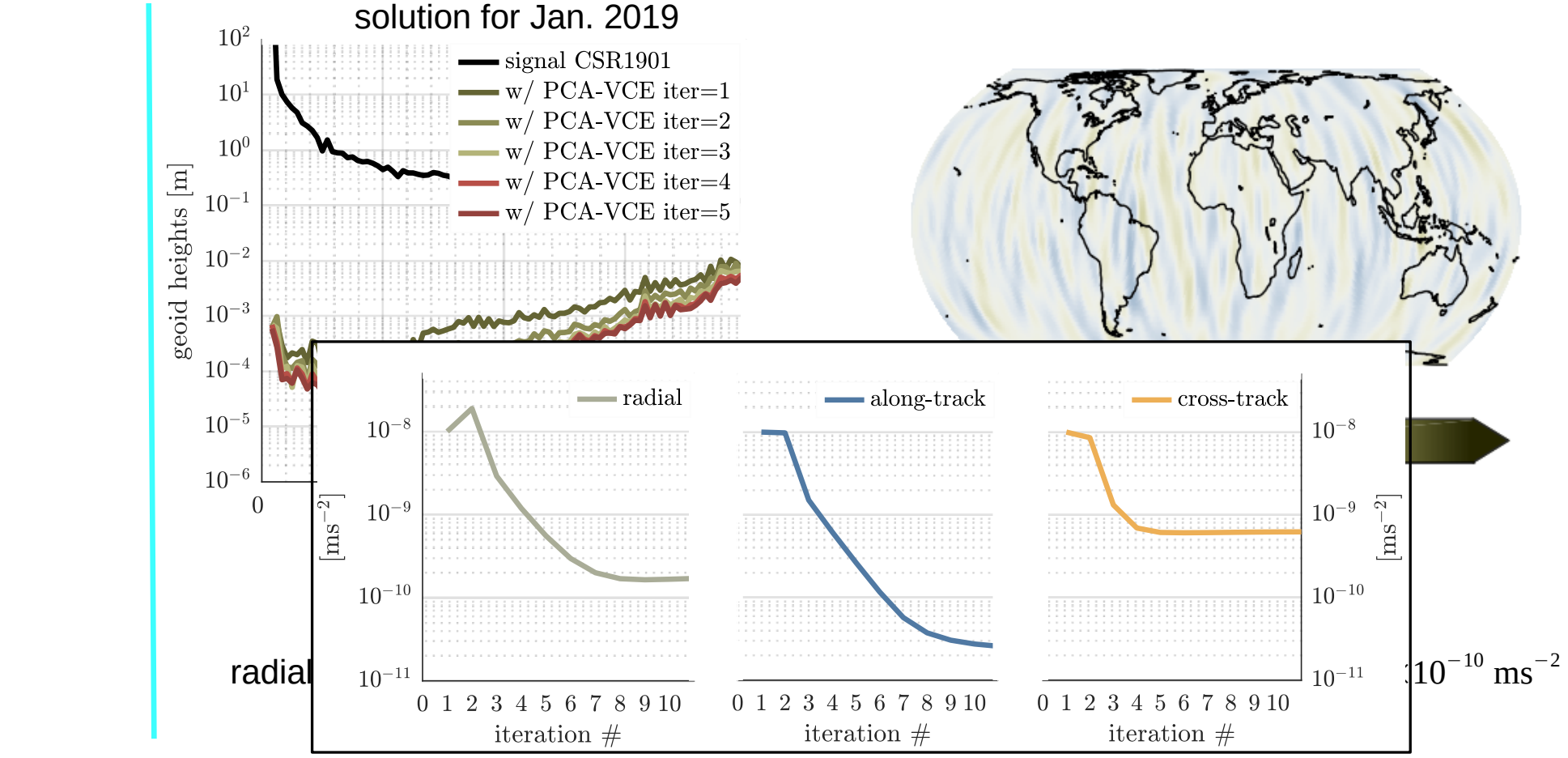
Results for VCE on constraints



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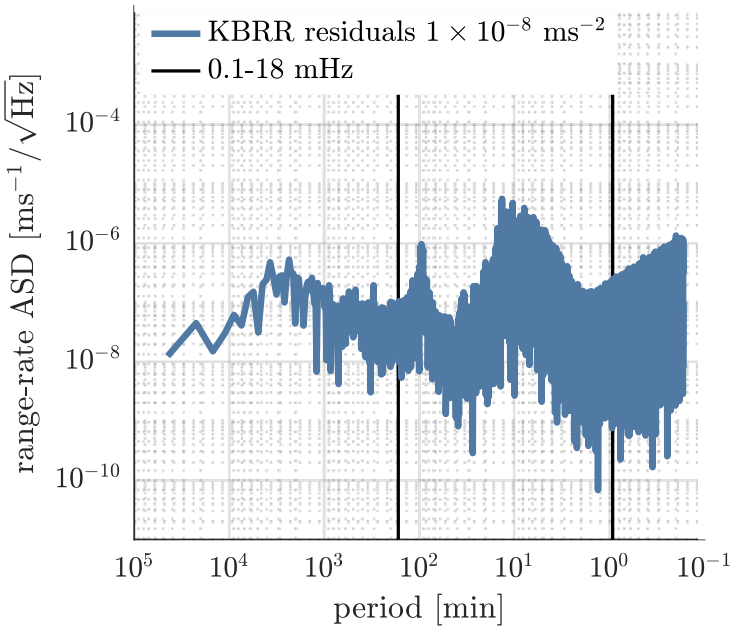
Results for VCE on constraints – post-fit residuals

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



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Results for VCE on constraints – post-fit residuals

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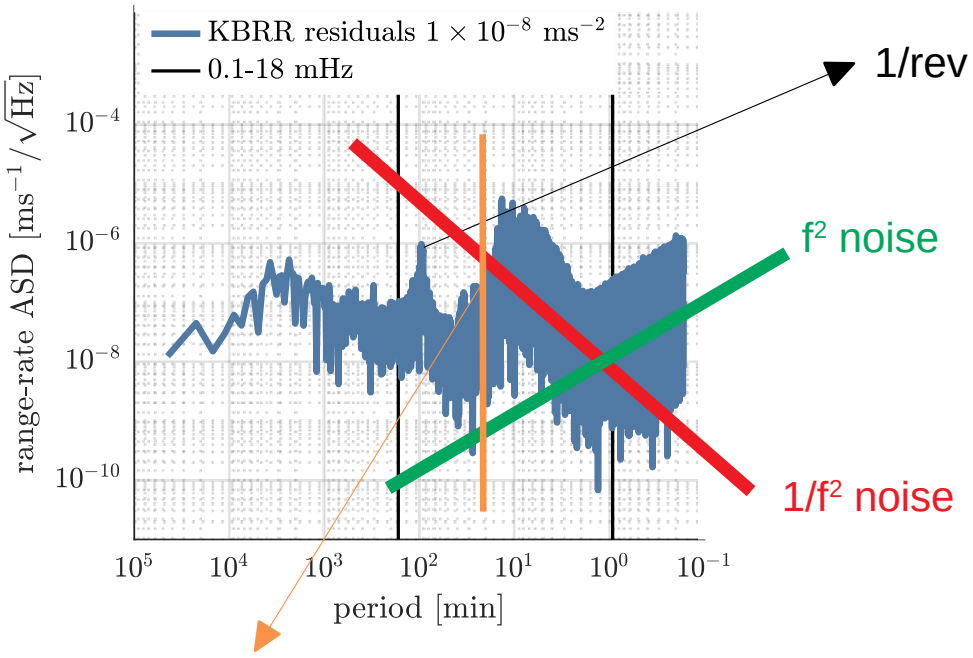

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Results for VCE on constraints – post-fit residuals

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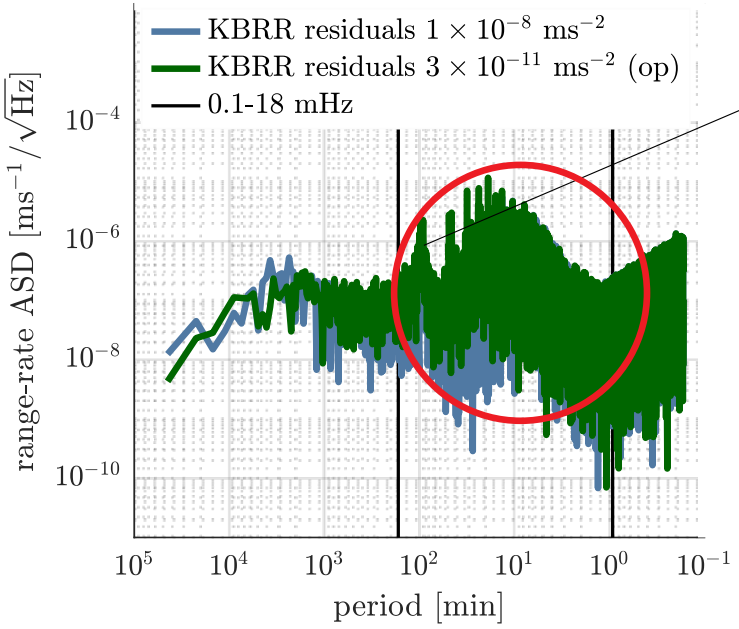

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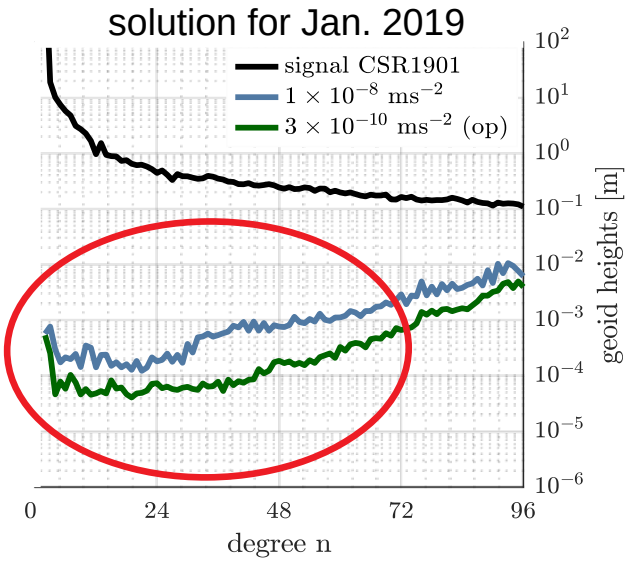

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Results for VCE on constraints – post-fit residuals

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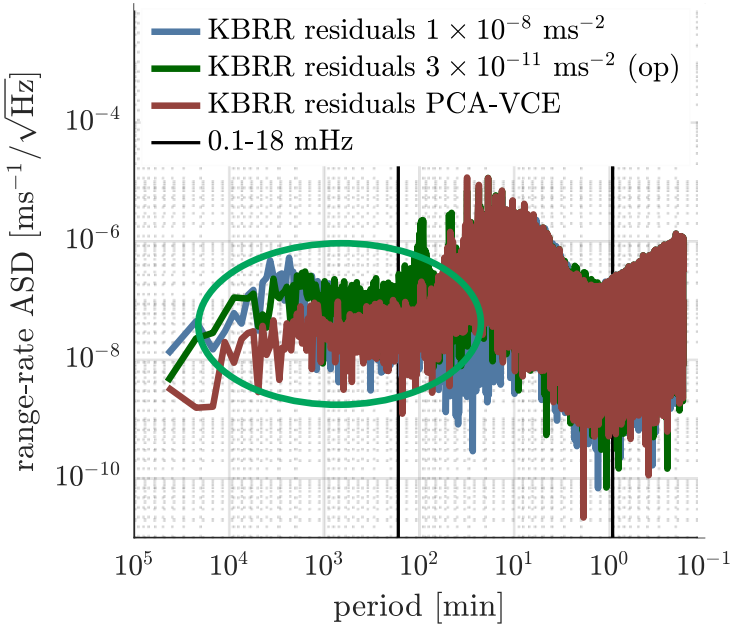
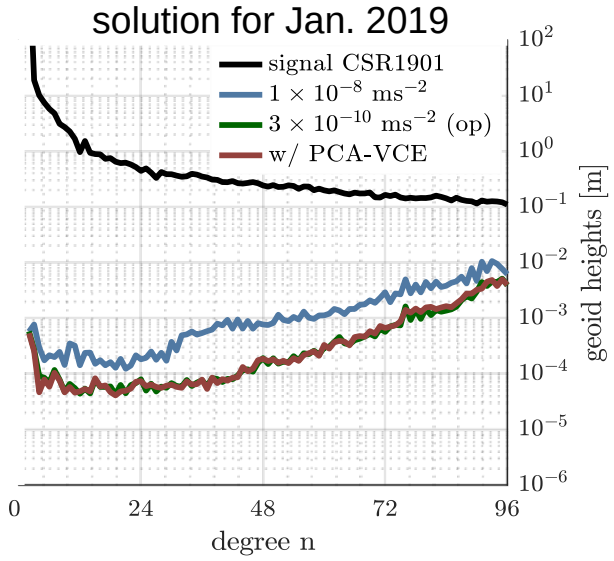
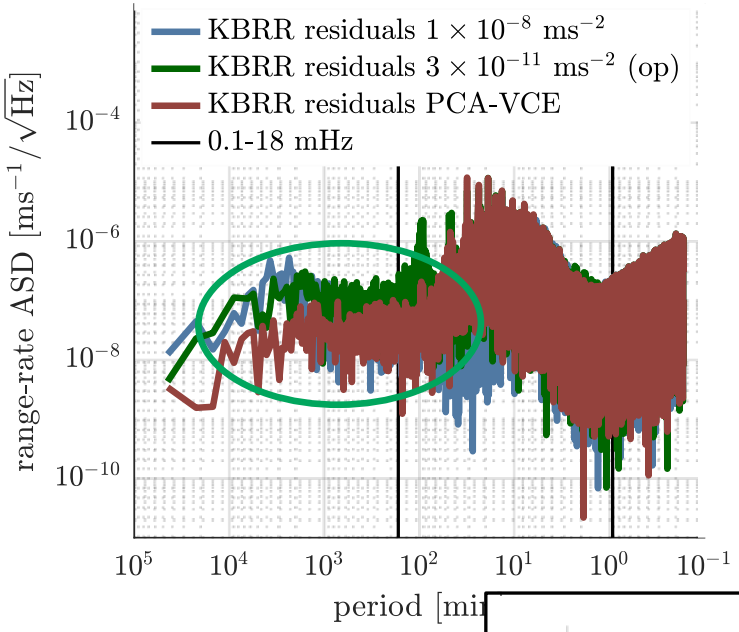


- almost flat spectrum for long periods
- indication for a “good” parametrisation of the respective signal



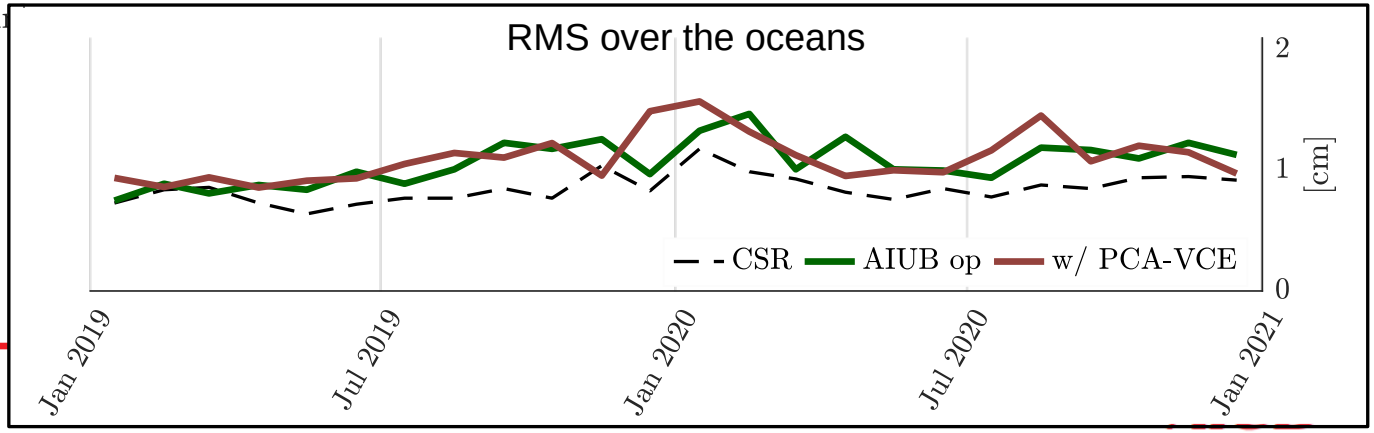
Results for VCE on constraints – summary

$$\hat{e} = l - A\,\hat{x}$$



- almost flat spectrum for long periods
- indication for a “good” parametrisation of the respective signal

- 12 months improved
- 12 months degraded



Results for VCE on constraints – summary

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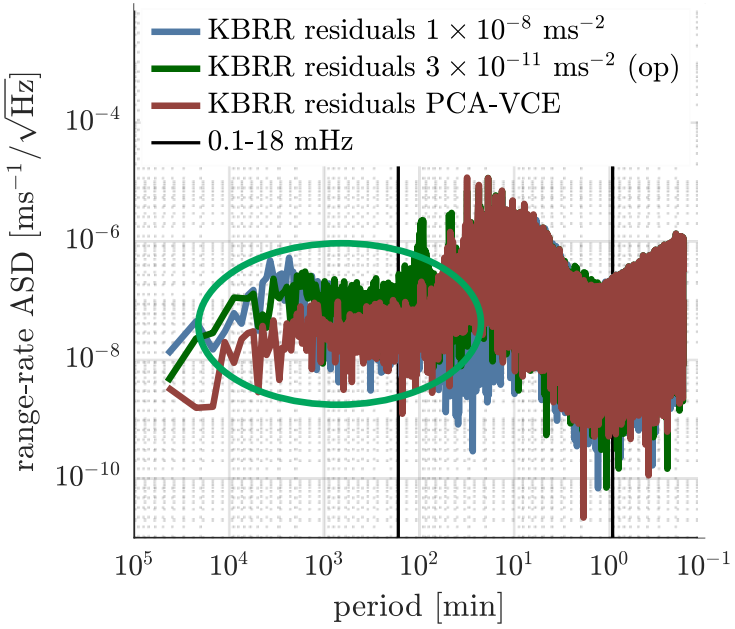

- observation-based approach
- computed together with the solution
- provides a good solution (if PCAs sample correctly)

- computational efficiency...
- observation-based – outliers

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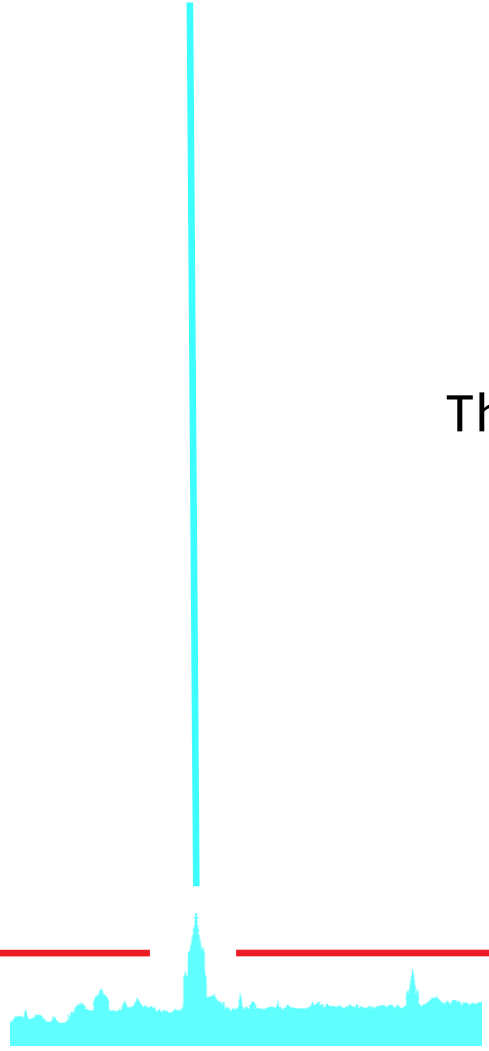
Thank you for your attention



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