

Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery

Martin Lasser, Ulrich Meyer, Daniel Arnold, Adrian Jäggi

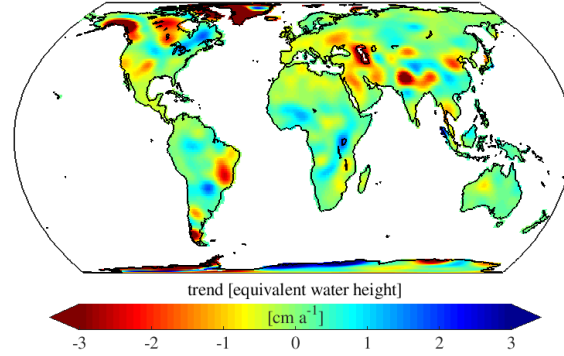
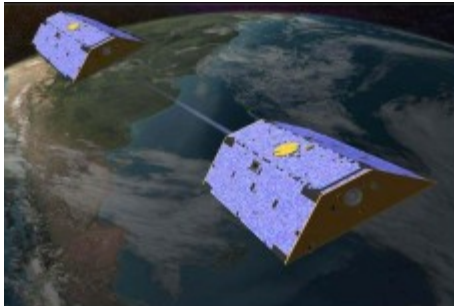
Astronomical Institute, University of Bern, Switzerland

EGU General Assembly 2022

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Vienna, Austria

Introduction

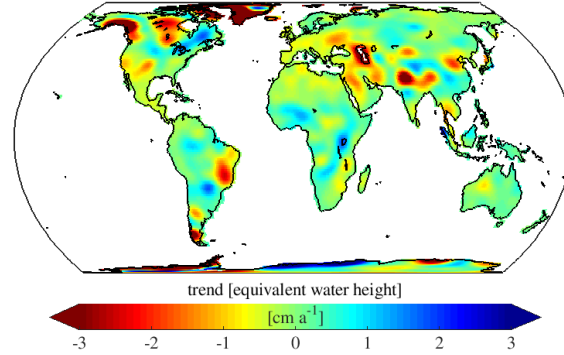
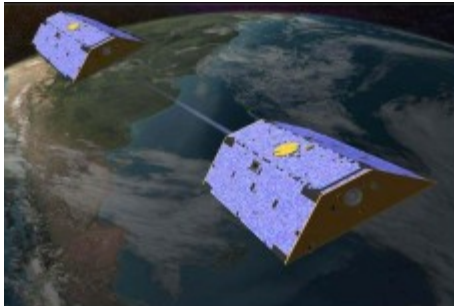


basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelerometer scaling 2x(3)

parameters per arc 24

Introduction



Perturbation theory [Kim, 2000]:
Errors in background models will (mostly) sum up in 1/rev
→ frequently used in the Celestial Mechanics Approach
[Beutler et al., 2010]

basic parametrisation:

- initial conditions $2 \times (6)$
- accelerometer bias $2 \times (3)$
- accelerometer scaling $2 \times (3)$

parameters per arc 24

additional parameters:

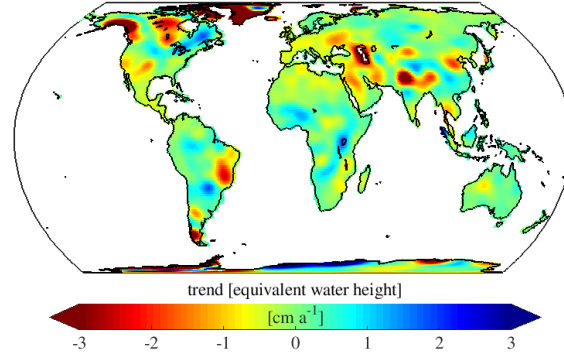
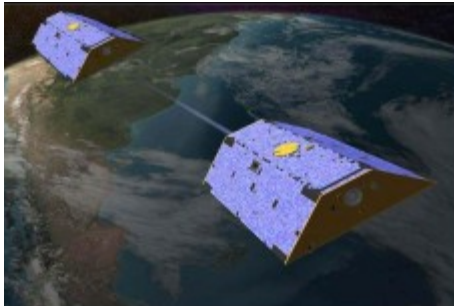
- 15 min PCA per satellite in
 - radial $2 \times (96)$
 - along-track $2 \times (96)$
 - cross-track $2 \times (96)$

parameters per arc 576

in daily arcs (30 days):

- 18000 parameters,
- 17280 for the noise model
- + gravity field

Introduction



Perturbation theory [Kim, 2000]:
Errors in background models will (mostly) sum up in 1/rev
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How to constrain their impact
to the correct magnitude?

basic parametrisation:

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parameters per arc 24

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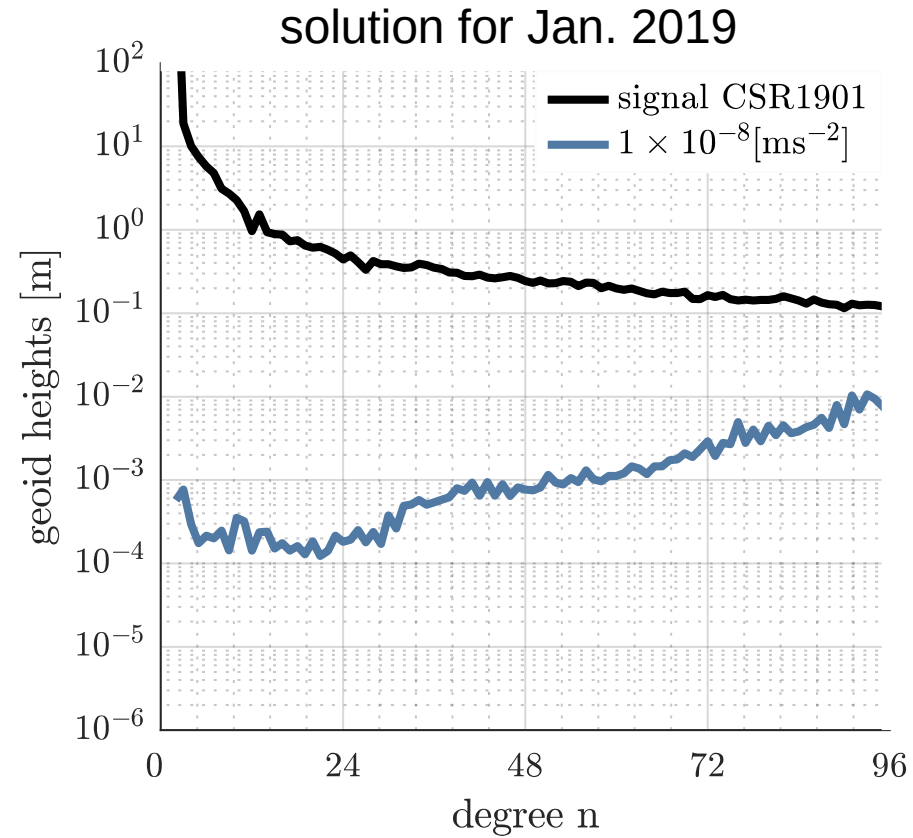
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Impact of different constraints

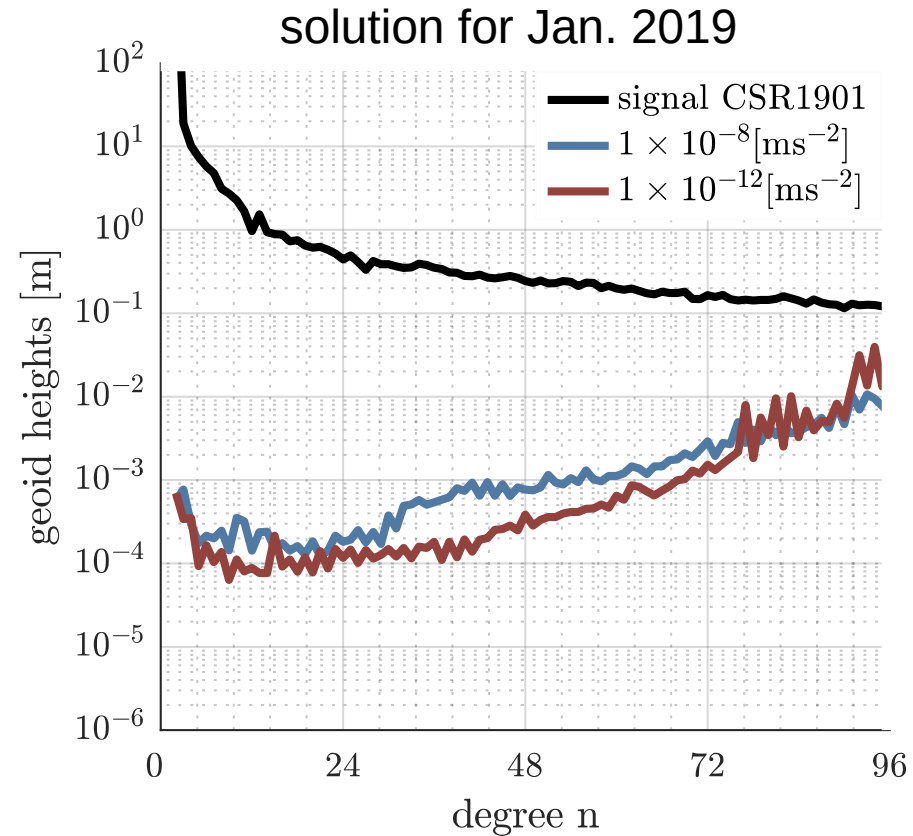


$$1 \times 10^{-8} \text{ ms}^{-2}$$

«loose» constraint

(gravity field signal absorbed in PCAs)

Impact of different constraints

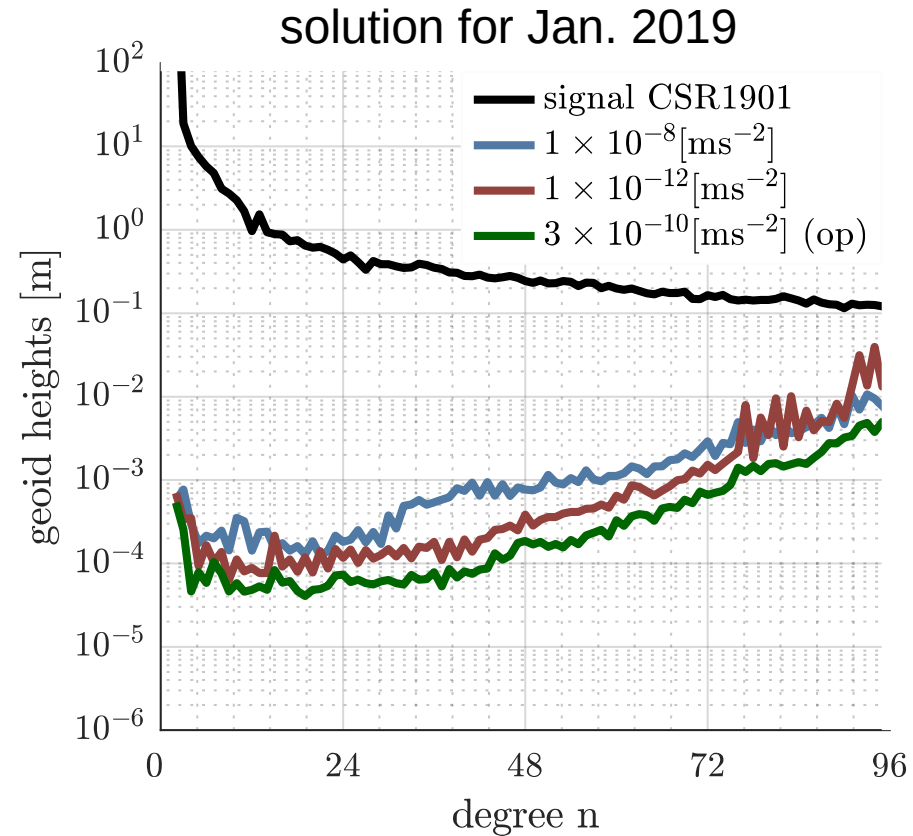


$$1 \times 10^{-12} \text{ ms}^{-2}$$

«tight» constraint

(not enough to absorb mis-modellings)

Impact of different constraints



$$3 \times 10^{-10} \text{ ms}^{-2}$$

«reasonable» balance

(applied in the operational solutions)

Constraining

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A})$$

and $\mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l}$ \longrightarrow $\hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{b}$

design matrix weight matrix vector of observations

Constraining

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A}) \quad \text{and} \quad \mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l} \quad \longrightarrow \quad \hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{b}$$

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A} + \mathbf{W})$$

$$\mathbf{N} = \begin{bmatrix} \text{dark blue} & \text{blue} & \text{blue} & \text{light green} & \text{blue} & \text{teal} \\ \text{blue} & \text{dark blue} & \text{blue} & \text{blue} & \text{light green} & \text{blue} \\ \text{blue} & \text{blue} & \text{dark blue} & \text{blue} & \text{blue} & \text{light green} \\ \text{light green} & \text{blue} & \text{blue} & \text{dark blue} & \text{blue} & \text{blue} \\ \text{blue} & \text{light green} & \text{blue} & \text{blue} & \text{dark blue} & \text{blue} \\ \text{teal} & \text{blue} & \text{light green} & \text{blue} & \text{blue} & \text{dark blue} \end{bmatrix} + \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

$$\begin{bmatrix} \text{orange} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{matrix} \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \end{matrix} \frac{\sigma_0^2}{\sigma_{PCA}^2},$$

$$\sigma_{PCA}^2 = \text{e.g., } 3 \times 10^{-10} \text{ ms}^{-2}$$

Variance Component Estimation

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A}) \quad \text{and} \quad \mathbf{b} = \mathbf{A}^T \mathbf{P} \mathbf{l} \quad \longrightarrow \quad \hat{\mathbf{x}} = \mathbf{N}^{-1} \mathbf{b}$$

$$\mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A} + \mathbf{W})$$

$$\mathbf{l} = \begin{bmatrix} \text{green} \\ \text{light green} \\ \text{dark green} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

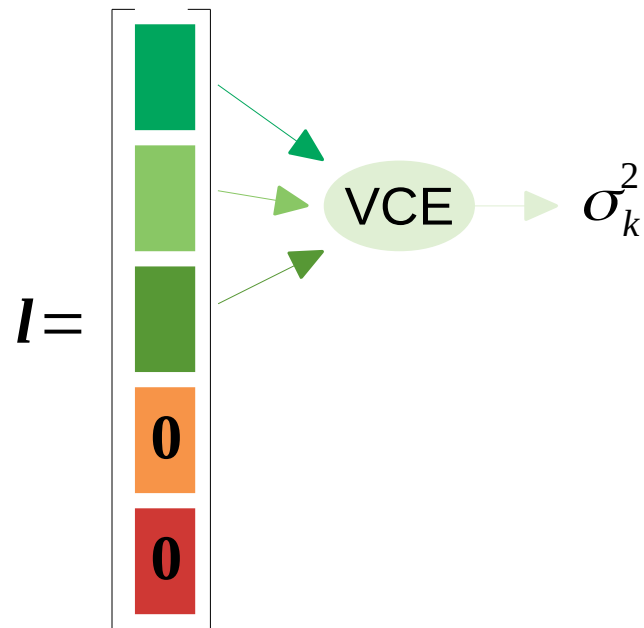
M. Lasser, U. Meyer, D. Arnold, A. Jäggi: Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery, EGU General Assembly 2022, 26 May, 2022

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VCE: Each group of observations gets a weight based on its contribution to the final solution

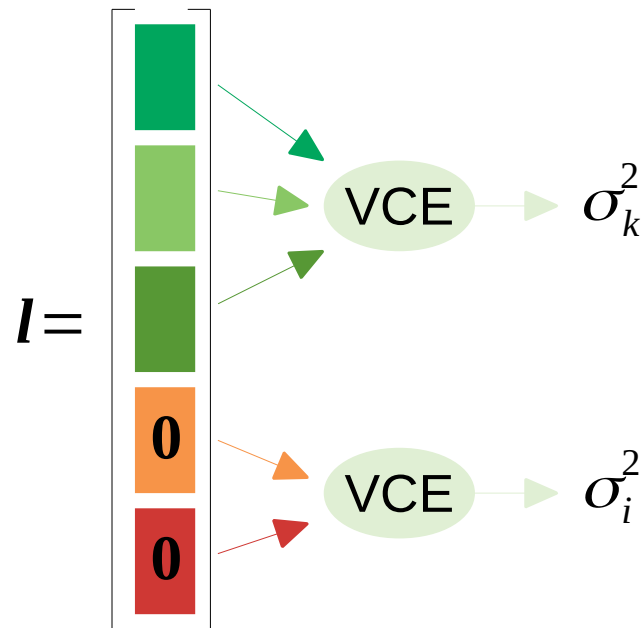


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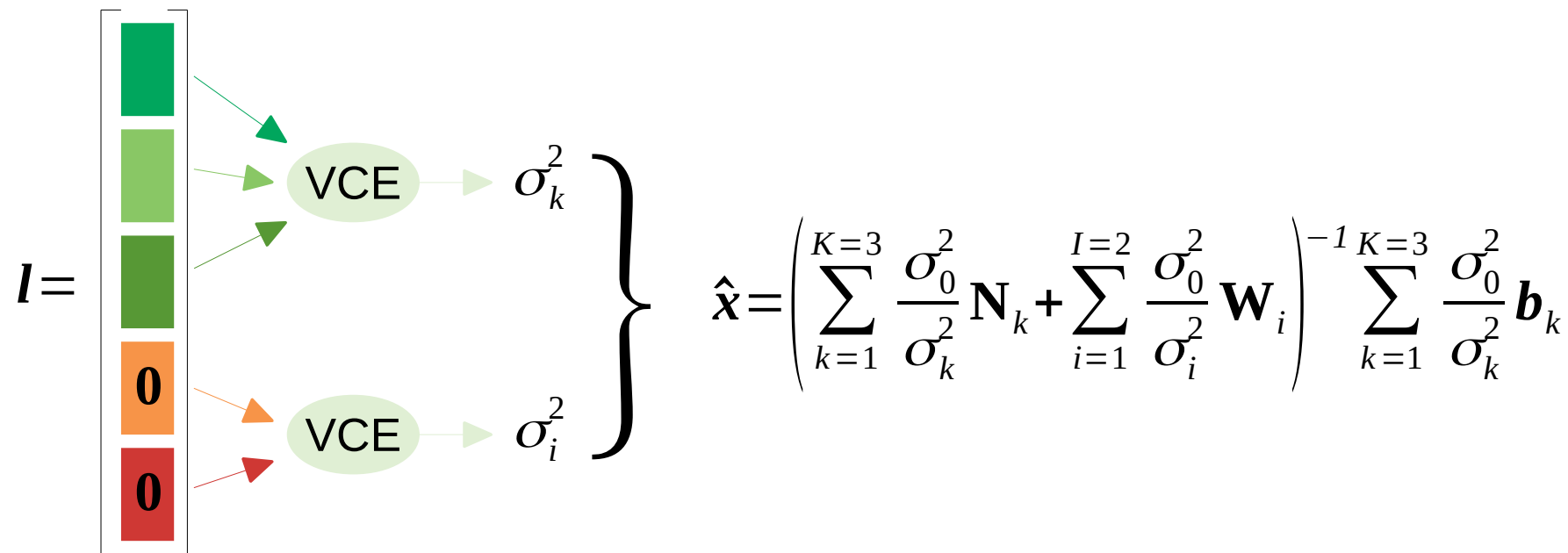


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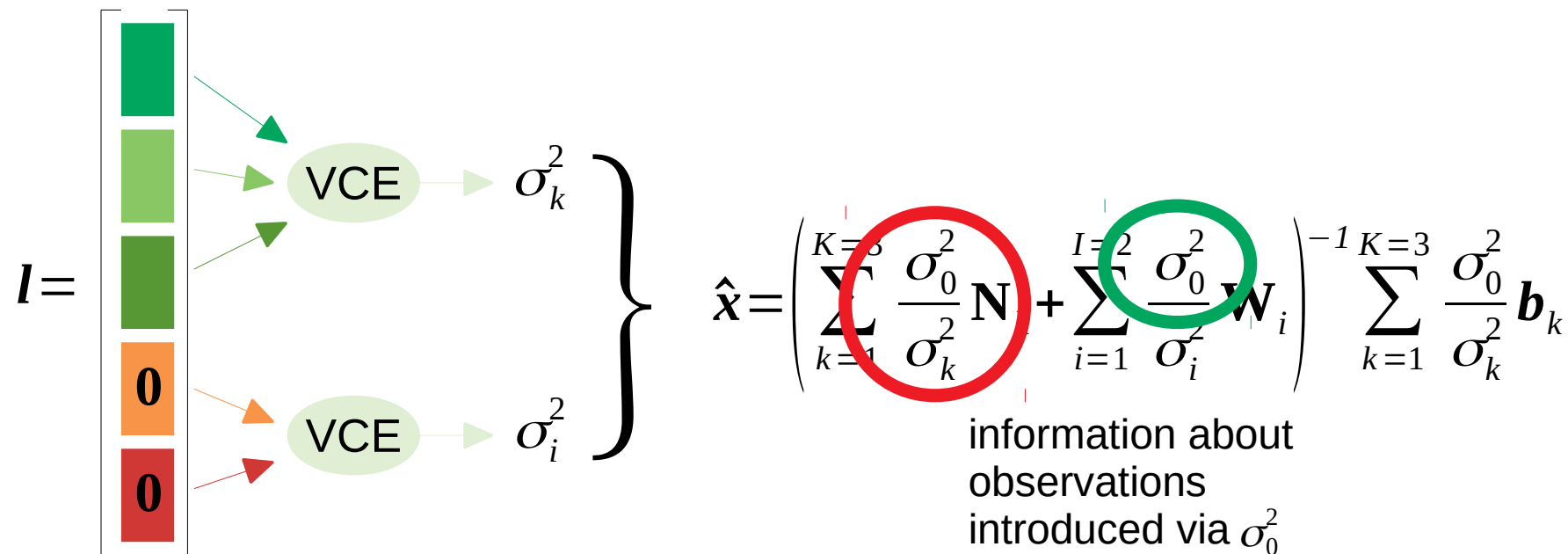


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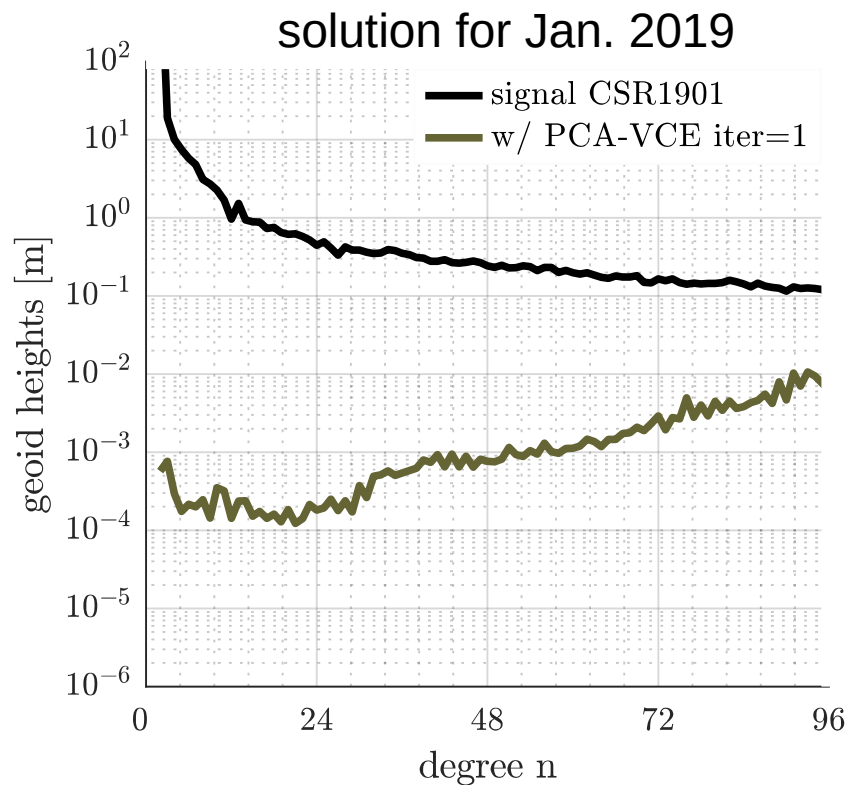
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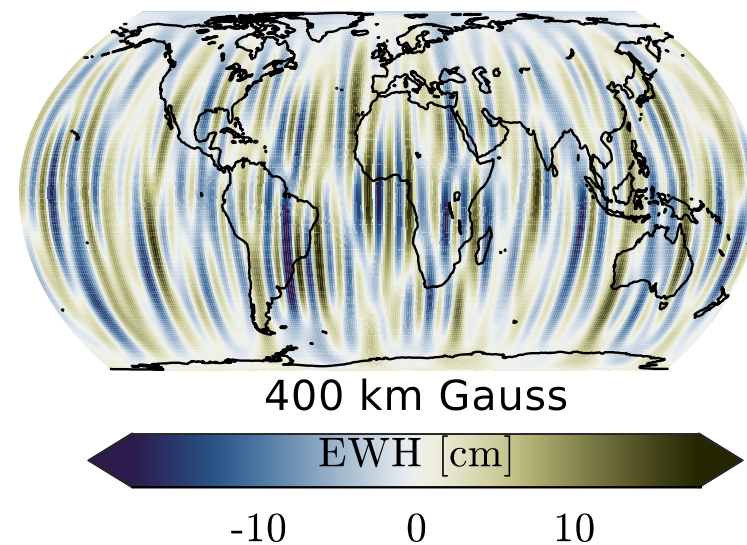


Results

M. Lasser, U. Meyer, D. Arnold, A. Jäggi: Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery, EGU General Assembly 2022, 26 May, 2022

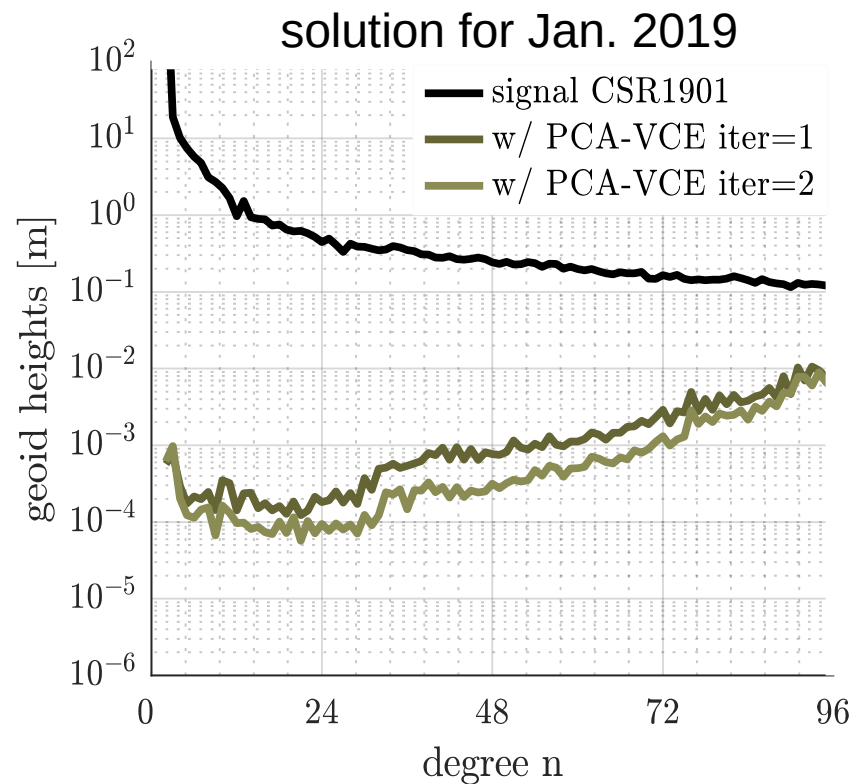


	ms^{-2}
radial	1×10^{-8}
along-track	1×10^{-8}
cross-track	1×10^{-8}

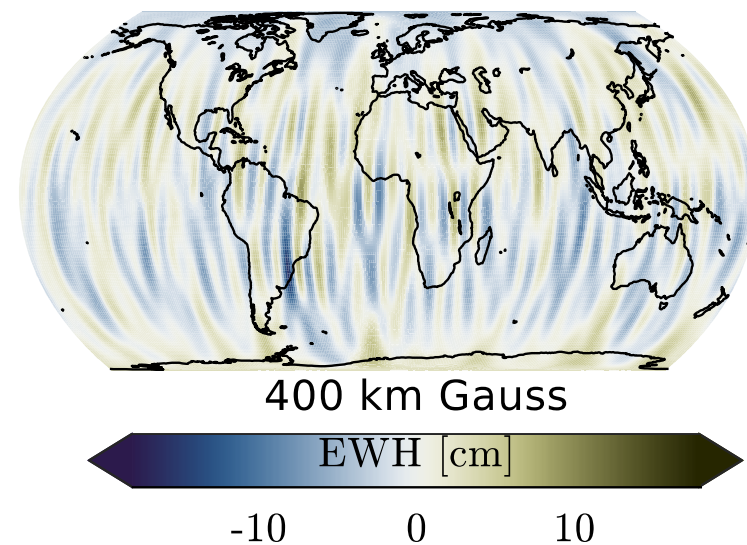


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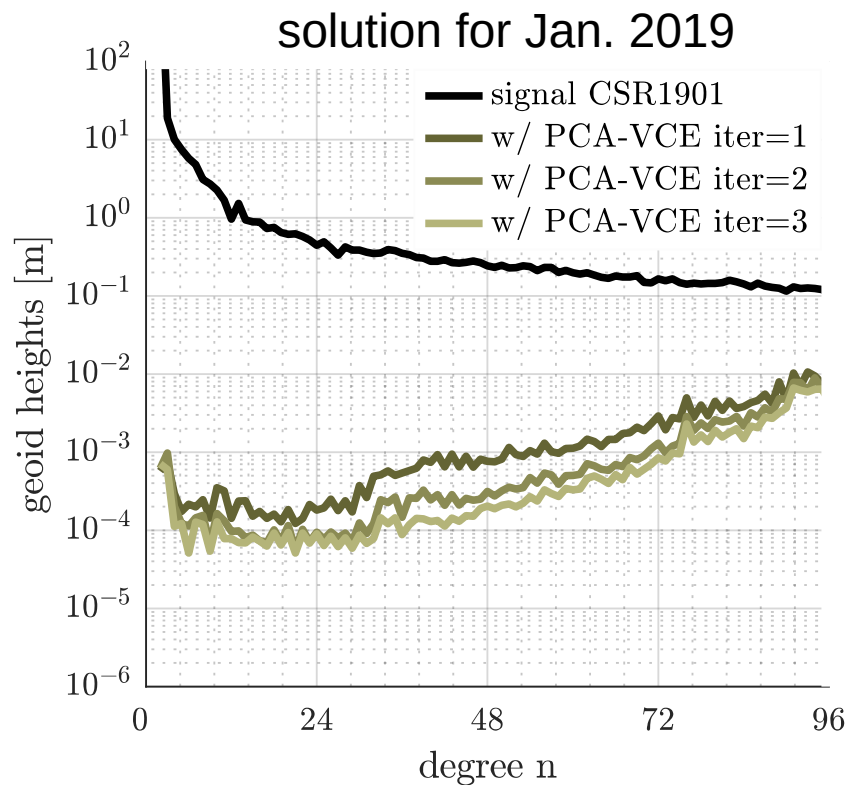


	ms^{-2}
radial	1.9×10^{-8}
along-track	9.8×10^{-9}
cross-track	8.6×10^{-9}

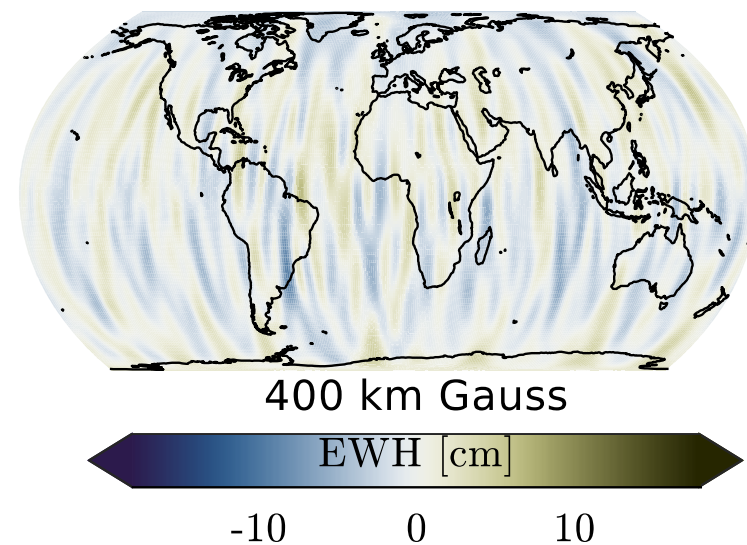


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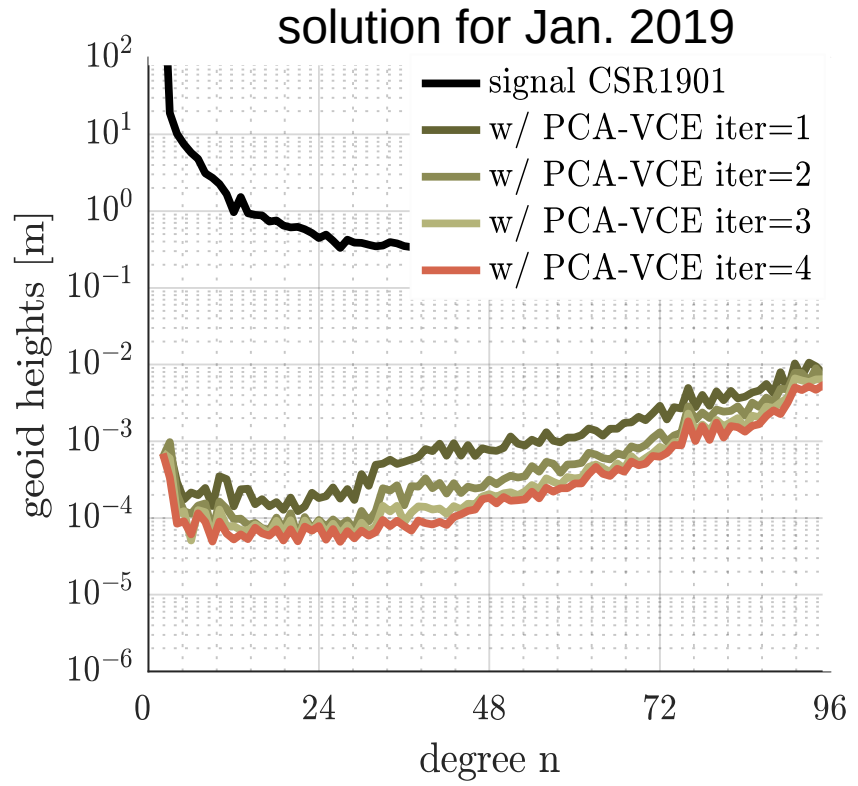


	ms^{-2}
radial	2.9×10^{-9}
along-track	1.5×10^{-9}
cross-track	1.3×10^{-9}

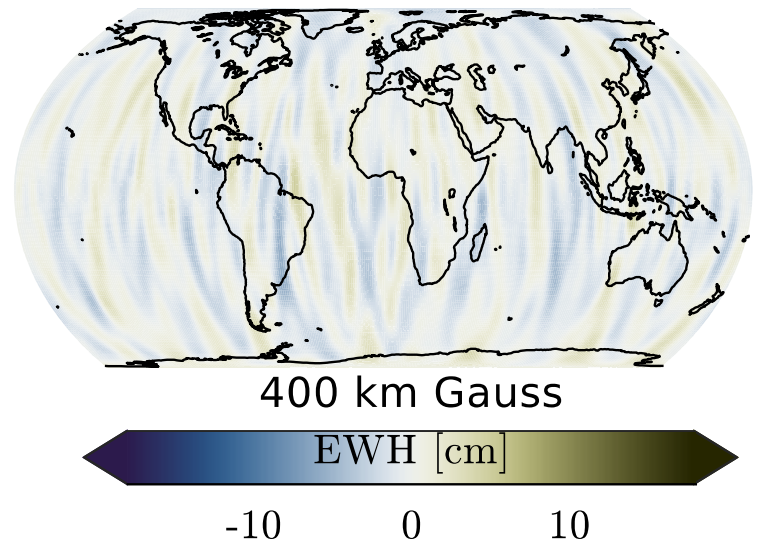


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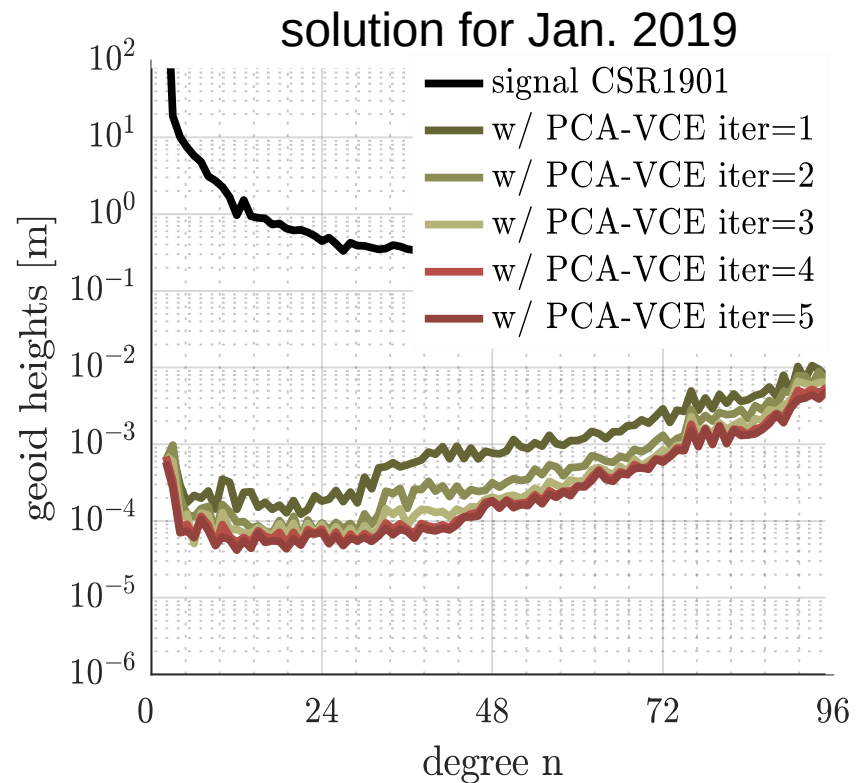


	ms^{-2}
radial	1.2×10^{-9}
along-track	6.2×10^{-10}
cross-track	6.9×10^{-10}

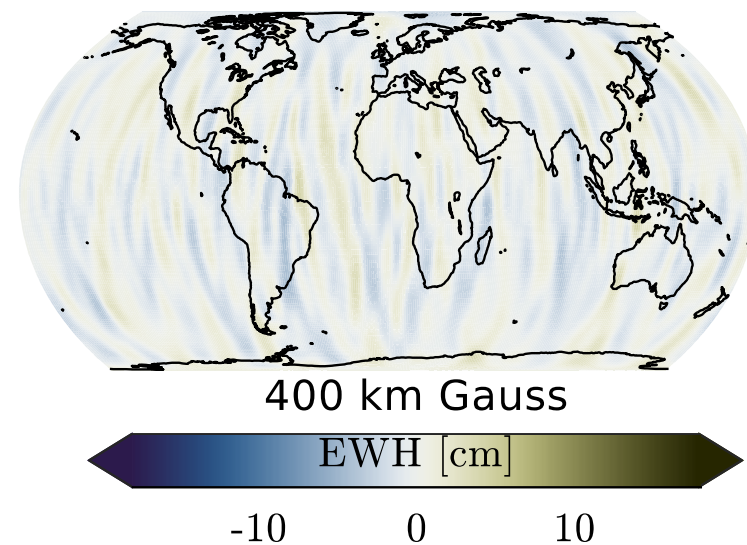


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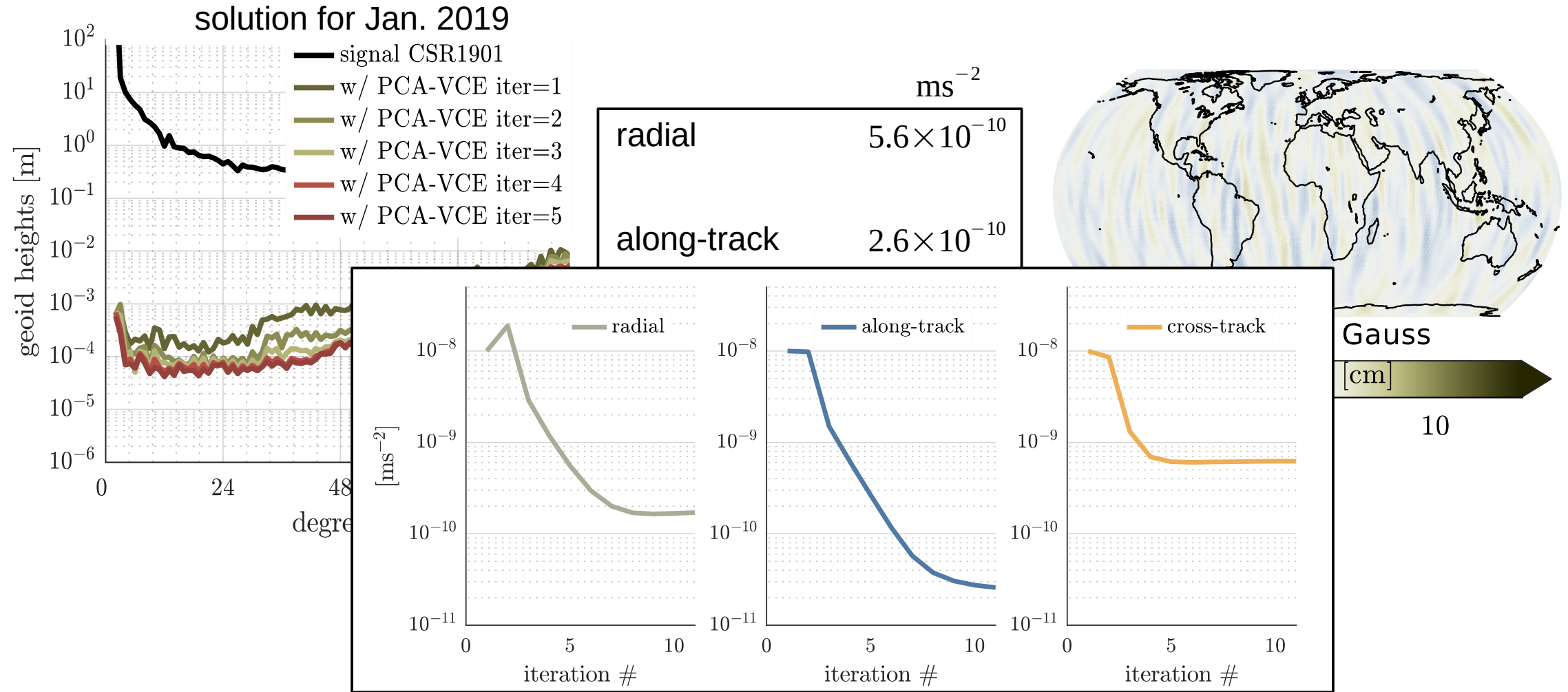


	ms^{-2}
radial	5.6×10^{-10}
along-track	2.6×10^{-10}
cross-track	6.1×10^{-10}



Results

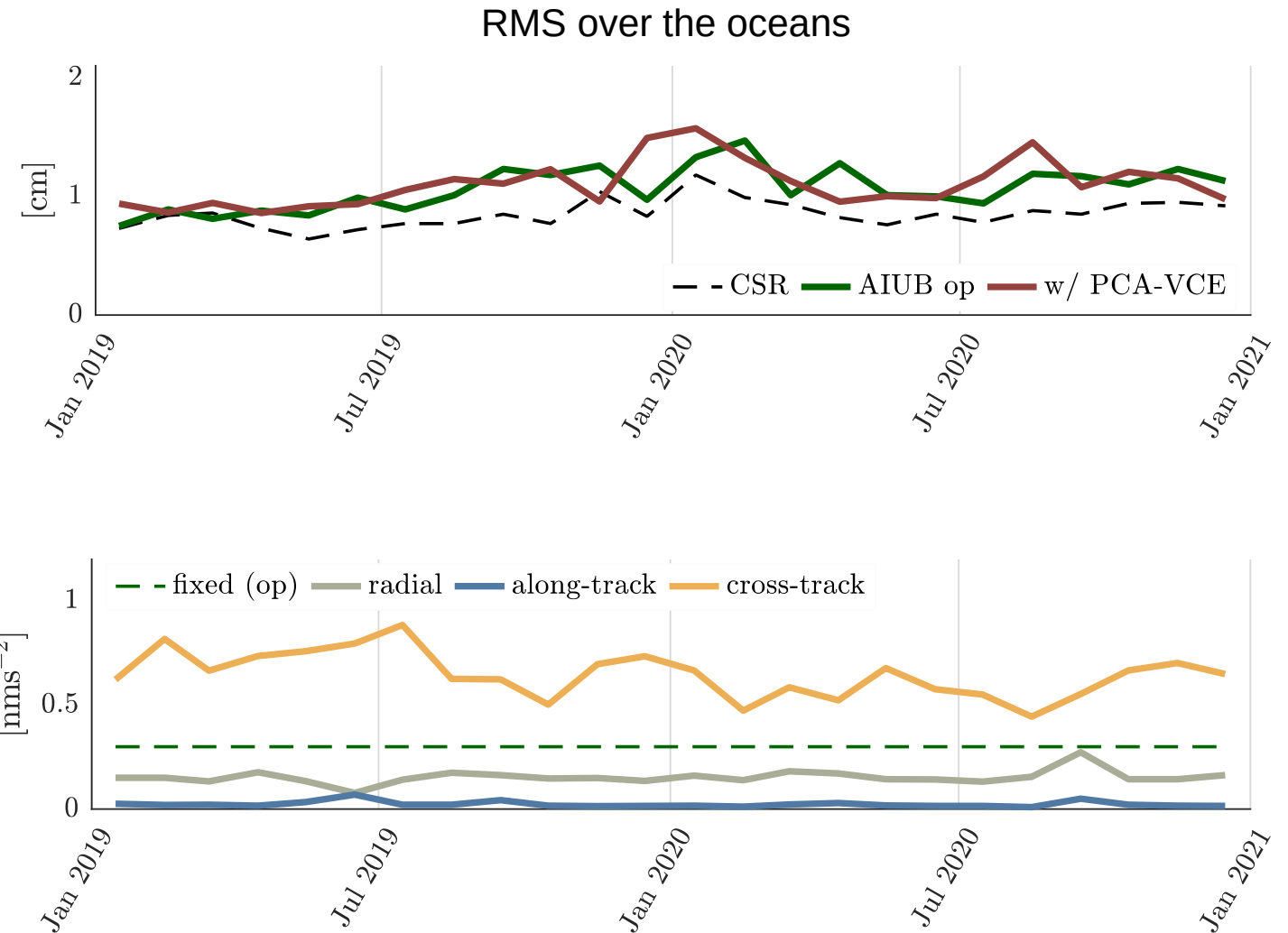
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Conclusion

- observation-based approach
- computed together with the solution
- provides a good solution (if PCAs sample correctly)
- improvement...

- computational efficiency?
- observation-based – outliers
- improvement...



References

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