

# Empirical SRP model for the orbit normal attitude mode

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# Introduction

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- CODE MGEX (COM) orbit and clock solution since 2012 (see doi: [10.1007/s00190-016-0968-8](https://doi.org/10.1007/s00190-016-0968-8) for description; changes from 2015 on summarized in next IUGG report); SRP model: ECOM2
- BeiDou2 (BDS2) MEO+IGSO since late 2013; QZSS since early 2014
- BDS2 and QZS-1: Orbit-Normal (ON) attitude during eclipses
- ECOM models designed for Yaw-steering (YS); not suitable for ON-mode ( $\Rightarrow$  very poor orbit quality during ON periods)
- Other groups: ECOM plus box-wing a priori model (Montenbruck et al. 2017, Zhao et al. 2018) for individual satellites or additional along-track acceleration parameters (e.g., Guo et al. 2017)
- Our goal: definition of an ECOM suitable for the ON-mode (to be used stand-alone or together with an a priori model)

# Attitude modes of GNSS satellites

| GNSS                            | YS | Ecl. law<br>(small $\beta$ ) | ON                         | Other |
|---------------------------------|----|------------------------------|----------------------------|-------|
| GPS                             | X  | x                            |                            |       |
| GLONASS                         | X  | x                            |                            |       |
| Galileo                         | X  | x                            |                            |       |
| BDS2 MEO, IGSO                  | X  | x                            | x ( $ \beta  < 4^\circ$ )  |       |
| BDS3 MEO, IGSO                  | X  | x                            |                            |       |
| QZSS IGSO                       | X  | x                            | x ( $ \beta  < 17^\circ$ ) |       |
| IRNSS                           |    |                              |                            | X     |
| GEO (SBAS, BDS, QZSS, non GNSS) |    |                              | X                          |       |

(see doi 10.1016/j.asr.2015.06.019 for detailed information)

- MEO and IGSO satellites (BDS2, QZS-1): ON-mode is a special case of an eclipse attitude law (applied to avoid rapid noon and midnight turns at small  $\beta$ -angles)
- GEO satellites keep ON permanently ( $|\beta| < (23.5^\circ + i)$ )

# Solar Radiation Pressure (SRP) - definitions

SRP acceleration due to illumination of a surface (without re-radiation)  
according to Milani (1987):

$$\mathbf{a}_{tot} = \mathbf{a}_\alpha + \mathbf{a}_\rho + \mathbf{a}_\delta$$

$$\mathbf{a}_\alpha = -PF \cos \theta \alpha \mathbf{e}_D \quad \text{absorption (coeff. } \alpha \text{)}$$

$$\mathbf{a}_\rho = -PF 2 \cos^2 \theta \rho \mathbf{e}_{SN} \quad \text{specular reflection (coeff. } \rho \text{)}$$

$$\mathbf{a}_\delta = -PF \cos \theta \left( \delta \mathbf{e}_D + \frac{2}{3} \delta \mathbf{e}_{SN} \right) \quad \text{diffuse reflection (coeff. } \delta \text{)}$$

$PF$ : Pre-factor; function of solar flux, speed of light, AMR

$\mathbf{e}_D$ : Unit vector in Sat.-Sun direction

$\mathbf{e}_W$ : Unit vector normal to orbital plane

with  $\mathbf{e}_{SN}, \mathbf{e}_{SPN}$ : Unit vectors normal to surface  $i$  and to solar panel

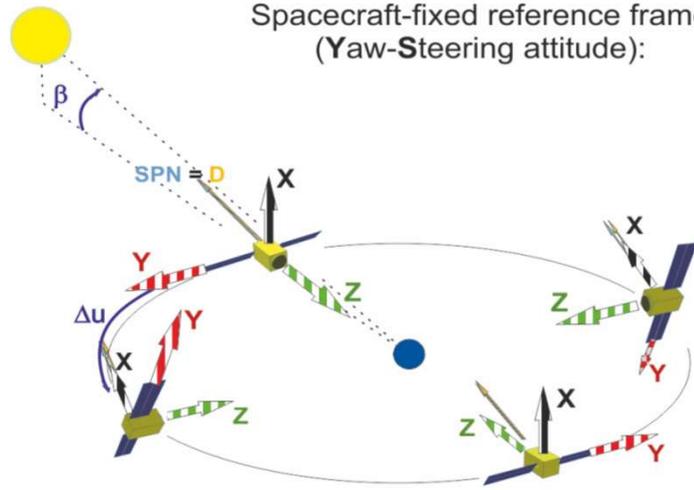
$\theta$ : Angle enclosed by  $\mathbf{e}_D$  and  $\mathbf{e}_{SN}$

$\beta$ : Elevation of the Sun above the orbital plane

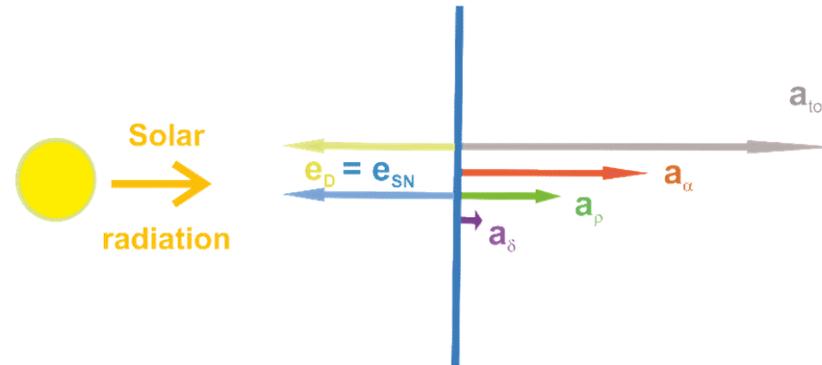
$\Delta u$ : Difference between arguments of latitude of SC and Sun

# SRP during YS mode

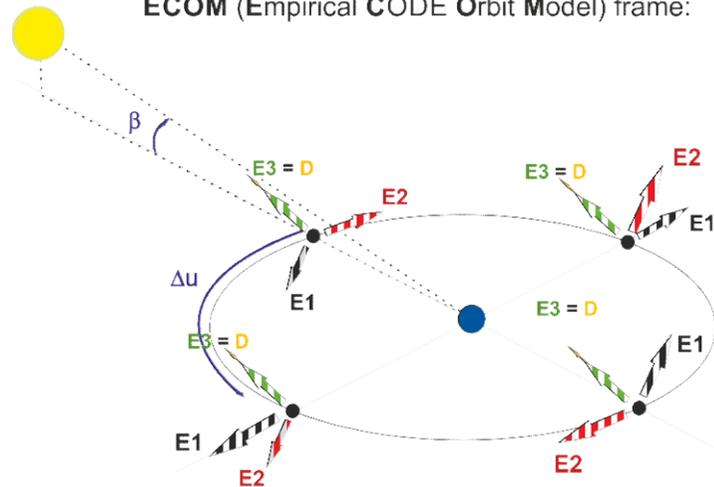
Spacecraft-fixed reference frame (Yaw-Steering attitude):



Acceleration due to solar radiation (Yaw-Steering attitude):

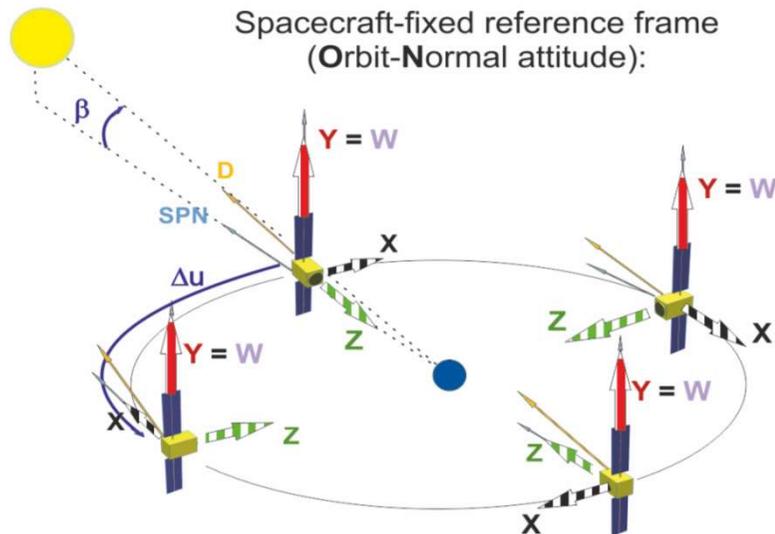


ECOM (Empirical CODE Orbit Model) frame:

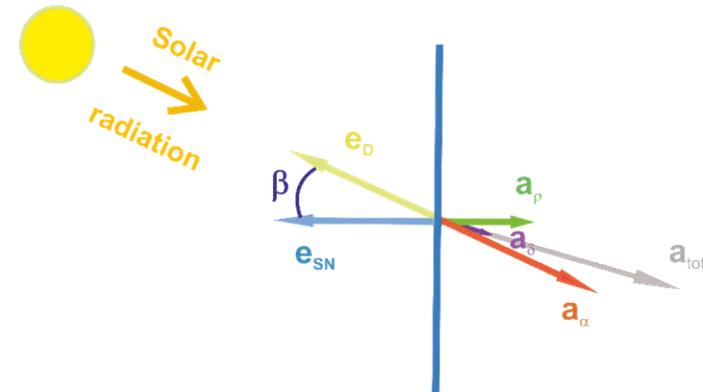


- Solar panel (SP) plane is always parallel to the terminator plane and normal to the vector Satellite-Sun ( $D$ ) ( $\Rightarrow \theta = 0$ )
- SRP due to SP: covered by one parameter in  $D$
- SC's Y-sides: are never illuminated ( $\Rightarrow E2=0$ ; exception: Y-bias)
- SC body (+Z, -Z, +X panels): causes additional SRP signal, periodic w.r.t.  $\Delta u$  in  $D$  and  $E1$

# SRP during ON mode



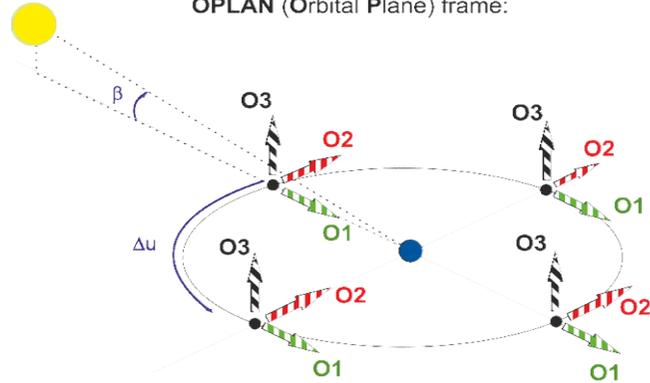
Acceleration due to solar radiation (Orbit-Normal attitude):



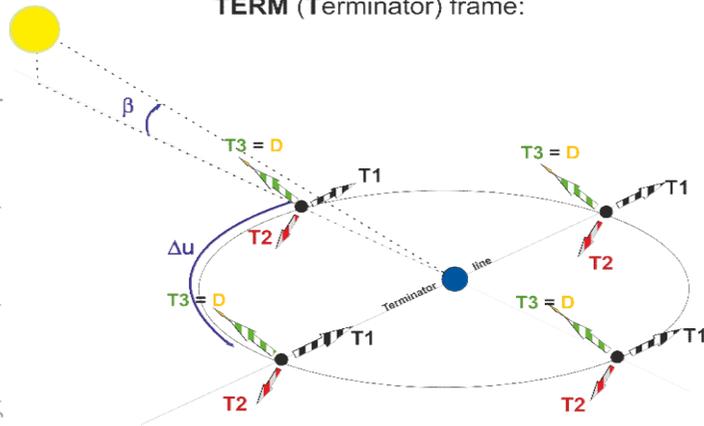
- SP-normal (**SPN**) deviates from the vector **D** ( $\Rightarrow \theta \approx \beta$ )
- Power generation and SRP ( $a_{tot}$ ) reduced - compared to the YS-mode
- SRP force vector due to SP is in the plane spanned by **D**, **Y**, and **SPN** (if all energy would be absorbed, the vector would point in **-D** direction)
- SRP due to SC body causes additional SRP signal, which is periodic w.r.t.  $\Delta u$  and  $\beta$

# Coordinate systems for modelling SRP

OPLAN (Orbital Plane) frame:

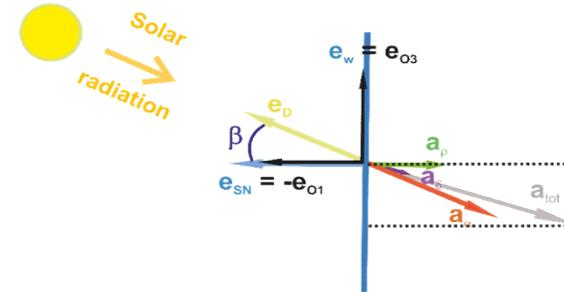


TERM (Terminator) frame:

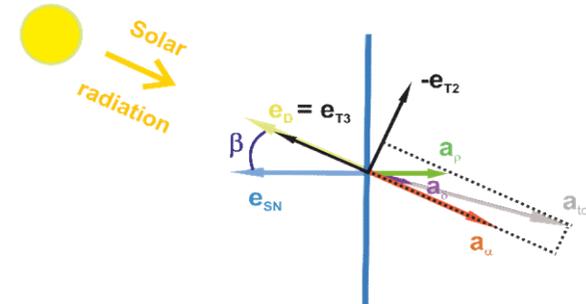


$$\begin{aligned} \mathbf{e}_{T_1} &= \frac{\mathbf{e}_D \times \mathbf{e}_W}{|\mathbf{e}_D \times \mathbf{e}_W|} \\ \mathbf{e}_{T_2} &= \mathbf{e}_D \times \mathbf{e}_{T_1} \\ \mathbf{e}_{T_3} &= \mathbf{e}_D \end{aligned}$$

Acceleration due to solar radiation (Orbit-Normal attitude), and its projection into an Orbital Plane Fixed Reference Frame (OPLAN)



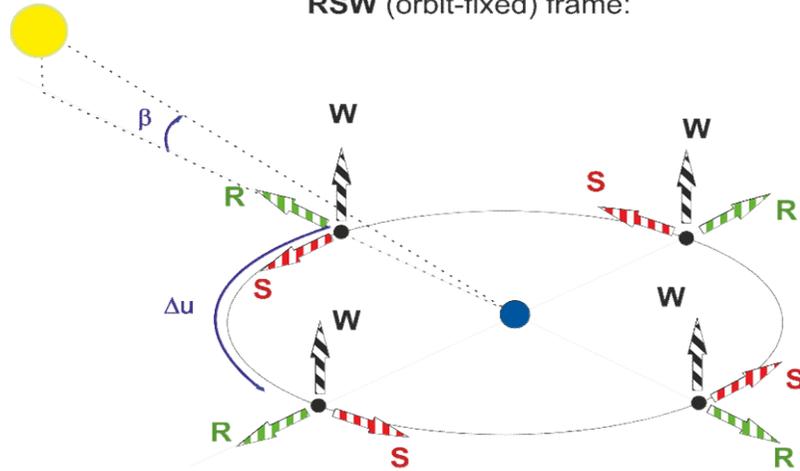
Acceleration due to solar radiation (Orbit-Normal attitude), and its projection into the Terminator Reference Frame (TERM)



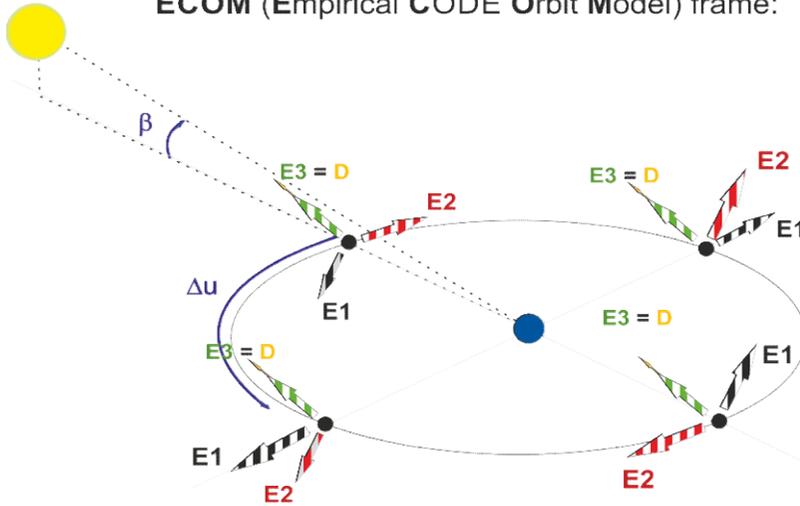
- SRP due to SP can be absorbed by 2 parameters, which are constant w.r.t.  $\Delta u$ , but not w.r.t.  $\beta \Rightarrow$  both frames are suitable in practice
- TERM better suited for physical interpretation (**T2** component depends solely on reflected solar radiation)

# Coordinate systems for modelling SRP

RSW (orbit-fixed) frame:



ECOM (Empirical CODE Orbit Model) frame:



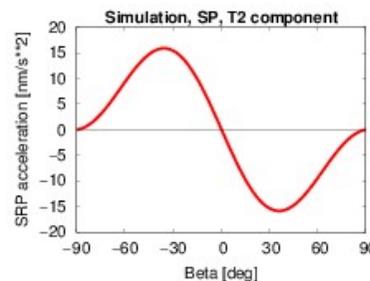
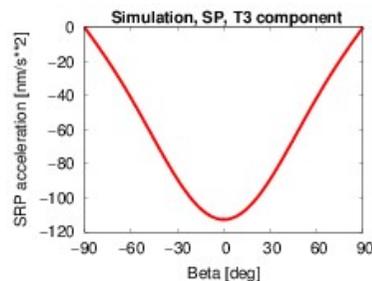
- OPLAN is special case of RSW
- TERM is special case of ECOM
- Difference: frame rotation  $\Rightarrow$  in RSW and ECOM one of the constant components is split up into two components, which are periodic w.r.t.  $\beta$  and  $\Delta u$ , respectively  $\Rightarrow$  more complex
- SRP due to SC bus is adding to it
- ECOM-frame offers at least operational advantages: the same basic ECOM model could be used for YS and ON modes  $\Rightarrow$  model switch by adding/removing constraints on the additional **E2**-parameters; physical interpretation of periodic **E2**-coeff. would be same as for **T2**

# SRP simulation

- Simulation of SRP (based on Milani, 1987) for SP and SC body separately with varying  $\Delta u$  and  $\beta$  angles
- Data basis: QZS-1 meta data released by Cabinet Office, Government of Japan (2017) and by Montenbruck (2017)
- Projection into different possible SRP model frames (RSW, ECOM, TERM, OPLAN); final decision in favor of TERM
- Fit of simulated accelerations with truncated Fourier series and selection of significant coefficients

- ECOM-TM:**
- Minimal parameterization for SP only
  - $\beta$  slowly changing in time  $\Rightarrow$  param. not constant  $\Rightarrow$  not suited for long arcs

$$\begin{aligned} T3(\Delta u) &= \mathbf{T30} \\ T2(\Delta u) &= \mathbf{T20} \\ T1(\Delta u) &= 0 \end{aligned}$$

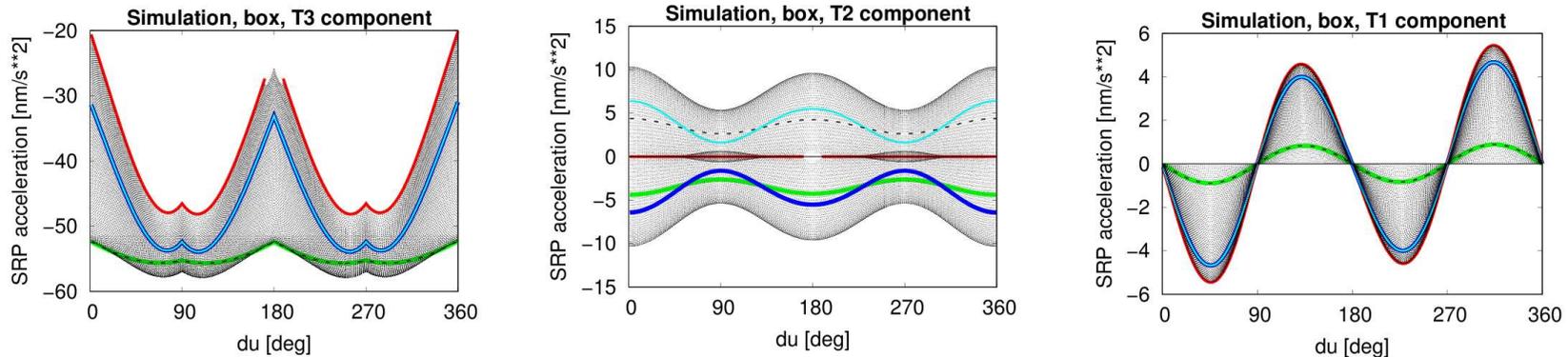


$$\begin{aligned} T3(\Delta u, \beta) &= \mathbf{T30C1b} \cos \beta \\ T2(\Delta u, \beta) &= \mathbf{T20S2b} \sin 2\beta \\ T1(\Delta u, \beta) &= 0 \end{aligned}$$

- ECOM-TBM:**
- ECOM-TM considering  $\beta$ -dependency  $\Rightarrow$  suitable for long arcs

# SRP model definition

SRP acceleration due to the surfaces of the SC body causes more complex signal:



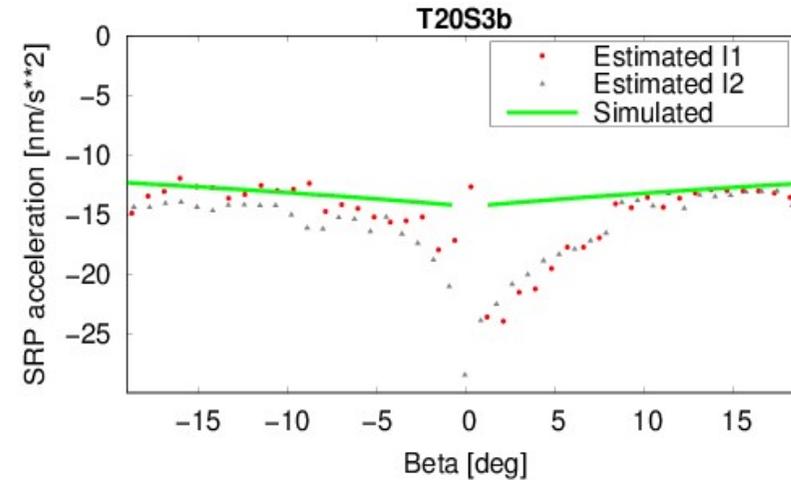
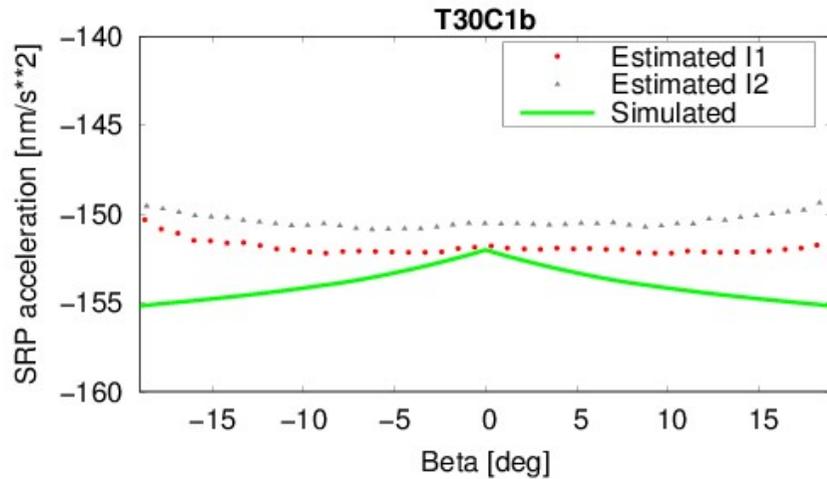
(Colors highlight  $\beta$ -angles: green:  $-80^\circ$ , blue:  $-30^\circ$ , red:  $0^\circ$ , cyan:  $+30^\circ$ , gray, dotted:  $+80^\circ$ )

- ECOM-TB:**
- Full model (covering SP + SC body) with 9 SRP parameters
  - Stochastic pulses/accelerations ( $\Rightarrow$  ECOM-TBP) or an a priori model may be added

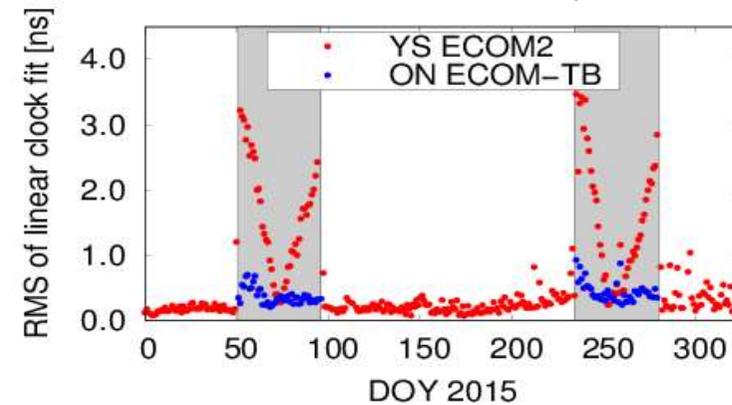
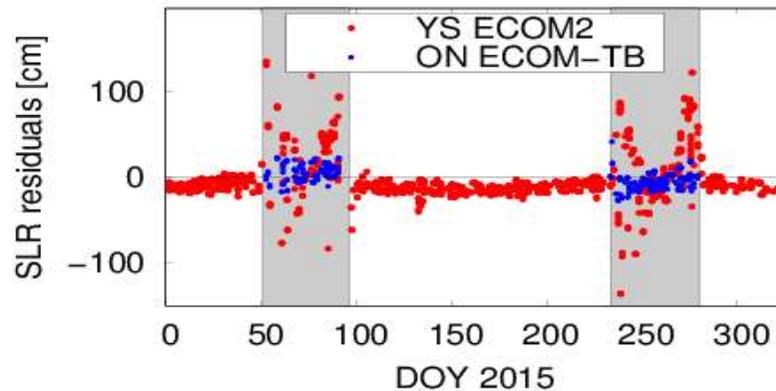
$$\begin{aligned}
 T3(\Delta u, \beta) &= T30C1b \cos \beta & + & T3C2uC1b \cos 2\Delta u \cos \beta \\
 & & + & T3S2uC1b \sin 2\Delta u \cos \beta \\
 & & + & T3C4uC1b \cos 4\Delta u \cos \beta \\
 & & + & T3S4uC1b \sin 4\Delta u \cos \beta \\
 T2(\Delta u, \beta) &= T20S3b \sin 3\beta & + & T2C2uS2b \cos 2\Delta u \sin 2\beta \\
 & & + & T2S2uS2b \sin 2\Delta u \sin 2\beta \\
 T1(\Delta u, \beta) &= & + & T1S2uC1b \sin 2\Delta u \cos \beta
 \end{aligned}$$

# Application to POD of QZS-1

Agreement between simulated and estimated (QZS-1) ECOM-TB coefficients:



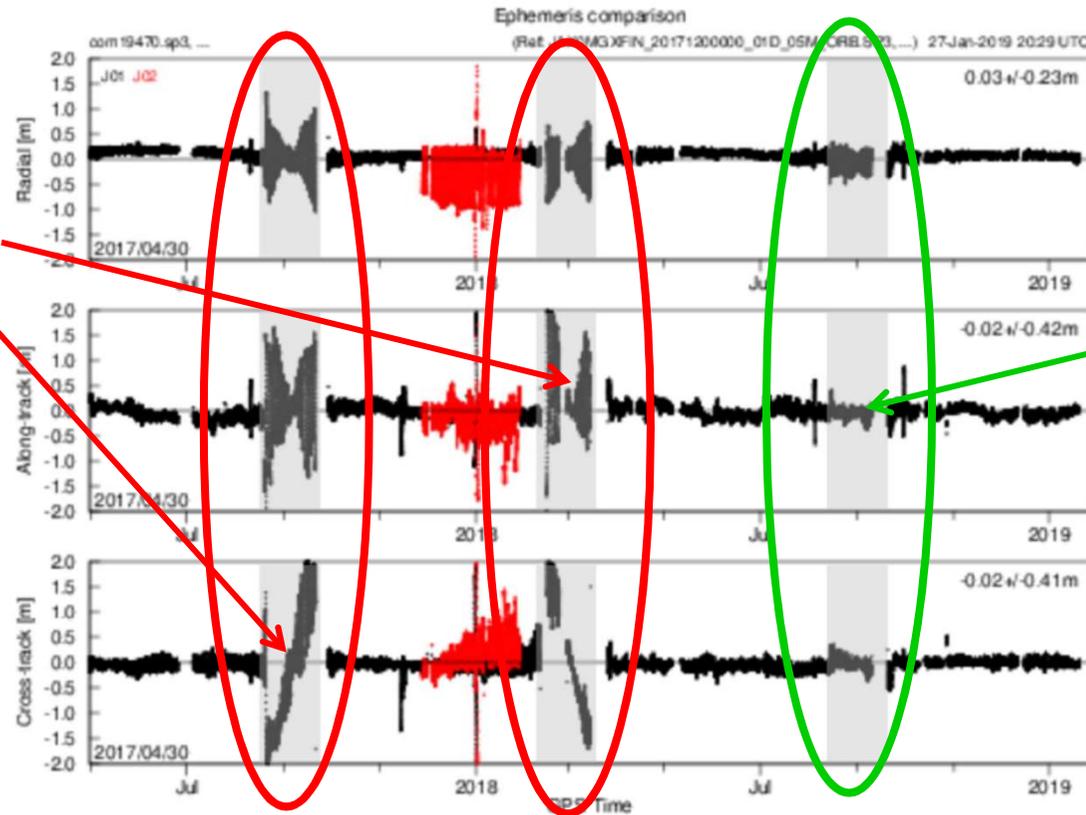
(Time intervals: I1: DOY 357/2014 to 108/2015; I2: DOY 230 - 294/2015)



Significant reduction of SLR residuals and CLK-fit RMS during ON-periods (QZS-1, 3-day long arc solution)

# External Validation

Comparison between QZSS orbits (QZS-1 in black) from the MGEX solutions “JAXA” and “COM”. Time windows with ON mode are shaded in gray (screenshot taken from <http://mgex.igs.org/analysis/>):



ECOM2 used for POD @COM

ECOM-TB used for POD @COM

⇒ Significantly better agreement, since COM replaced ECOM2 by ECOM-TB

# Internal Validation

Orbit validation with SLR residuals (SLR), linear fit of satellite clock corrections (CLK), orbit misclosures (OMC), and 3-day long-arc fit (ORB-fit) for BDS2 IGSO (BI), BDS2 MEO (BM), and QZS-1 (Q1) satellites with different SRP models during ON-periods in late 2014 to late 2015:

| Vali.-Method | SLR, IQR [cm] |      |      | CLK-fit, median [ns] |      |      | OMC, median [cm] |      |      | ORB-fit, med. [cm] |      |      |
|--------------|---------------|------|------|----------------------|------|------|------------------|------|------|--------------------|------|------|
| Sat.-System  | BI            | BM   | Q1   | BI                   | BM   | Q1   | BI               | BM   | Q1   | BI                 | BM   | Q1   |
| ECOM2        | 20.5          | 21.0 | 62.0 | 1.72                 | 1.61 | 1.43 | 55.9             | 29.2 | 42.4 | 23.0               | 10.1 | 14.1 |
| ECOM-TB      | -             | -    | 15.2 | -                    | -    | 0.35 | -                | -    | 14.2 | -                  | -    | 5.6  |
| ECOM-TBP     | -             | 12.2 | -    | -                    | 0.69 | -    | -                | 9.8  | -    | -                  | 6.8  | -    |
| ECOM-TBMP    | 12.2          | -    | -    | 0.72                 | -    | -    | 27.1             | -    | -    | 44.0               | -    | -    |

- **QZS-1:** improvement by factor of 3 - 4 (ECOM-TB vs. ECOM2)
- **BDS2 MEO:** improvement by factor 2 with ECOM-TBP (pulses in R,S,W every 12h added)
- **BDS2 IGSO:** minimal model (2 SRP param., no periodic terms) plus pulses (ECOM-TBMP); improvement by factor 2 over ECOM2 for most validation methods; poor long-arc fit, because model is highly un-physical; difficulties to determine full ECOM-TB in a long arc POD

# Summary

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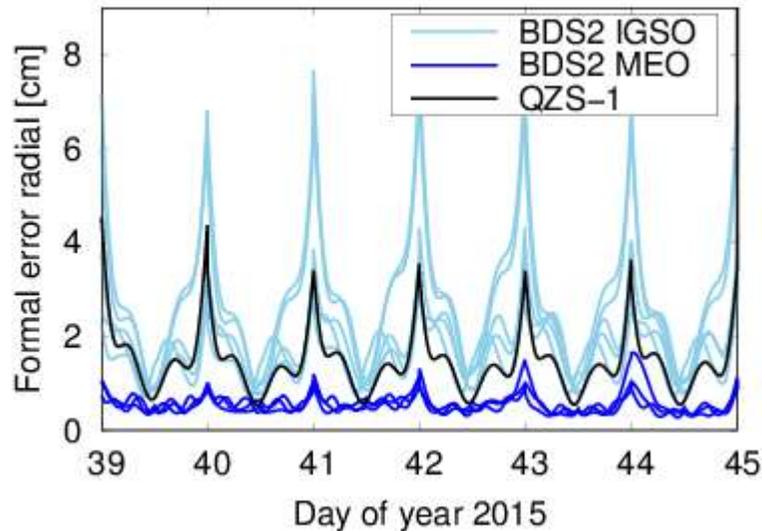
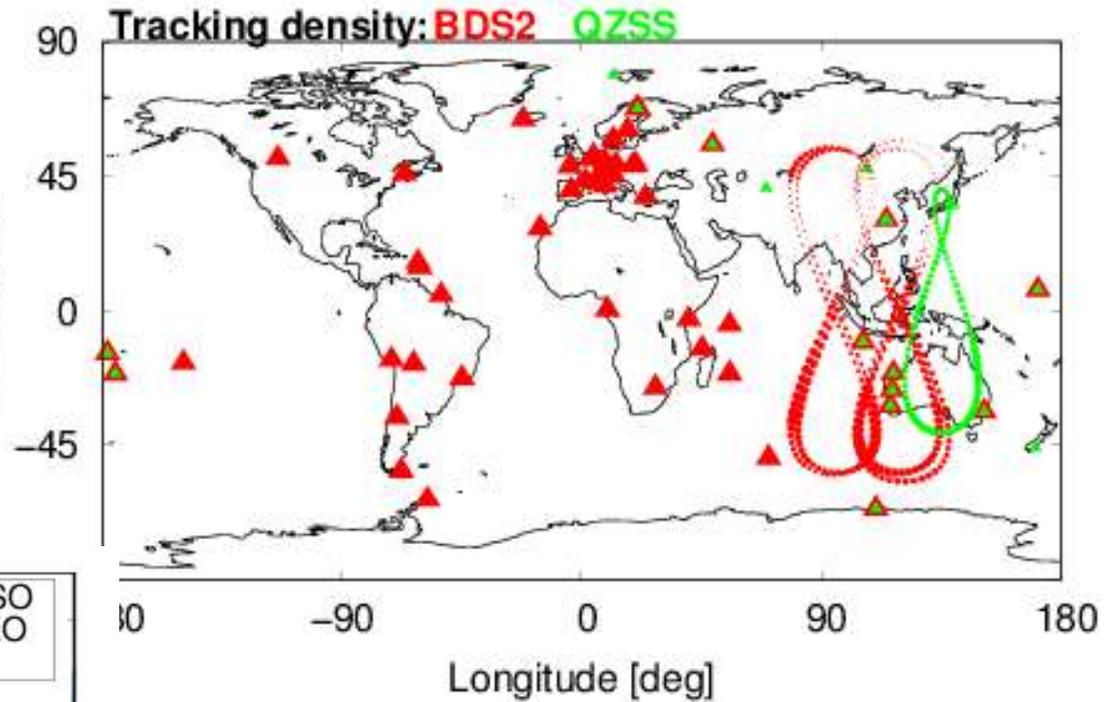
- Terminator reference frame (TERM) defined as a suitable coordinate system for empirical modelling of SRP accelerations during ON-mode
- Definition of the ECOM-TB (i.e., an ECOM using the TERM frame and considering the  $\beta$ -angle) model 'family'
- ECOM-TB performs well for POD of QZS-1
- Problems with POD of BDS2 IGSOs - especially for long-arcs (pulses needed)
- Activated for COM since Summer 2018
- Paper containing model details: submitted to ASR
- Combination with a priori models not yet tested

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Thank you  
for  
your interest!

# Orbit errors

Ground stations contributing to and ground-tracks of **BDS2 (IGSO)** and **QZS-1** orbits. The width of the lines corresponds with the number of ground stations with view to the satellite at the given location (elevation cutoff 45°; early 2015).



Formal error of radial orbit component (in cm) of QZS-1, **BDS2 MEO**, and **BDS2 IGSO** orbits (YS and ECOM2 applied to all satellites)