Single-receiver ambiguity fixing for GPS-based precise orbit determination of low Earth orbiters

Using CODE’s new clock and phase bias products

Daniel Arnold 1  Stefan Schaer 1,2  Arturo Villiger 1
Rolf Dach 1  Adrian Jäggi 1

1 Astronomical Institute, University of Bern, Switzerland
2 Swiss Federal Office of Topography, Wabern, Switzerland

EGU General Assembly 2019, Session G2.4
Vienna, Austria
April 8, 2019
Motivation (1)

- GNSS-based Precise Orbit Determination (POD) of Low Earth Orbiters (LEOs) has become a standard application for high-quality GNSS products
- Processing of dual-frequency GNSS carrier phase data enables the absolute positioning of LEOs with (sub-)cm accuracy in post processing
  → crucial, e.g., for altimetry satellites
### Code and phase observation eqs. for satellite $s$, receiver $r$, freq. $i$

$$P^s_{r;i} = \rho^s_r + I^s_{r;i} + c(\delta t_r - \delta t^s) + c(d^s_{r;i} - d^i_i)$$

$$L^s_{r;i} = \rho^s_r - I^s_{r;i} + c(\delta t_r - \delta t^s) + c(\phi^s_{r;i} - \phi^i_i) + \lambda^i_i \omega^s_r + \lambda^i_i N^{s}_{r;i}$$

- $P^s_{r;i}$: code observation
- $L^s_{r;i}$: phase observation
- $\rho^s_r$: geometric distance
- $I^s_{r;i}$: ionospheric correction
- $d^s_{r;i}, \phi^s_{r;i}$: receiver code/phase bias
- $d^i_i, \phi^i_i$: satellite code/phase bias
- $\lambda^i_i$: carrier wavelength
- $\omega^s_r$: phase windup
- $N^{s}_{r;i}$: integer phase ambiguity

- Fixing ambiguities to their integer values stabilizes solution
- When not modeling phase biases, their effect will be absorbed by ambiguity parameters $\rightarrow$ not integers anymore
- Classical ambiguity resolution approach: Form double differences
Motivation (3)

- Double-difference processing of space baselines has been proven successful and beneficial for relative POD of LEO constellations, e.g., GRACE
Motivation (3)

- Double-difference processing of space baselines has been proven successful and beneficial for relative POD of LEO constellations, e.g., GRACE
- Double-difference processing of space-ground baselines is very costly in computational terms if all correlations shall be modeled
Motivation (3)

- Double-difference processing of space baselines has been proven successful and beneficial for relative POD of LEO constellations, e.g., GRACE
- Double-difference processing of space-ground baselines is very costly in computational terms if all correlations shall be modeled
- Usual LEO POD is based on Precise Point Positioning (PPP), where GNSS satellite orbits and clock corrections from an external global solution are introduced

Code and phase observation eqs. for satellite $s$, receiver $r$, freq. $i$

\[
\begin{align*}
P_{r;i}^s &= \rho_r^s + I_{r;i}^s + c(\delta t_r - \delta t_s^s) + c(d_{r;i} - d_i^s) \\
L_{r;i}^s &= \rho_r^s - I_{r;i}^s + c(\delta t_r - \delta t_s^s) + c(\phi_{r;i} - \phi_i^s) + \lambda_i \omega_r^s + \lambda_i N_{r;i}^s
\end{align*}
\]
**Motivation (3)**

- Double-difference processing of space baselines has been proven successful and beneficial for relative POD of LEO constellations, e.g., GRACE
- Double-difference processing of space-ground baselines is very costly in computational terms if all correlations shall be modeled
- Usual LEO POD is based on Precise Point Positioning (PPP), where GNSS satellite orbits and clock corrections from an external global solution are introduced
- Undifferenced ambiguity resolution in PPP mode requires satellite phase biases as well

---

**Code and phase observation eqs. for satellite $s$, receiver $r$, freq. $i$**

\[
\begin{align*}
P_{r; i}^s & = \rho_r^s + I_{r; i}^s + c(\delta t_r - \delta t^s) + c(d_{r; i} - d_i^s) \\
L_{r; i}^s & = \rho_r^s - I_{r; i}^s + c(\delta t_r - \delta t^s) + c(\phi_{r; i} - \phi_i^s) + \lambda_i \omega_r^s + \lambda_i N_{r; i}^s
\end{align*}
\]
New CODE clock and phase bias product

- Since GPS week 2009 (July 2018) CODE (Center for Orbit Determination in Europe) produces a high-quality signal-specific phase bias product

<table>
<thead>
<tr>
<th>Bias</th>
<th>SVN</th>
<th>PRN</th>
<th>Station name</th>
<th>Obs</th>
<th>yyyy mm dd hh mm ss</th>
<th>yyyy mm dd hh mm ss</th>
<th>Value (ns)</th>
<th>RMS (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>C1C</td>
<td>2007 04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.52254</td>
<td>0.00610</td>
<td></td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>C1W</td>
<td>2007 04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>-0.00000</td>
<td>0.00025</td>
<td></td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>C2W</td>
<td>2007 04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>-0.00000</td>
<td>0.00025</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L1C</td>
<td>2007 04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.16431</td>
<td>0.00000</td>
<td></td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L1W</td>
<td>2007 04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.16431</td>
<td>0.00000</td>
<td></td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L2C</td>
<td>2007 04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.24524</td>
<td>0.00000</td>
<td></td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L2W</td>
<td>2007 04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.24524</td>
<td>0.00000</td>
<td></td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L2X</td>
<td>2007 04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.24524</td>
<td>0.00000</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
New CODE clock and phase bias product

- Since GPS week 2009 (July 2018) CODE (Center for Orbit Determination in Europe) produces a high-quality signal-specific phase bias product
- The Bernese GNSS Software has been extended to introduce these biases, and the new CODE rapid, final, and MGEX clock corrections are based on a fully consistent ambiguity-fixed processing (ambiguity-float clocks → extract phase biases → fix ambiguities and re-estimate clocks)

<table>
<thead>
<tr>
<th>Bias</th>
<th>SVN</th>
<th>PRN</th>
<th>Station name</th>
<th>Obs</th>
<th>yyyy mm dd hh mm ss</th>
<th>yyyy mm dd hh mm ss</th>
<th>Value (ns)</th>
<th>RMS (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>C1C</td>
<td>2007</td>
<td>04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.52254</td>
<td>0.00610</td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>C1W</td>
<td>2007</td>
<td>04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>-0.00000</td>
<td>0.00025</td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>C2W</td>
<td>2007</td>
<td>04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>-0.00000</td>
<td>0.00025</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L1C</td>
<td>2007</td>
<td>04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.16431</td>
<td>0.00000</td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L1W</td>
<td>2007</td>
<td>04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.16431</td>
<td>0.00000</td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L2C</td>
<td>2007</td>
<td>04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.24524</td>
<td>0.00000</td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L2W</td>
<td>2007</td>
<td>04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.24524</td>
<td>0.00000</td>
</tr>
<tr>
<td>OSB</td>
<td>G032</td>
<td>G01</td>
<td>L2X</td>
<td>2007</td>
<td>04 01 00 00 00</td>
<td>2007 04 02 00 00 00</td>
<td>0.24524</td>
<td>0.00000</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
New CODE clock and phase bias product

Final Clocks (AC solutions compared to IGS Final)

(weekly means)

Clock Std Dev [ps]

Time [GPS weeks]


COD EMR ESA GFZ GRG JPL MIT NGS SIO IGR
Test scenario

Using the new CODE products, we
- test undifferenced ambiguity resolution (AR) for POD of
  - GRACE-A/B, April 2007
  - Sentinel-3A/B, September 2018
- compare its performance to double-difference processing, including AR
Test scenario

Using the new CODE products, we

- test undifferenced ambiguity resolution (AR) for POD of
  - GRACE-A/B, April 2007
  - Sentinel-3A/B, September 2018
- compare its performance to double-difference processing, including AR
Test scenario

Using the new CODE products, we
- test undifferenced ambiguity resolution (AR) for POD of
  - GRACE-A/B, April 2007
  - Sentinel-3A/B, September 2018
- compare its performance to double-difference processing, including AR
- demonstrate benefit of undifferenced AR for Swarm POD for June 2018 to March 2019
Methods (1)

Computation of **reduced-dynamic** and **kinematic** orbits using Bernese GNSS Software v5.3

- **Reduced-dynamic** orbit:
  - 6 initial conditions
  - constant accelerations in radial (R), along-track (T) and cross-track (N) direction
  - 6-min piecewise constant accelerations (constrained) in R,T,N
  - no explicit non-gravitational force modeling

- **Kinematic** orbit: epoch-wise 3-dimensional position (+ clocks)

- **Double-difference processing**:
  - reduced-dynamic orbit of GRACE-A / Sentinel-3A is reference
  - relative orbit parameters for GRACE-B / Sentinel-3B estimated
  - relative empirical accelerations are only rather loosely constrained $(1 \cdot 10^{-8} \text{ m/s}^2)$
Methods (2)

Melbourne-Wubbena linear combination of code and phase observations, fix wide-lane ambiguities

Reduced-dynamic POD: ionosphere-free linear combination of phase observations, introduce fixed wide-lane ambiguities, fix narrow-lane ambiguities

Kinematic POD: introduce fixed ambiguities
AR success rate

Percentage of fixed narrow-lane ambiguities for zero-difference (ZD) and double-difference (DD) processing:

![Graph showing AR success rate over time]

- Average: 97.1%
- GRACE-A ZD: 99.1%
- GRACE-B ZD: 92.2%
- GRACE-A/B DD: 96.0%
AR success rate

Percentage of fixed narrow-lane ambiguities for zero-difference (ZD) and double-difference (DD) processing:

![Graph showing AR success rate over dates in September 2018 for Sentinel-3, with data points for Sentinel-3A ZD, Sentinel-3B ZD, and Sentinel-3A/B DD]
Internal orbit consistency

Differences between reduced-dynamic and kinematic orbits:

GRACE-A

\[ \Delta R \text{ [mm]} \]

\[ \Delta T \text{ [mm]} \]

\[ \Delta N \text{ [mm]} \]

Hour of day 07/091
Internal orbit consistency

Differences between reduced-dynamic and kinematic orbits:

**GRACE-A**

![Graph showing differences between reduced-dynamic and kinematic orbits for GRACE-A](image)

- **ΔR [mm]**
- **ΔT [mm]**
- **ΔN [mm]**

Hour of day 07/091
Internal orbit consistency

GRACE RD–KN (float vs fixed), 3D

Average: 19.7 mm
28.3 mm
9.8 mm
12.7 mm

RMS of orb. diff. [mm]

Date in 2007

April
01
04
07
10
13
16
19
22
25
28

01
04
07
10
13
16
19
22
25
28

April

GRACE–A float
GRACE–B float
GRACE–A ZD fixed
GRACE–B ZD fixed
Internal orbit consistency

Sentinel–3 RD–KN (float vs fixed), 3D

Average: 21.0 mm
21.6 mm
10.1 mm
10.6 mm

RMS of orb. diff. [mm]

Date in 2018

01 Sep 04 Sep 07 Sep 10 Sep 13 Sep 16 Sep 19 Sep 22 Sep 25 Sep 28 Sep

Sentinel–3A float
Sentinel–3B float
Sentinel–3A ZD fixed
Sentinel–3B ZD fixed
K-band validation

K-band residual = difference between computed range and range derived from ultra-precise inter-satellite K-band measurement.

External orbit validation!
K-band validation

K-band residual = difference between computed range and range derived from ultra-precise inter-satellite K-band measurement.

External orbit validation!

GRACE–A/B red.–dyn. KBR residuals

Date in 2007

Average: 5.54 mm
1.82 mm
1.32 mm

KBR residuaIs [mm]

April 01
April 04
April 07
April 10
April 13
April 16
April 19
April 22
April 25
April 28
SLR validation

SLR residual = difference between computed range and range derived from Satellite Laser Ranging (SLR) measurement.

External orbit validation!
SLR validation

SLR residual = difference between computed range and range derived from Satellite Laser Ranging (SLR) measurement.

External orbit validation!

<table>
<thead>
<tr>
<th>Orbits</th>
<th>Float</th>
<th>ZD AR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>red.-dyn.</td>
<td>kin.</td>
</tr>
<tr>
<td>GRACE-A</td>
<td>+0.5/15.5</td>
<td>+1.5/16.6</td>
</tr>
<tr>
<td>GRACE-B</td>
<td>+0.9/12.1</td>
<td>-0.5/16.9</td>
</tr>
<tr>
<td>Sentinel-3A</td>
<td>-6.0/11.5</td>
<td>-6.5/14.7</td>
</tr>
<tr>
<td>Sentinel-3B</td>
<td>-2.9/12.4</td>
<td>-4.3/15.2</td>
</tr>
</tbody>
</table>

Mean values and standard deviations in mm of SLR residuals over April 2007 (GRACE) and September 2018 (Sentinel-3), respectively. No parameters estimated, station coordinates according to SLRF2008 (GRACE) and SLRF2014 (Sentinel-3) introduced. SLR data of 12 stations used. 20 cm outlier threshold, 10° elevation cutoff.
Swarm POD (1)

- Initially, Swarm GPS data were affected by *half-cycle ambiguities*, hindering successful AR
- Fixed for the reprocessed level-1 Swarm GPS data (Montenbruck et al., 2017)
Initially, Swarm GPS data were affected by *half-cycle ambiguities*, hindering successful AR.

Fixed for the reprocessed level-1 Swarm GPS data (Montenbruck et al., 2017)

![Swarm POD (1)](image)
• Initially, Swarm GPS data were affected by *half-cycle ambiguities*, hindering successful AR
• Fixed for the reprocessed level-1 Swarm GPS data (Montenbruck et al., 2017)

Swarm (solvable)

Counting only those NLs where the WLs could be resolved
Internal orbit consistency:

Swarm–A RD–KN (float vs fixed), 3D

Average: 19.9 mm  
11.2 mm

RMS of orb. diff. [mm]
SLR residuals (mean and standard deviation) in mm (statistics computed as for GRACE and Sentinel-3):

<table>
<thead>
<tr>
<th>Orbits</th>
<th>Float</th>
<th>ZD AR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>red.-dyn.</td>
<td>kin.</td>
</tr>
<tr>
<td>Swarm-A</td>
<td>+6.4/12.2</td>
<td>+5.2/16.2</td>
</tr>
<tr>
<td>Swarm-B</td>
<td>+4.6/12.8</td>
<td>+3.8/16.9</td>
</tr>
<tr>
<td>Swarm-C</td>
<td>+4.9/12.2</td>
<td>+4.1/15.8</td>
</tr>
</tbody>
</table>
Summary and conclusion

- CODE operationally produces an observation-specific phase bias product
- The new CODE rapid, final, and MGEX clock corrections are based on ambiguity-fixed processing
- Tested undifferenced ambiguity fixing for LEO POD of GRACE, Sentinel-3 and Swarm. Beneficial for internal orbit consistency, as well as for absolute orbit quality (K-band and SLR residuals)
- A test data set including phase biases for GPS week 2026 (4–10 November 2018) will be provided to interested users. Please write an email to code@aiub.unibe.ch
Summary and conclusion

- CODE operationally produces an observation-specific phase bias product
- The new CODE rapid, final, and MGEX clock corrections are based on ambiguity-fixed processing
- Tested undifferenced ambiguity fixing for LEO POD of GRACE, Sentinel-3 and Swarm. Beneficial for internal orbit consistency, as well as for absolute orbit quality (K-band and SLR residuals)
- A test data set including phase biases for GPS week 2026 (4-10 November 2018) will be provided to interested users. Please write an email to code@aiub.unibe.ch

Thank you very much!
Ambiguity resolution strategy (1)

1. Form Melbourne-Wubbena linear combination of pseudo-range $P_{r;i}^s$ and carrier phase $L_{r;i}^s$ observations:

$$
MW(L_{r;i}^s, P_{r;i}^s) = \frac{f_1 L_{r;i}^s - f_2 L_{r;i}^s}{f_1 - f_2} - \frac{f_1 P_{r;i}^s + f_2 P_{r;i}^s}{f_1 + f_2}
$$

$$
= \lambda_{wl} N_{r;wl}^s + c MW(\phi_{r;i}, d_{r;i}) - c MW(\phi_i^s, d_i^s),
$$

where $\lambda_{wl} = c/(f_1 - f_2) \approx 86$ cm and $N_{r;wl}^s = N_{r;1}^s - N_{r;2}^s$.

2. Form satellite differences

$$
MW(L_{r;i}^{s1}, P_{r;i}^{s1}) - MW(L_{r;i}^{s2}, P_{r;i}^{s2}) =
$$

$$
\lambda_{wl}(N_{r;wl}^{s1} - N_{r;wl}^{s2}) - c \left[ MW(\phi_i^{s1}, d_i^{s1}) - MW(\phi_i^{s2}, d_i^{s2}) \right],
$$

introduce satellite code and phase biases and resolve wide-lane ambiguity differences, no fixing for reference satellite.

3. Process ionosphere-free linear combination of phase observations,

\[ L_{r;\text{if}}^s = \frac{f_1^2 L_{r;1}^s - f_2^2 L_{r;2}^s}{f_1^2 - f_2^2} \]

\[ = \rho_r^s + c(\delta t_r - \delta t_s) + c(\phi_{r;\text{if}} - \phi_{\text{if}}^s) \]

\[ + \lambda_{\text{nl}} \left( N_{r;1}^s + \frac{\lambda_{\text{wl}}}{\lambda_2} N_{r;\text{wl}}^s \right) + \lambda_{\text{nl}} \omega_r^s , \]

where \( \lambda_{\text{nl}} = c/(f_1 + f_2) \approx 11 \text{ cm}.\)

4. Form satellite differences, introduce satellite phase biases, wide-lane ambiguities \( N_{r;\text{wl}}^s \) and resolve narrow-lane ambiguities \( N_{r;1}^s \).
CODE observation-specific biases

- CODE follows a so-called CC-OSB (common clocks and observable-specific signal biases) approach
- OSB values are provided in Bias-SINEX V1.00
- Easy to use and applicable for all applications
GRACE & Sentinel-3: Internal orbit consistency

GRACE RD–KN (float vs fixed), radial

Average: 13.7 mm
18.6 mm
8.5 mm
11.3 mm

RMS of orb. diff. [mm]

Date in 2007

01 April 04 April 07 April 10 April 13 April 16 April 19 April 22 April 25 April 28 April
GRACE & Sentinel-3: Internal orbit consistency

GRACE RD–KN (float vs fixed), along–track

Average: 10.8 mm
15.6 mm
4.2 mm
4.8 mm

Date in 2007

01 April
04 April
07 April
10 April
13 April
16 April
19 April
22 April
25 April
28 April
GRACE & Sentinel-3: Internal orbit consistency

GRACE RD–KN (float vs fixed), cross–track

Average: 8.9 mm
14.3 mm
2.3 mm
2.9 mm

Date in 2007

RMS of orb. diff. [mm]

01 April 04 April 07 April 10 April 13 April 16 April 19 April 22 April 25 April 28 April
GRACE & Sentinel-3: Internal orbit consistency

Sentinel–3 RD–KN (float vs fixed), radial

Average: 15.1 mm
15.6 mm
8.9 mm
9.3 mm

RMS of orb. diff. [mm]

Date in 2018
GRACE & Sentinel-3: Internal orbit consistency

Sentinel-3 RD-KN (float vs fixed), along-track

Average: 11.6 mm
12.1 mm
4.1 mm
4.3 mm

RMS of orb. diff. [mm]

Date in 2018

Sentinel-3A float
Sentinel-3B float
Sentinel-3A ZD fixed
Sentinel-3B ZD fixed
GRACE & Sentinel-3: Internal orbit consistency

Sentinel-3 RD–KN (float vs fixed), cross–track

Average:  8.7 mm
          8.7 mm
          2.5 mm
          2.6 mm

RMS of orb. diff. [mm]
GRACE: K-band validation

**GRACE–A/B kin. KBR residuals**

- **Average:** 14.32 mm
- **4.74 mm**
- **2.70 mm**

Date in 2007

- Float
- ZD fixed
- DD fixed

KBR residuals [mm]

Date in 2007

01 April
04 April
07 April
10 April
13 April
16 April
19 April
22 April
25 April
28 April

Astronomical Institute, University of Bern
Swarm: Internal orbit consistency

Swarm–B RD–KN (float vs fixed), 3D

Average: 19.9 mm
10.7 mm

RMS of orb. diff. [mm]

Date

Swarm–B float
Swarm–B fixed

Jun 2018
Jul 2018
Aug 2018
Aug 2018
Sep 2018
Oct 2018
Nov 2018
Dec 2018
Jan 2019
Feb 2019
Mar 2019

2018
2018
2018
2018
2018
2018
2018
2018
2019
2019
2019
2019
2019
2019

Daniel Arnold: Single-receiver ambiguity fixing for GPS-based precise orbit determination of low Earth orbiters
Swarm: Internal orbit consistency

Swarm–C RD–KN (float vs fixed), 3D

Average: 20.4 mm
          11.6 mm

Date

RMS of orb. diff. [mm]