



# Empirical modeling of the topside ionosphere and plasmasphere using LEO GPS-TEC

Lucas Schreiter<sup>1,3</sup>, Ludger Scherliess<sup>2,3</sup>, Claudia Stolle<sup>3</sup>, Daniel Arnold<sup>1</sup>, and Adrian Jäggi<sup>1</sup>

<sup>1</sup> University of Bern, Astronomical Institute, Bern, Switzerland

<sup>2</sup> Center for Atmospheric and Space Sciences, Utah State University, United States of America

<sup>3</sup> GFZ Potsdam, Section 2.3 Geomagnetism, Germany

## Slant TEC computation

Relative slant TEC can be derived from the geometry-free linear combination of two GPS phase observables  $L_1$  and  $L_2$ :

$$L_{gf} = L_1 - L_2 \approx \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) \cdot 40.3 \int_{LEO}^{GPS} N_e dl + C_{ARC}$$

where  $f_1$  and  $f_2$  are the two GPS frequencies, the integral is the slant TEC and  $C_{ARC}$  is an unknown constant offset for each connected arc containing biases and ambiguity parameters.

Prior to the relative sTEC computation cycle-slips and outliers are removed using the Melbourne-Wuebbena linear combination and ionosphere-free phase residuals obtained from POD.

Furthermore, an initial leveling of the relative sTEC is performed using the code observations for an ambiguity estimation. However, the final leveling is performed together with the estimation of the model parameters in the least-squares adjustment.

## Electron density from sTEC

The model is an empirical model for the electron density, valid for three hour time windows. The coordinate system is magnetic local time ( $LT$ ) and magnetic latitude ( $mLat$ ). We assume an exponential decrease with altitude for the electron density  $N_e$ :

$$N_e(h, mlat, mLT) = \begin{cases} N_{1000} \cdot e^{-\frac{(h-h_m)}{H_1}}, & h < 2000 \text{ km} \\ N_{1000} \cdot e^{-\frac{(2000 \text{ km} - h_m)}{H_1} - \frac{(h-2000 \text{ km})}{H_2}}, & h \geq 2000 \text{ km} \end{cases}$$

The parameters are the reference electron density  $N_{1000}$  at 1000 km, the scale height  $H_1$  from LEO altitude to 2000 km, and the scale height  $H_2$  from 2000 km to GPS altitude and  $h_m$  is set to 1000 km. These are represented by the exponential of a spherical harmonics expansion of degree 5 ( $N_{1000}$ ) and 4 ( $H_1, H_2$ ):

$$N_{1000}(mLat, mLT) = \exp \left\{ \sum_{i=0}^5 \sum_{j=-i}^i c_{ij} f_{ij}(mLat, mLT) \right\}$$

$$H_1(mLat, mLT) = \exp \left\{ \sum_{i=0}^4 \sum_{j=-i}^i d_{ij} f_{ij}(mLat, mLT) \right\}$$

$$H_2(mLat, mLT) = \exp \left\{ \sum_{i=0}^4 \sum_{j=-i}^i e_{ij} f_{ij}(mLat, mLT) \right\}$$

where  $f_{ij}$  are the spherical harmonic base functions (degree  $i$ , order  $j$ ) and  $c_{ij}$ ,  $d_{ij}$ ,  $e_{ij}$  are the corresponding coefficients. For deriving sTEC from the model a Gauss-Legendre quadrature is used with the line of sight sampled at each 10 nodes below and above 2000 km. This implies that the integration may be expressed by a linear operator  $L$ . Therefore we can use the equation:

$$sTEC = L \cdot D \cdot x + bias,$$

where sTEC is the vector of the observed sTEC values,  $L$  is the integration matrix,  $x$  contains the coefficients, and  $D$  evaluates the electron density at the sampled points. The bias term contains the arc-wise P1-P2 biases to be estimated.

To estimate the model parameters and biases all observations from Sentinel 1A, 1B, 2A, 2B, and 3A (0.1 Hz) from within a 3-hour time window are used in a least-squares adjustment.

Figure 1 shows the resulting electron content for 3/22/2017 (doy 081) at a reference time of 01:30 UT. The vTEC values are obtained by numerical integration of the model densities in vertical direction. The peak value of 13 TECU appears close to noon.

## LEO GPS-TEC

Low earth orbit (LEO) missions are usually equipped with a dual-frequency GPS receiver for precise orbit determination (POD). The two frequencies can be used to remove the first order ionospheric content from the signal, but they may also be used to obtain the total electron content between GPS receiver and GPS satellite (slant TEC or sTEC).

For our study, we focus on five LEO satellites from ESA's Sentinel mission: Sentinel 1A, 1B, 2A, 2B, 3A. These Satellites are in a near-polar circular sun-synchronous orbit with an inclination of about 98 degrees and an initial altitude of 693 km for Sentinel 1, 786 km for Sentinel 2 and 814 km for Sentinel 3. The local times are 6 LT/18 LT (1 A/B) and 10 LT/22 LT (2 A/B, 3A).

Sentinel 1A, 1B, 2A, and 2B provide GPS data with a 0.1 Hz sampling, whereas Sentinel 3A uses a 1 Hz sampling.

## Plasmasphere observations

The plasmasphere is a torus-shaped region of cold ( $\sim 1$  eV) and relatively dense plasma coupled to the magnetic field. It is located above the ionosphere. Usually, the transition altitude is defined by the altitude, where  $H^+$  ions become the most prominent ion species. For most applications, the lower boundary is set to approximately 1000 km. Most of the current knowledge and empirical models are based on satellite observations and usually cover L-values larger 4 (Zhelavskaya et al., 2017). Usually, ground-based GPS is used for the computation of TEC maps. The whole slant TEC is mapped into a single layer, usually assumed at 350 km - 450 km. This is obviously not suited for the plasmasphere.

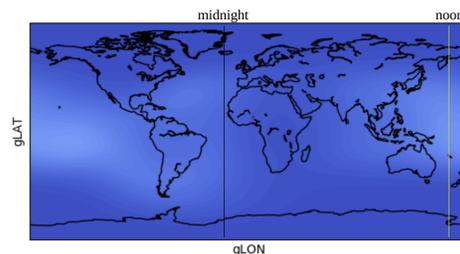
We exclusively use the TEC derived from the LEO POD GPS antenna. Due to the relatively high altitude of the Sentinel satellites, we ensure that our observations are mainly affected by plasmaspheric TEC.

## LEO GPS-TEC for plasmasphere

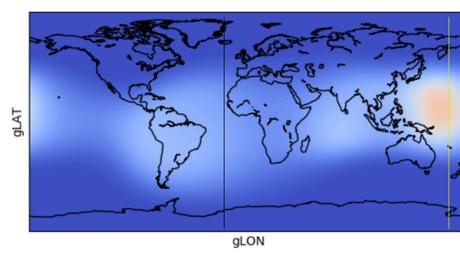
We model the Plasmaspheric TEC in three dimensions using a 3-hour time windows. Our parameters, developed in magnetic local time (mLT) and magnetic latitude (mLat) using a spherical harmonic expansion, are a reference electron density at 1000 km, and two scale heights, one from Satellite altitude to 2000 km and a second from 2000 km to GPS altitude. We also estimate the P1-P2 receiver bias for each connected arc of GPS carrier phase observations.

We will use the model to generate plasmaspheric TEC maps and generate phase center variation (PCV) maps for the geometry-free linear combination. We show that plasmaspheric TEC may reach more than 10 TECU, whereas the scatter in the P1-P2 bias is at a level of about 5 TECU, and the geometry-free PCV's are at the centimeter level ( $1 \text{ cm} \sim 0.095 \text{ TECU}$ ).

Plasmaspheric TEC, 2017, doy 081, 01:30UT  
from 1000 km to 2000 km



from 2000 km to 20000 km



from 1000 km to 20000 km

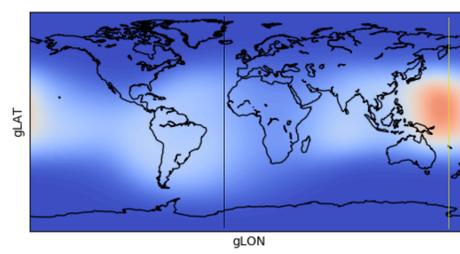


Figure 1: Plasmaspheric TEC, 3/22/2017, 01:30 UT. Parameters estimated based on GPS observations from 00:00 UT to 03:00 UT

## P1-P2 receiver bias estimation

The P1-P2 GPS satellite biases was taken from the Center for Orbit Determination in Europe (CODE). However, the P1-P2 receiver bias needs to be estimated empirically. In many cases it is assumed to be constant for each day (LEO's) or month (ground stations) and determined using a zero minimum condition (Schaer and Steigenberger, 2006, Heise et al., 2005).

This condition might lead to an inconsistency when using TEC values from different LEO's at different altitude. For our approach we don't assume the bias to be constant. Therefore we estimated an individual P1-P2 receiver bias for each phase arc. Figure 2 shows the biases for a 3-hour time window. For each Sentinel satellite the biases show a scatter of a few TECU about a constant value.

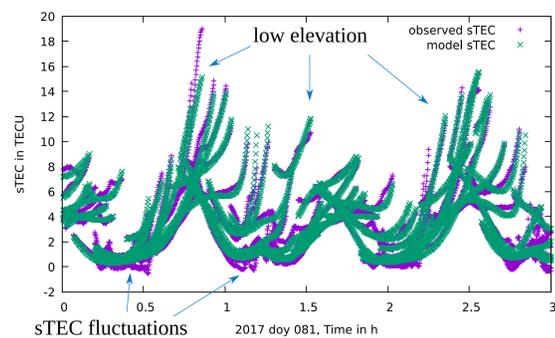


Figure 2: Arc-wise estimated P1-P2 receiver bias for each GPS phase arc between 00:00 UT and 03:00 UT, 3/22/2017.

## Elevation dependent errors

GPS data quality is known to be degraded at low elevations. This may be due to multipath, higher order ionospheric effects and ray bending on centimeter level (Hoque and Jakowski 2008).

Figure 4 shows the residuals for the 3-hour time window of the observed-model sTEC values as a function of the elevation of the GPS satellites. The largest values can be observed at low elevations, and the residuals tend to become smaller with high elevation.

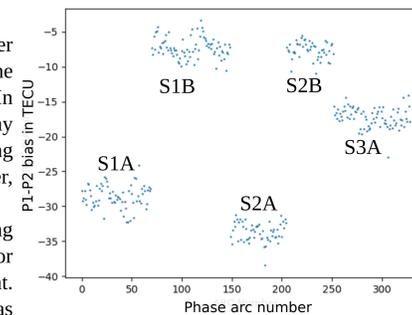


Figure 3: Observed and model slant TEC values for Sentinel 1A. Observed values are shifted by the estimated P1-P2 bias.

## Model and observed sTEC

Figure 3 shows a comparison of the observed sTEC values (purple) to the modeled sTEC values for Sentinel 1A from 00:00 UT to 03:00 UT of doy 081, year 2017.

It can be seen, that the model is capable of reproducing the trends, but it is not capable of reproducing sTEC fluctuations as they occur at epochs with small TEC values.

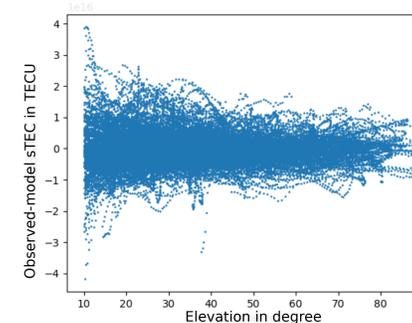


Figure 4: Differences between observed and model slant TEC as a function of the elevation of the GPS satellites. All five Sentinels were used. Time window 00:00 UT to 03:00 UT, 3/22/2017.

## Phase center variations

Phase center variation (PCV) maps can be used to describe satellite and antenna specific elevation and azimuth dependent errors. For the ionosphere-free linear combination PCV maps may be computed using the residual stacking approach (Peter et al., 2017). The ionosphere-free residuals to an orbit model are binned and averaged as a function of elevation and azimuth expressed in the antenna reference frame. Using our topside ionosphere and plasmasphere model, we can employ the same approach to derive a PCV map for the geometry-free linear combination by assuming no geometry-free PCV for the GPS transmitter antennas. In turn one may use both PCV maps to derive single frequency  $L_1$  and  $L_2$  PCV maps.

For this study, we use April 2017 to check for systematic differences between model and observations. The results are shown in fig. 5. As expected the PCV maps for Sentinel 1A and 1B, as well as for Sentinel 2A and 2B, look similar since the pairs have the same satellite, receiver, and antenna design.

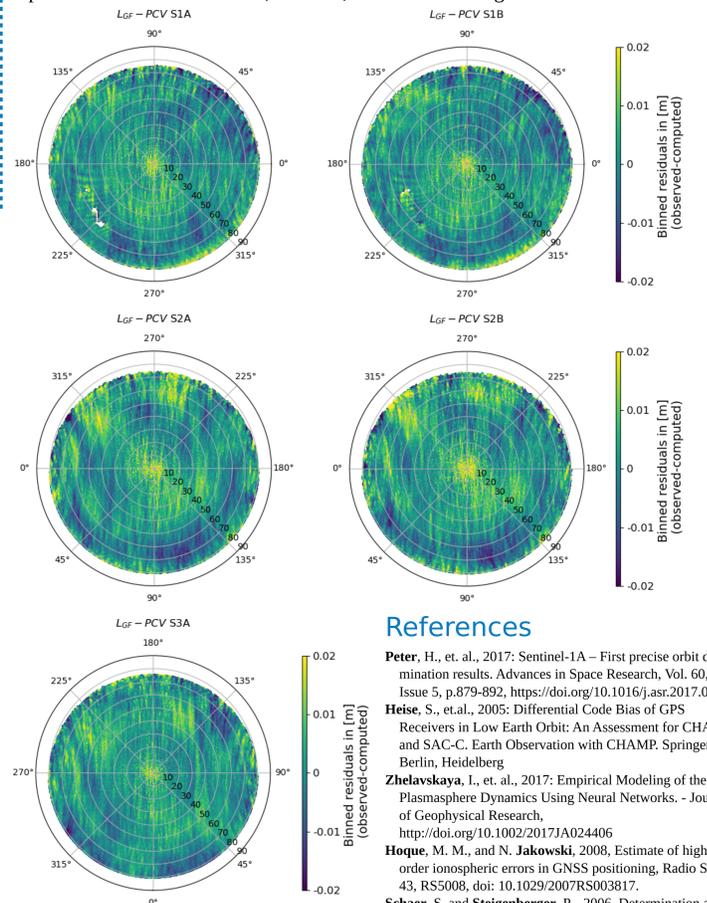


Figure 5: Geometry-free PCV maps

## References

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## Conclusions

Using Sentinel GPS-TEC is an efficient way to monitor the topside ionosphere and plasmaspheric TEC. The model presented may be used to estimate the slant TEC to improve ground GPS-TEC maps and subtract topside and plasmasphere TEC from ground or LEO GPS-TEC.

The LEO receiver bias should not be assumed to be constant since a scatter of a few TEC is large compared to the plasmaspheric electron content.

Also generating geometry-free PCV maps may be used in combination with ionosphere-free PCV maps in order to derive single frequency  $L_1$  and  $L_2$  PCV maps.

## Contact address

Lucas Schreiter  
Astronomical Institute, University of Bern  
Sidlerstrasse 5  
3012 Bern (Switzerland)  
lucas.schreiter@aiub.unibe.ch

