Stochastic noise modelling of kinematic orbit positions in the Celestial Mechanics Approach

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Introduction

Gravity field models may be derived from kinematic orbit positions of Low Earth Orbiting (LEO) satellites equipped with onboard GPS receivers. An accurate description of the stochastic behaviour of the kinematic positions plays a key role to calculate high quality gravity field solutions. In the Celestial Mechanics Approach (Beutler et al., 2010) kinematic positions are used as pseudo-observations to estimate orbit parameters and gravity field coefficients simultaneously. So far, a simplified stochastic model based on epoch-wise covariance information, which may be efficiently derived in the kinematic point positioning process, has been applied.

We extend this model by using the fully populated covariance matrix (see Jäggi et al., 2011), covering correlations over 50 minutes. As this purely mathematical error propagation cannot describe noise characteristics introduced by the original observations, we sophisticate our model by deriving empirical covariances from the residuals of an orbit fit of the kinematic positions based on a priori force field. In this poster we use a fixed a priori force field.

We process GRACE data from April 2007 to derive gravity field solutions up to degree and order 70. Two different orbit parametrisations are adopted, with EGM2008 (Pavlis et al., 2012) serving as a priori gravity field up to degree and order 160: A purely dynamical orbit (six Keplerian elements and additional accelerometer calibration parameters) and a reduced-dynamic orbit with additional constrained piecewise constant accelerations set up within intervals of 15 minutes. The resulting gravity fields are solved on a monthly basis using daily orbital arcs.

type of covariance	
mathematical	(epoch-wise)
mathematical	(full over 50 min)
empirical	(full over 50 min)

The solutions based on the epoch-wise covariance information for the reduced-dynamic and the dynamic orbit are shown in Fig. 1. The gravity field solution based on the dynamic orbit parametrisation is degraded because of a deficient force model. The gravity field solution based on the reduced-dynamic parametrisation (red) is of good quality and represents the classical parametrisation used so far in the context of the Celestial Mechanics Approach. In both cases the formal errors (dashed lines) of the solution do not reflect the true accuracy assessed by the differences to the (superior) GOCO05s (Mayer-Gürr et al., 2015) gravity field model (solid line). Hence, the stochastic description of the pseudo-observations needs to be expanded.

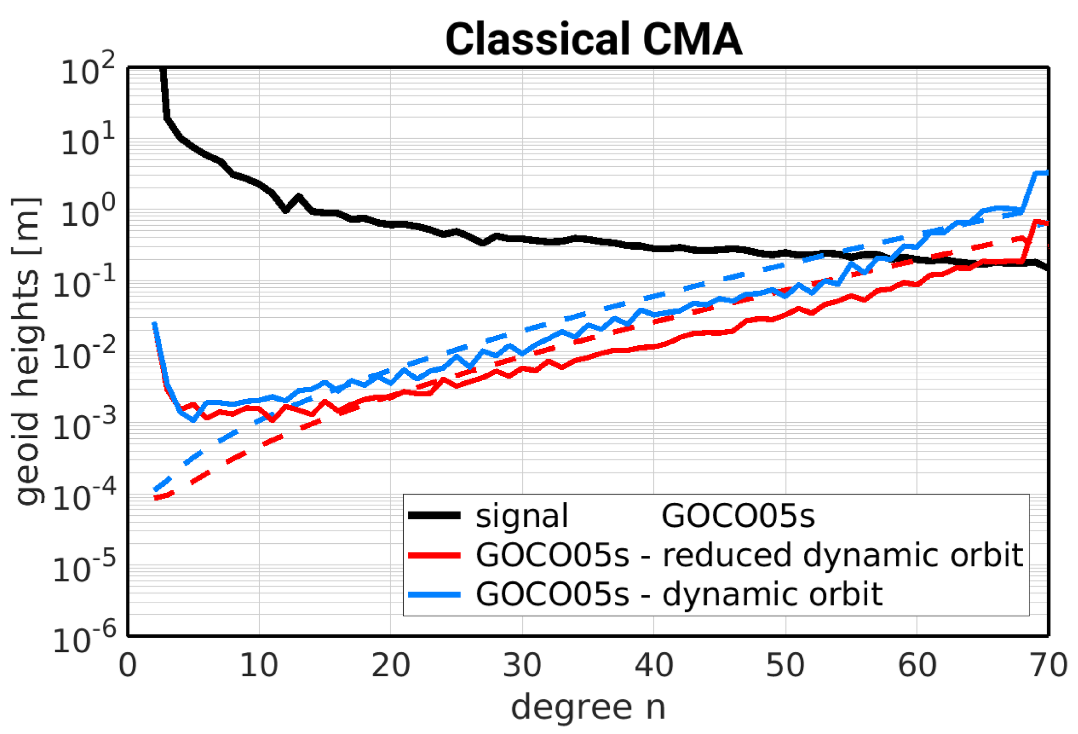

Figure 1: Monthly GRACE GPS-only gravity field solution based on a reduced-dynamic orbit (red) with constrained piecewise constant accelerations set up every 15 minutes and a dynamic orbit (blue). The solid lines depict the degree variance differences to GOCO05s, the dashed lines denote the formal errors from the least squares adjustment.

Mathematical error propagation

We derive the covariance matrix $\mathbf{C}_{kk}$ from the inverse normal equation matrix, which is the result of a formal mathematical error propagation from the phase observations to the unknown kinematic positions (Eq. 1).

$$\mathbf{C}_{kk} = \mathbf{R}\mathbf{C}_{ll}\mathbf{R}^{\mathrm{T}}, \qquad \mathbf{R} = (\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P} \tag{1}$$

$\mathbf{C}_{ll}$ denotes the covariance matrix of the original phase observations. White noise is generally assumed for $\mathbf{C}_{ll}$. The propagation matrix $\mathbf{R}$ stems from the kinematic point positioning with $\mathbf{A}$ being the first design-matrix and $\mathbf{P}$ the weight matrix derived from $\mathbf{C}_{ll}$.
The GPS carrier phase ambiguities are the only parameters in $\mathbf{A}$ connecting different epochs when using GPS phase data. Consequently, deficiencies in the modelling of the GPS phase data may be propagated through the ambiguities over several epochs, which is also reflected by $\mathbf{C}_{kk}$. Since $\mathbf{C}_{kk}$ only depends on the observation scenario but not on the actual observations, any degradation of positions due to GPS data quality issues (including GPS orbits and clocks) is not reflected in this type of covariance information.

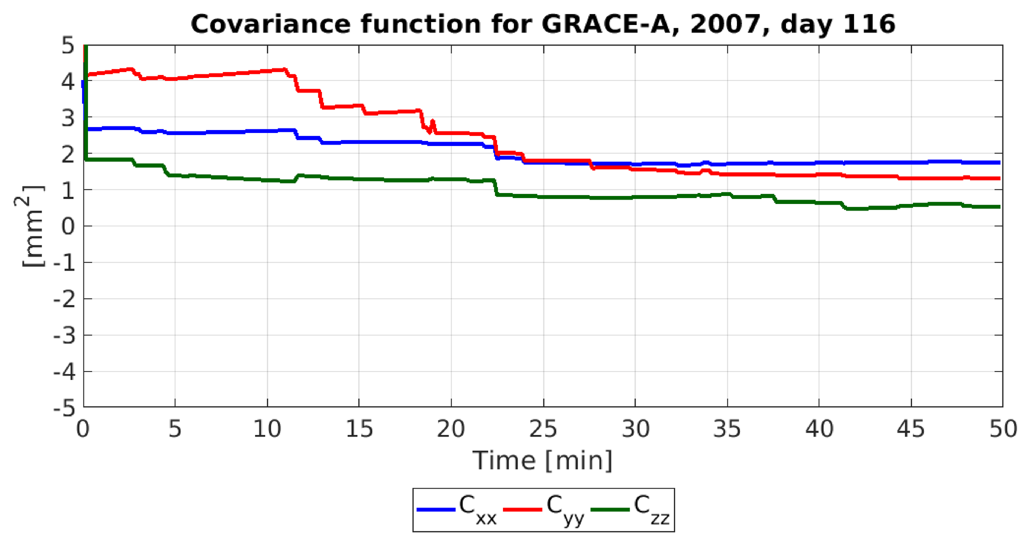

Figure 2: Covariance function over 50 minutes for kinematic positions (first 50 minutes of day 116, 2007). Only the covariances between the same coordinate over time are shown. The jumps occur due to the set up of new ambiguities (changes in the observed constellation).

Weighting the kinematic positions according to $\mathbf{C}_{kk}$ from Eq. (1) leads to more realistic formal errors in the gravity field recovery process (Fig. 3). Orbit residuals become larger because long-period variations of the pseudo-observations are no longer (erroneously) fitted by the parameters of the orbit model but (correctly) interpreted as a consequence of the ambiguity-induced correlations in time (Jäggi et al., 2011). The very low degrees still reveal deficiencies in the stochastic modelling.

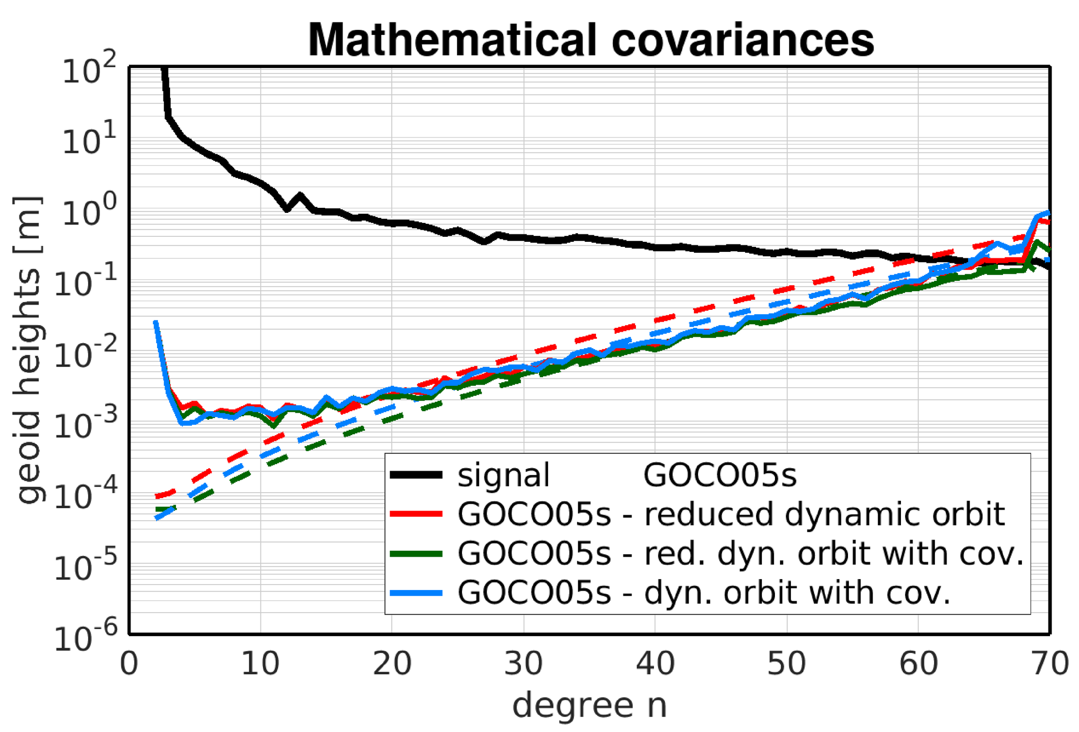

Figure 3: Monthly GRACE GPS-only gravity field solution. Covariances considering correlations over 50 minutes from the purely mathematical error propagation in the determination of the kinematic positions are used to weight the kinematic positions.

Empirical covariances

The residuals of an orbit fit w.r.t. to the kinematic positions reflect all model (functional and stochastic model) and data deficiencies. Consequently, deriving the covariances from the residuals leads to a stochastic description of the entire physical system. The covariance function for a certain time interval Δt_k is defined as the auto-correlation between the respective residuals $\mathbf{e}$ (Eq. 2).

$$cov(\Delta t_k) = \frac{1}{n}\sum_{i=0}^{n}\mathbf{e}(t_i)\mathbf{e}(t_i + \Delta t_k) \tag{2}$$

As we do not want to model short term variations within an arc, we use the residuals of a whole month to derive a mean covariance function. For a better comparison with the mathematical error propagation, 50 minutes of correlations are taken into account, however, depending on the parametrisation it may take much longer until the correlation vanishes. Fig. 4 depicts the empirical covariance function for the reduced-dynamic parametrisation. When plotting the covariance function in the local orbital frame (not shown), the largest correlations occur in the radial direction, while the other directions play a minor role.

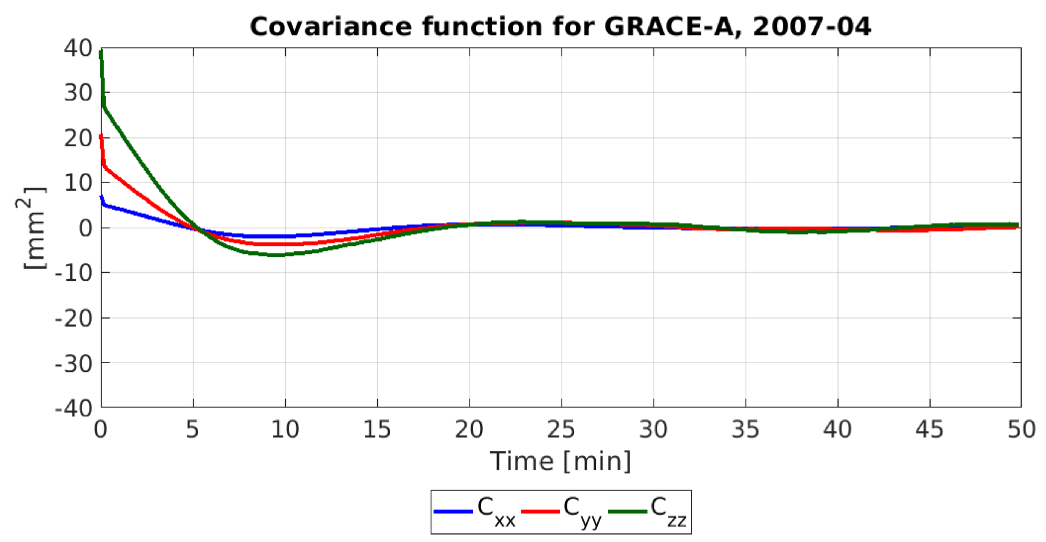

Figure 4: Empirical covariance function derived from reduced-dynamic orbit residuals over one month in for a period of of 50 minutes.

Using the empirical covariances to weight the kinematic positions in the gravity field recovery process, one can find formal errors much closer to the degree variance differences. As the use of empirical covariances takes only correlations in the residuals into account, their shape is highly dependent on the orbit parametrisation. Using mathematical covariances in addition to empirical ones does not significantly influence the results (not shown).

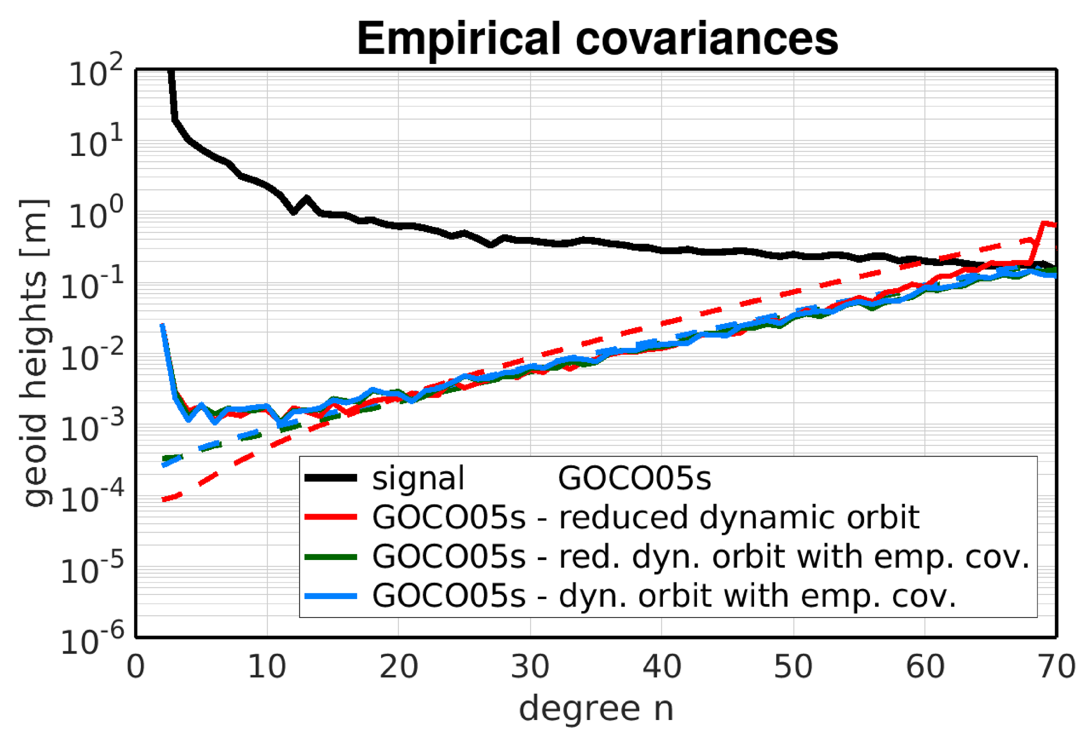

Figure 5: Monthly GRACE GPS-only gravity field solution. Empirical covariances based on the observation residuals are introduced in the gravity field determination process. These covariances are determined as a mean function over one month and include correlations over 50 minutes.

Undifferenced ambiguity fixed positions

Introducing undifferenced ambiguity fixed kinematic positions (Arnold et al., 2019) potentially helps the improve to gravity field determination process as indicated by the performance of the K-band validation (Fig. 6).

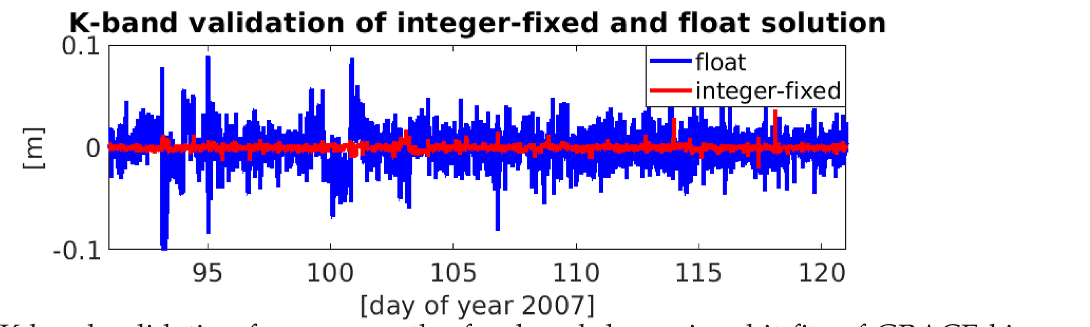

Figure 6: K-band validation for one month of reduced-dynamic orbit fits of GRACE kinematic positions using epoch-wise covariances.

Due to the ambiguity fixing the number of ambiguity parameters is significantly reduced, only the unresolved ambiguities are remaining in the system. This implies that the kinematic positions are almost uncorrelated in time (compare Fig. 7 below to Fig. 2).

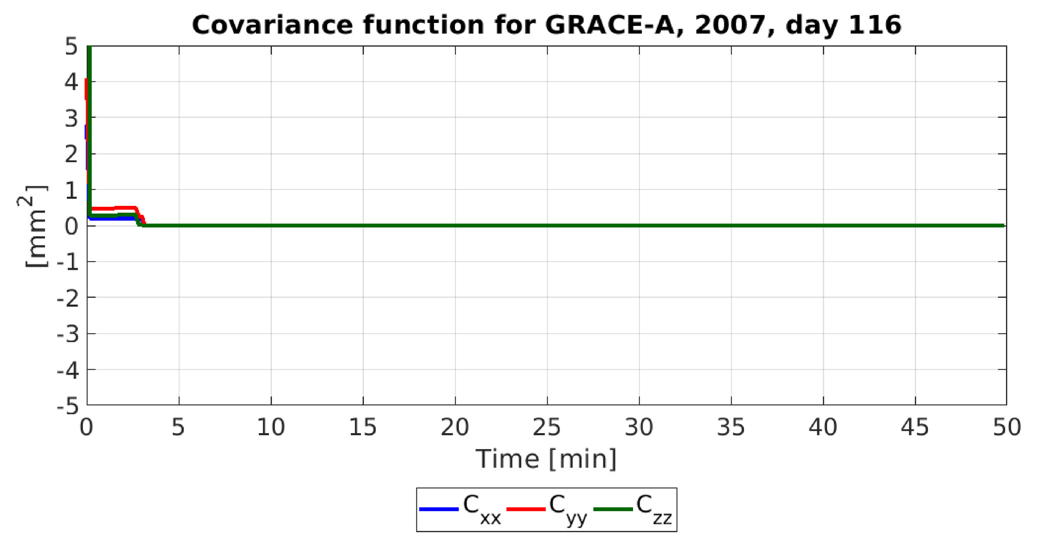

Figure 7: Covariance function over 50 minutes for undifferenced ambiguity fixed kinematic positions (first 50 minutes of day 116, 2007).

When using this kind of covariance information for gravity field recovery, the solution is at the level of the gravity field solutions shown in Fig. 1. The ambiguity fixed positions do not outperform a float solution when calculating a gravity field solution.

Conclusions

Kinematic positions, which may be used as pseudo-observations for gravity field recovery, are correlated in time due to the ambiguities in the original GPS phase observations. Not considering these correlations degrades a gravity field solution. Taking these correlations into account can be achieved either by the covariance information of the kinematic positions over longer time spans or by empirical covariances derived from residuals of an orbit fit w.r.t. to the kinematic positions.

References

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