GNSS Satellite Orbit Modelling
Theory and Practice

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NGK Summer School
29. August – 01. September 2016, Båstad, Sweden

Overview

Effect of Orbit Errors on GNSS Solutions

Errors in baseline components due to orbit errors

following Bauršima, 1983:

$$\Delta X_l(\mathrm{m}) \approx \frac{l}{d} \cdot \Delta X_{ORB}(\mathrm{m}) \approx \frac{l(\mathrm{km})}{25'000(\mathrm{km})} \cdot \Delta X_{ORB}(\mathrm{m})$$

Effect of Orbit Errors on GNSS Solutions

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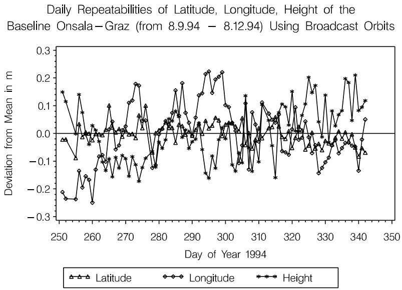
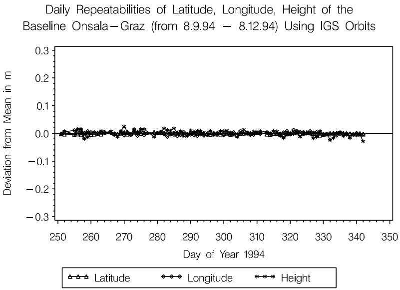
Orbit Error ΔX_{ORB}	Baseline Length l	Baseline Error $\frac{\delta X_{ORB}}{25'000\ km}$	Baseline Error ΔX_l
2.5 m	1 km	0.1 ppm	—
2.5 m	10 km	0.1 ppm	1 mm
2.5 m	100 km	0.1 ppm	10 mm
2.5 m	1000 km	0.1 ppm	100 mm
0.05 m	1 km	0.002 ppm	—
0.05 m	10 km	0.002 ppm	—
0.05 m	100 km	0.002 ppm	0.2 mm
0.05 m	1000 km	0.002 ppm	2 mm

Effect of Orbit Errors on GNSS Solutions

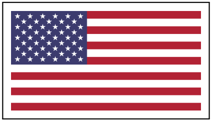
Errors in baseline components due to orbit errors





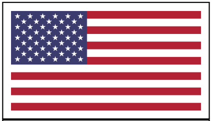
Repeatability (north, east, up) when processing 90 days of GPS observations at Graz (Austria) and Onsala (Sweden) (1200 km baseline) with broadcast orbits (left) and with IGS orbits (right).

GNSS: Global Navigation Satellite Systems



USA: GPS
Global Positioning System

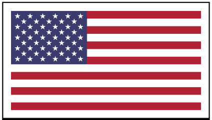
GNSS: Global Navigation Satellite Systems



USA: GPS
Global Positioning System

Russia: ГЛОНАСС
Глобальная навигационная спутниковая система

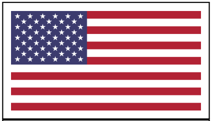
GNSS: Global Navigation Satellite Systems



USA: GPS
Global Positioning System

Russia: GLONASS
Global Satellite Navigation System

GNSS: Global Navigation Satellite Systems

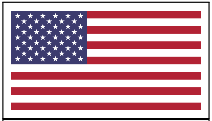


USA: GPS
Global Positioning System

Russia: GLONASS
Global Satellite Navigation System

Europe: Galileo

GNSS: Global Navigation Satellite Systems



USA: GPS
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Russia: GLONASS
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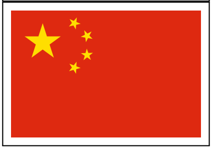
Europe: Galileo



P.R. of China: BeiDou

NAVSTAR GPS Block IIF Satellites

Approximate dimensions:
bus: $2 \times 2 \times 2.5\,\mathrm{m}$
solar panels: $3 \times 2.5 \times 2\,\mathrm{m}$
mass at launch: $\approx 1.6\,\mathrm{t}$

Pictures from the manufacturer Boeing and www.gps.gov .

GPS Constellation

Fact sheet

Orbital elements for GPS satellites

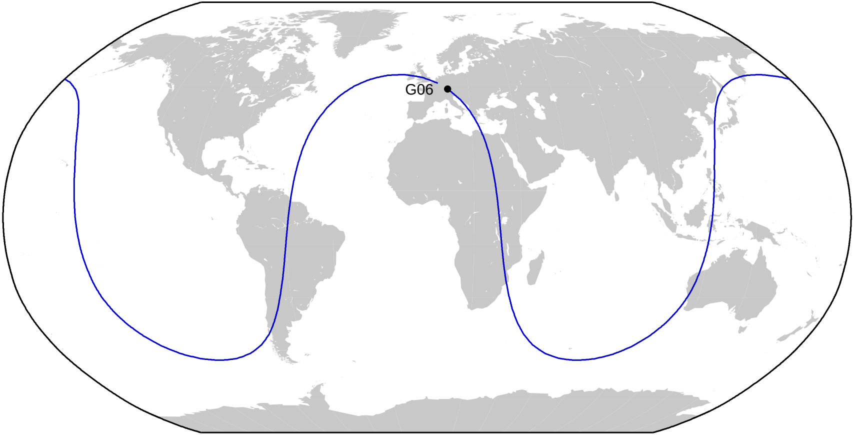
a:	26 560 km	
e:	0	(circular orbit)
i:	55°	

Distribution of orbital planes

Number	6	separated by $\Omega_i = \Omega_0 + n \cdot 60^\circ$
Satellites	4	unequally distributed

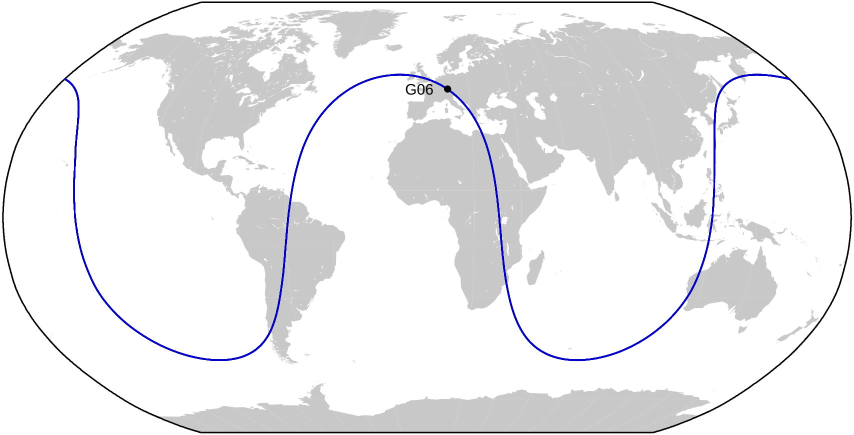
= 24 nominal constellation (today 32 active)

GPS Constellation



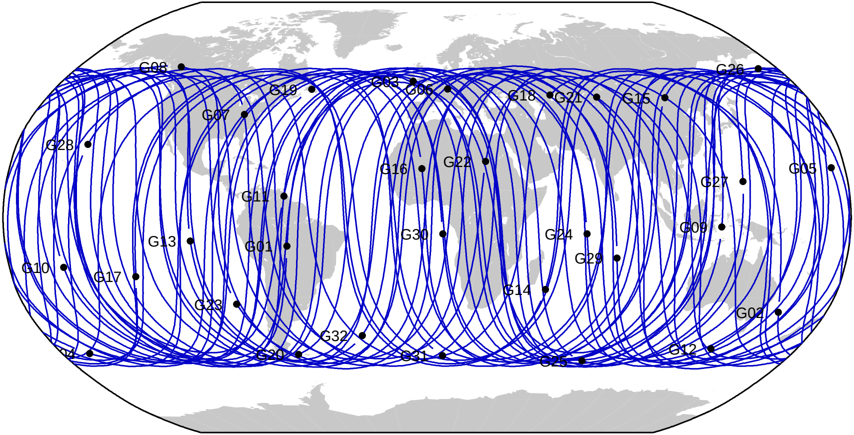
G06 for 1 day (09-May-2012)

GPS Constellation



G06 for 10 days (from 09-May-2012 to 18-May-2012)

GPS Constellation

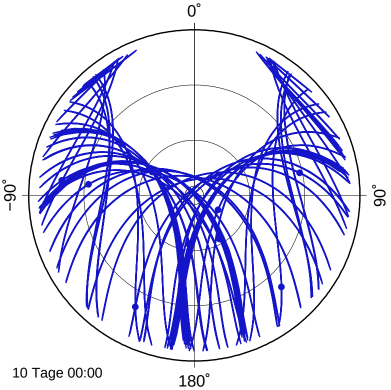


All GPS-satellites for 10 days (from 09-May-2012 to 18-May-2012)

GPS Constellation

- Revolution period $11^{\mathrm{h}}\ 58^{\mathrm{m}}$
  (same constellation after 2 revolutions within 1 sidereal day)

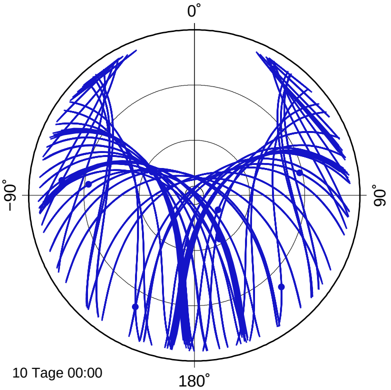
Elevation–Azimuth–Diagram for Zimmerwald



GPS Constellation

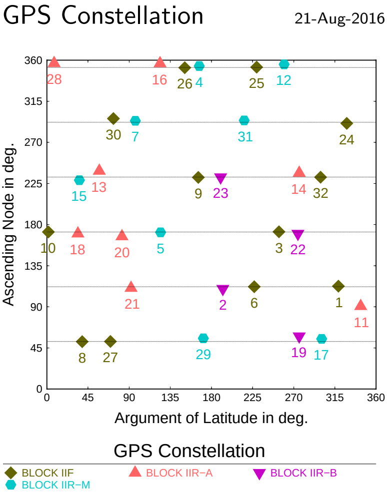
- Revolution period $11^{\mathrm{h}}\ 58^{\mathrm{m}}$
  (same constellation after 2 revolutions within 1 sidereal day)
- Repetition rates:
  same geometry: 1 sidereal day
  same constellation: 1 sidereal day

Elevation–Azimuth–Diagram for Zimmerwald



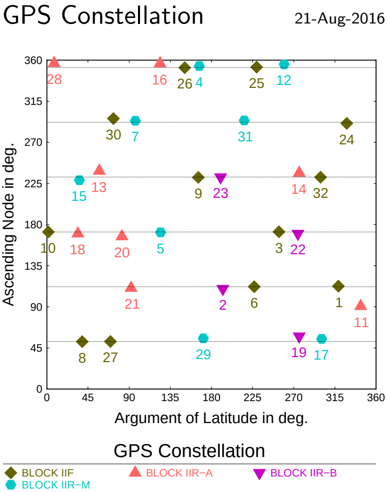
GPS Constellation

- Revolution period $11^{\mathrm{h}}\ 58^{\mathrm{m}}$
  (same constellation after 2 revolutions within 1 sidereal day)
- Repetition rates:
  same geometry: 1 sidereal day
  same constellation: 1 sidereal day
- Signals:
  Code: C1,
  P1, P2,
  Phase: L1, L2



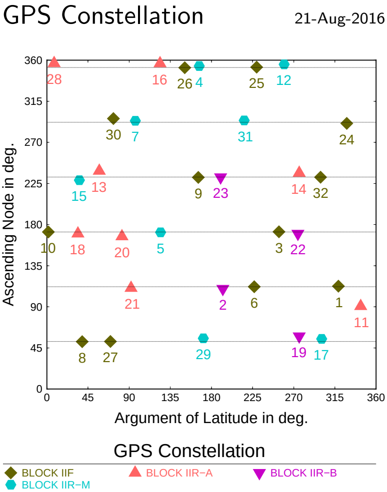
GPS Constellation

- Revolution period $11^{\mathrm{h}}\ 58^{\mathrm{m}}$
  (same constellation after 2 revolutions within 1 sidereal day)
- Repetition rates:
  same geometry: 1 sidereal day
  same constellation: 1 sidereal day
- Signals:
  Code: C1, C2 (since IIR–M), P1, P2,
  Phase: L1, L2 (L2C!!)



GPS Constellation

- Revolution period $11^{\mathrm{h}}\ 58^{\mathrm{m}}$
  (same constellation after 2 revolutions within 1 sidereal day)
- Repetition rates:
  same geometry: 1 sidereal day
  same constellation: 1 sidereal day
- Signals:
  Code: C1, C2 (since IIR–M),
  P1, P2, C5 (since IIF)
  Phase: L1, L2 (L2C!!), L5



GLONASS-M Satellites

Approximate dimensions:
bus: cylinder $2.4 \times 3.7\,\mathrm{m}$
solar panels: width of $7.2\,\mathrm{m}$
mass at launch: $\approx 1.5\,\mathrm{t}$

Pictures from http://cdn.satellitetoday.com and http://newspepper.su .

Fact sheet

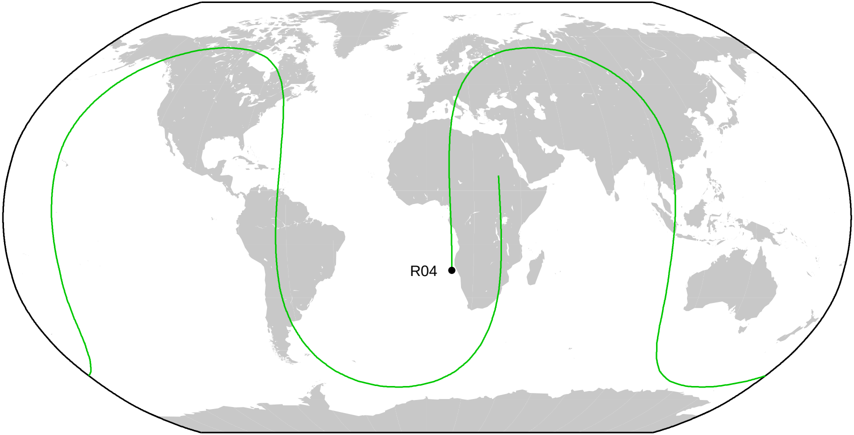
Orbital elements for GLONASS satellites

a:	25 500 km	
e:	0	(circular orbit)
i:	65°	

Distribution of orbital planes

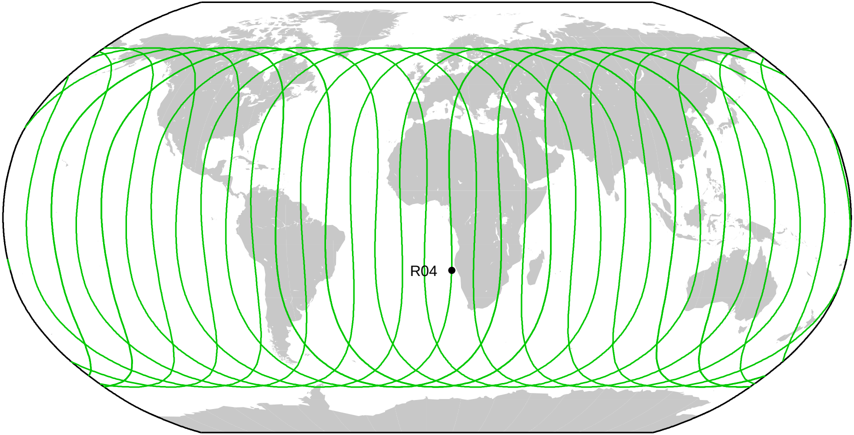
Number	3	separated by $\Omega_i = \Omega_0 + n \cdot 120^\circ$
Satellites	8	equally distributed
	= 24 nominal constellation	

GLONASS Constellation



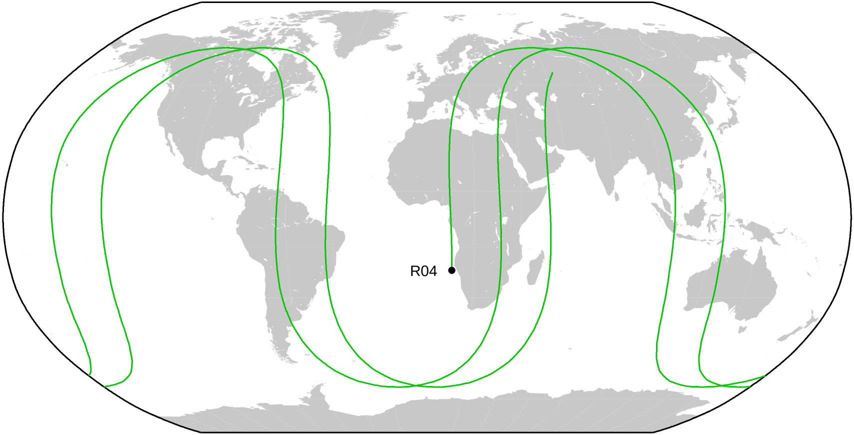
R04 for 1 day (09-May-2012)

GLONASS Constellation



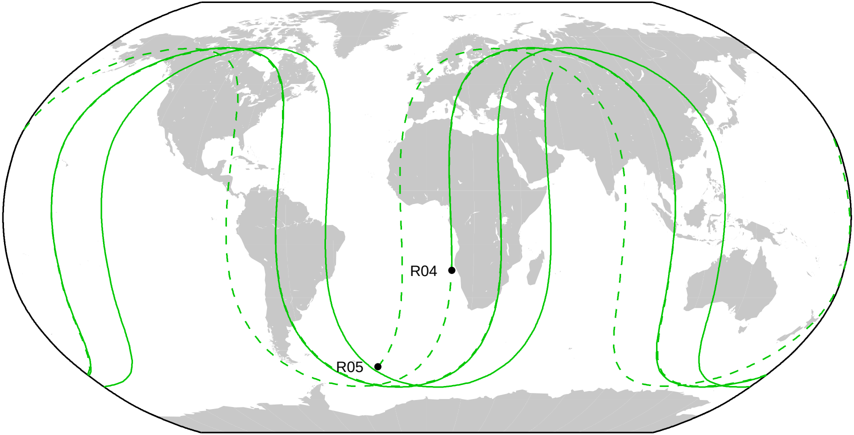
R04 for 10 days (from 09-May-2012 to 18-May-2012)

GLONASS Constellation



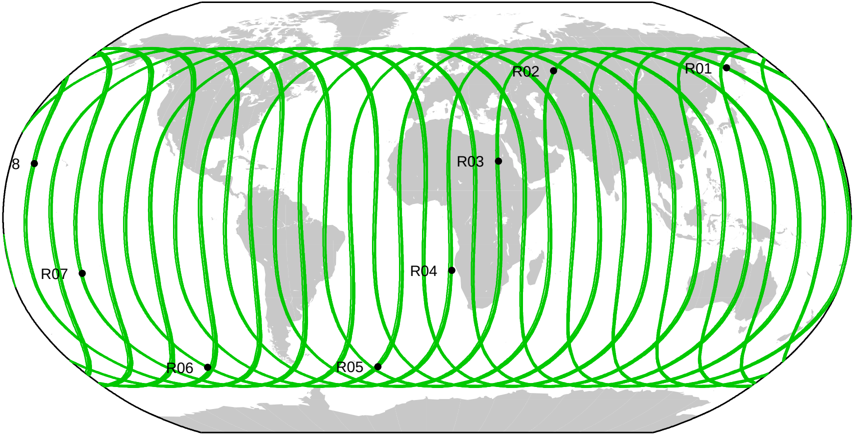
R04 for 2 days (from 09-May-2012 to 10-May-2012)

GLONASS Constellation



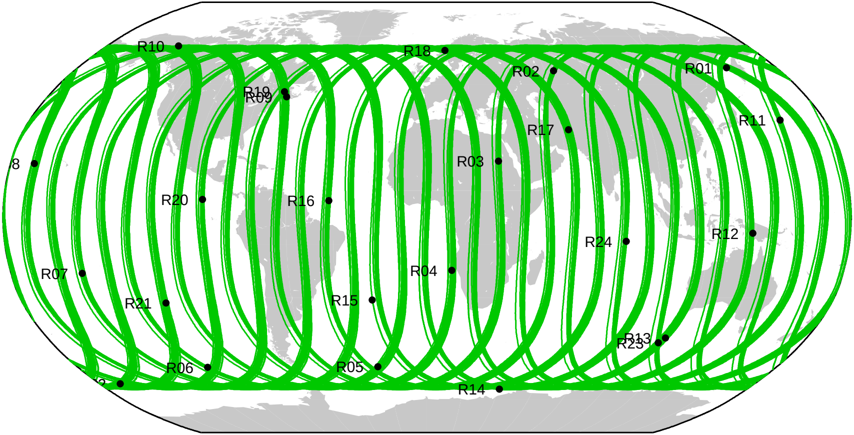
R04 and R05 for 2 days (from 09-May-2012 to 10-May-2012)

GLONASS Constellation



R01 to R08 for 10 days (from 09-May-2012 to 18-May-2012)

GLONASS Constellation

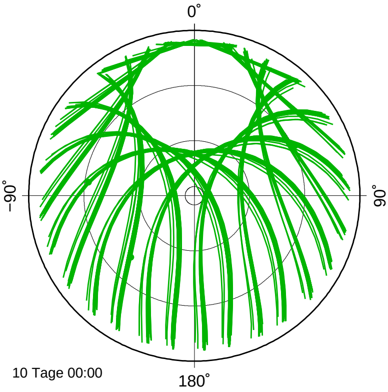


All GLONASS satellites for 10 days (from 09-May-2012 to 18-May-2012)

GLONASS Constellation

- Revolution period $11^{\mathrm{h}}\ 16^{\mathrm{m}}$
  (same constellation after 17 revolutions within 8 sidereal days)

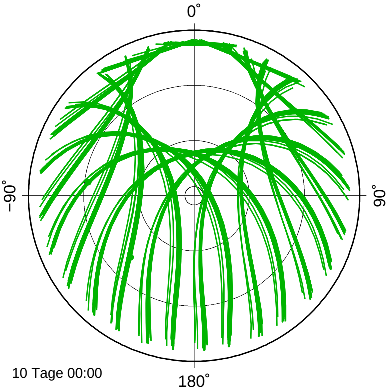
Elevation–Azimuth–Diagram for Zimmerwald



GLONASS Constellation

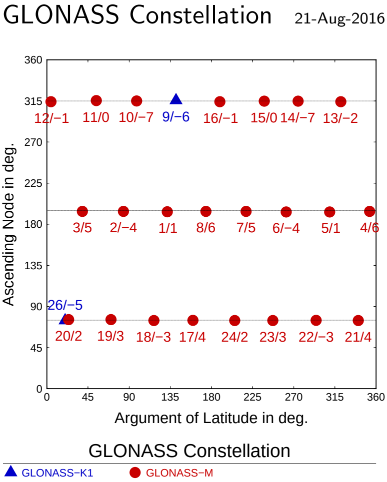
- Revolution period $11^{\mathrm{h}}\ 16^{\mathrm{m}}$
  (same constellation after 17 revolutions within 8 sidereal days)
- Repetition rates:
  same geometry:
  – same plane: 1 sidereal day
  – next plane: $\frac{1}{3}$ sidereal day
  same constellation: 8 sidereal days

Elevation–Azimuth–Diagram for Zimmerwald

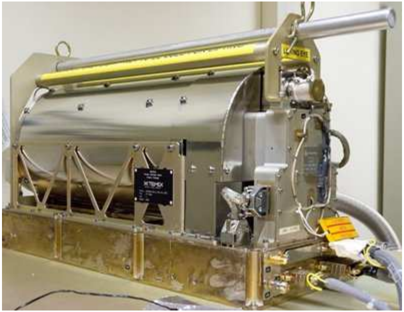


GLONASS Constellation

- Revolution period $11^{\mathrm{h}}\ 16^{\mathrm{m}}$
  (same constellation after 17 revolutions within 8 sidereal days)
- Repetition rates:
  same geometry:
  – same plane: 1 sidereal day
  – next plane: $\frac{1}{3}$ sidereal day
  same constellation: 8 sidereal days
- Signals:
  Code: C1, C2, P1, P2
  Phase: L1, L2



Galileo FOC Satellites



Approximate dimensions:

bus: $2.5 \times 1.1 \times 1.2\,\mathrm{m}$; solar panels: width tip-to-tip $14.5\,\mathrm{m}$

Mass at launch: $\approx 733\,\mathrm{kg}$

Pictures from ESA downloaded from `http://spaceflight101.com`.

Galileo Constellation

Fact sheet

Orbital elements for Galileo satellites

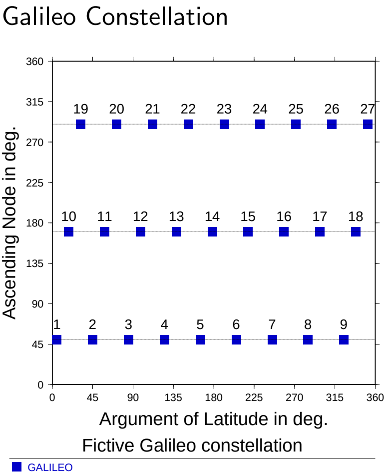
a:	30 000 km	
e:	0	(circular orbit)
i:	56°	

Distribution of orbital planes

Number	3	separated by $\Omega_i = \Omega_0 + n \cdot 120^\circ$
Satellites	9	equally distributed
	= 27 nominal constellation	

Galileo Constellation

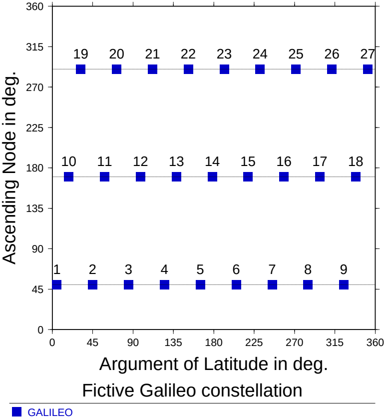
- Revolution period $13^{\mathrm{h}}\ 45^{\mathrm{m}}$
  (same constellation after 17 revolutions within 10 sidereal days)



Galileo Constellation

- Revolution period $13^{\mathrm{h}}\ 45^{\mathrm{m}}$
  (same constellation after 17 revolutions within 10 sidereal days)
- Repetition rates:
  same geometry/constellation: 10 sidereal days

Galileo Constellation



Fictive Galileo constellation

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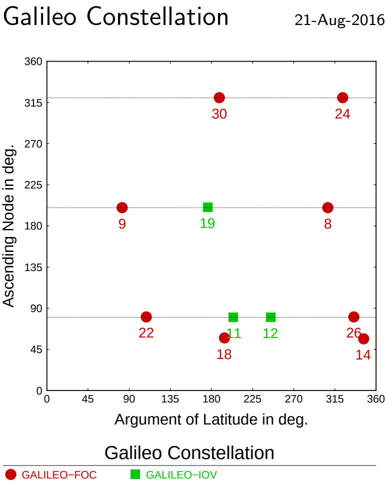
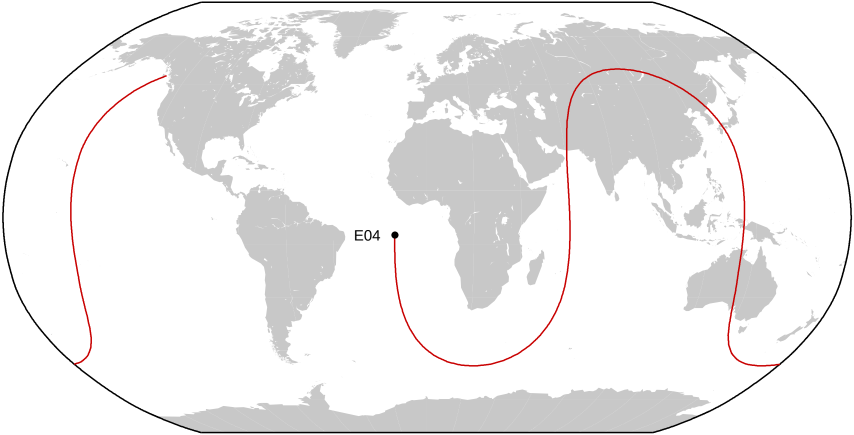
- Revolution period $13^{\mathrm{h}}\ 45^{\mathrm{m}}$
  (same constellation after 17 revolutions within 10 sidereal days)
- Repetition rates:
  same geometry/constellation: 10 sidereal days

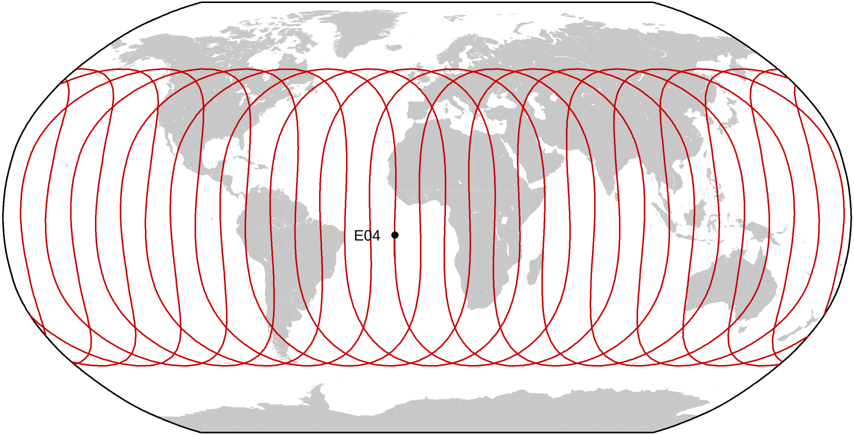


Galileo Constellation



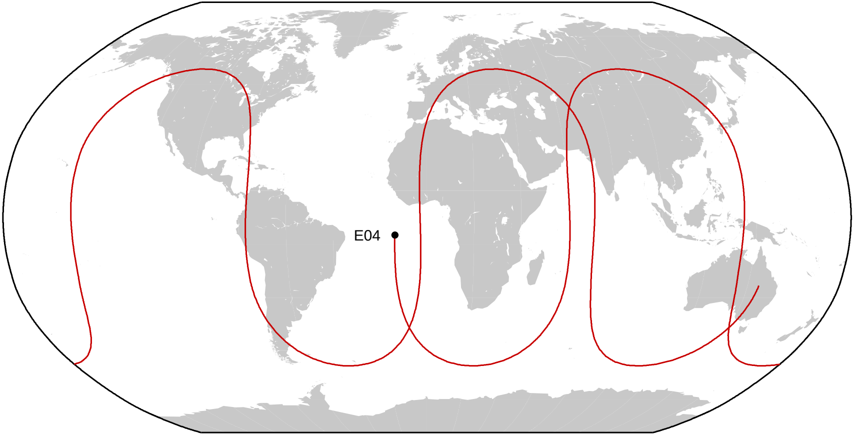
Fictive E04 for one day

Galileo Constellation



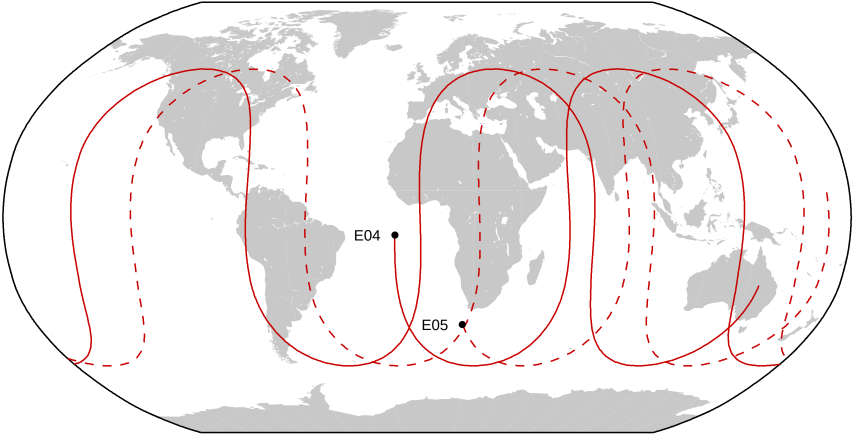
Fictive E04 for 10 days

Galileo Constellation



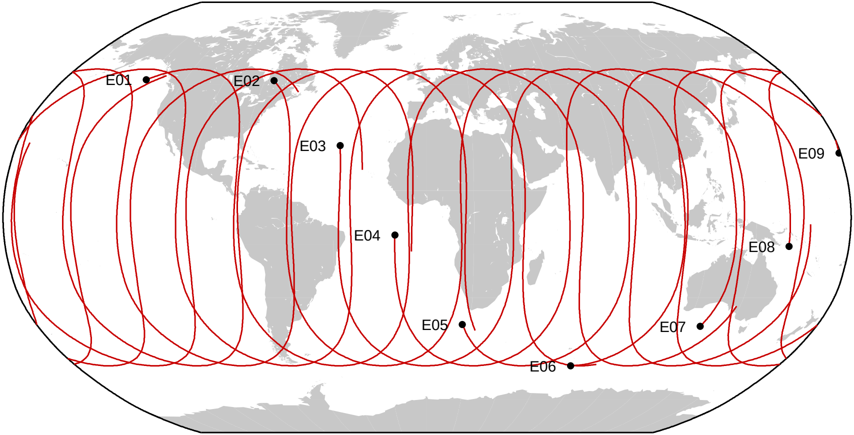
Fictive E04 for two days

Galileo Constellation



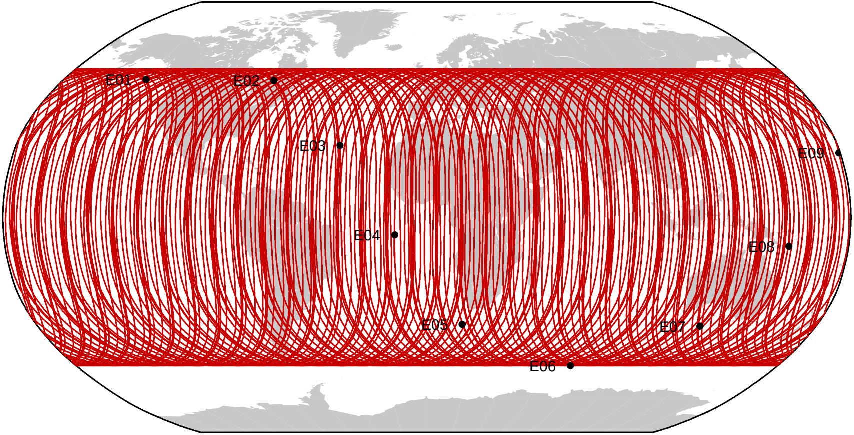
Fictive E04 and E05 for two days

Galileo Constellation



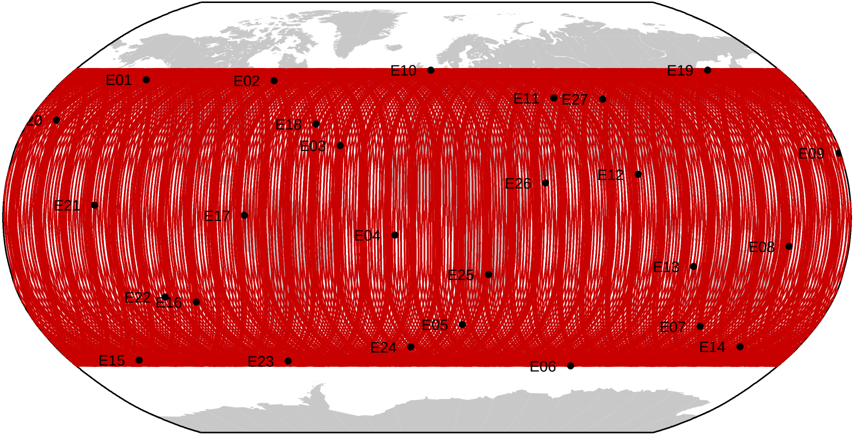
Fictive E01 to E09 for two days

Galileo Constellation



Fictive E01 to E09 for 10 days

Galileo Constellation



Fictive Galileo constellation for 10 days

BeiDou Constellation

Fact sheet (MEO)

Orbital elements for BeiDou satellites

a:	28 000 km	
e:	0	(circular orbit)
i:	55°	

Distribution of orbital planes

Number	3	separated by $\Omega_i = \Omega_0 + n \cdot 120^\circ$
Satellites	9	equally distributed
	= 27 nominal constellation	

Fact sheet (MEO)

Orbital elements for BeiDou satellites

a:	28 000 km	
e:	0	(circular orbit)
i:	55°	

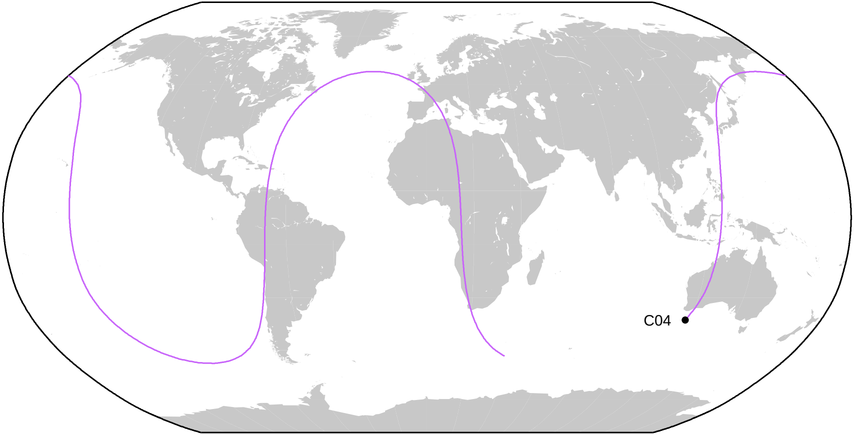
Distribution of orbital planes

Number	3	separated by $\Omega_i = \Omega_0 + n \cdot 120°$
Satellites	9	equally distributed
	= 27 nominal constellation	

Repetition rates

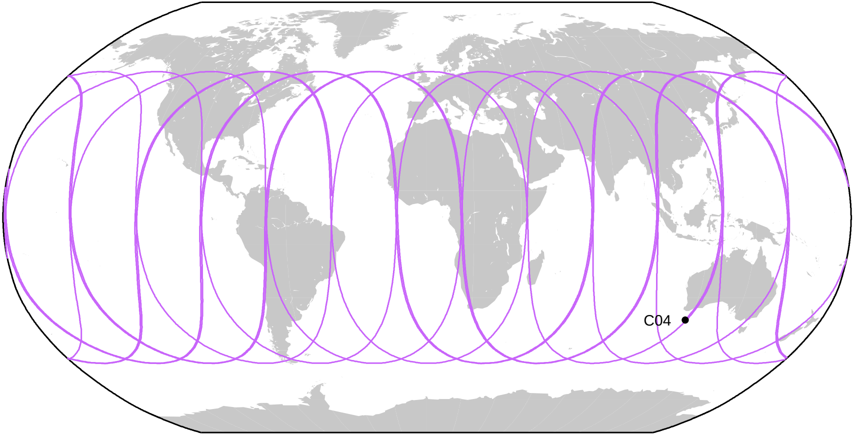
Revolution period	12 h 57 min
Constellation	after 17 revolutions within 7 sidereal days

BeiDou Constellation



Fictive C04 for one day

BeiDou Constellation



Fictive C04 for 10 days

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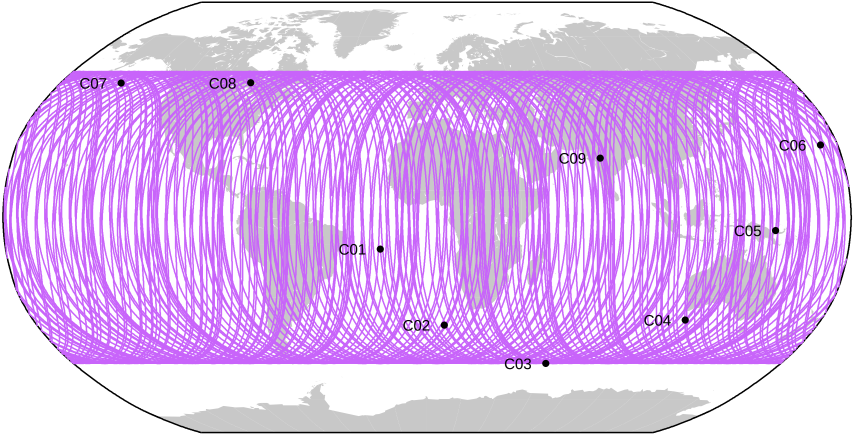
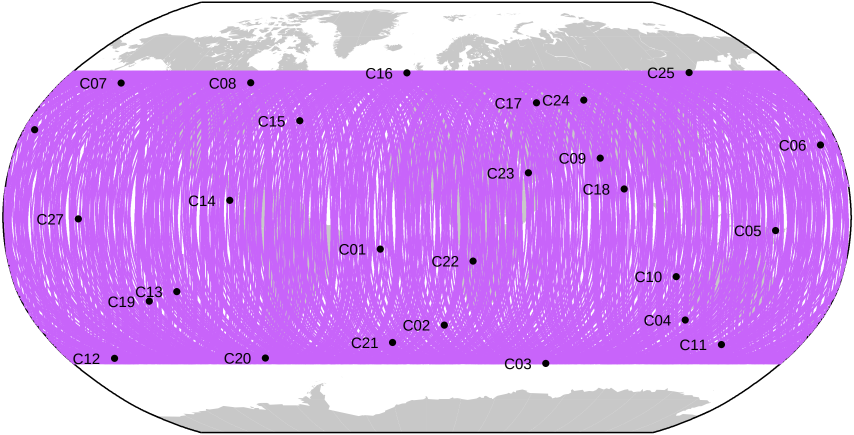
BeiDou Constellation



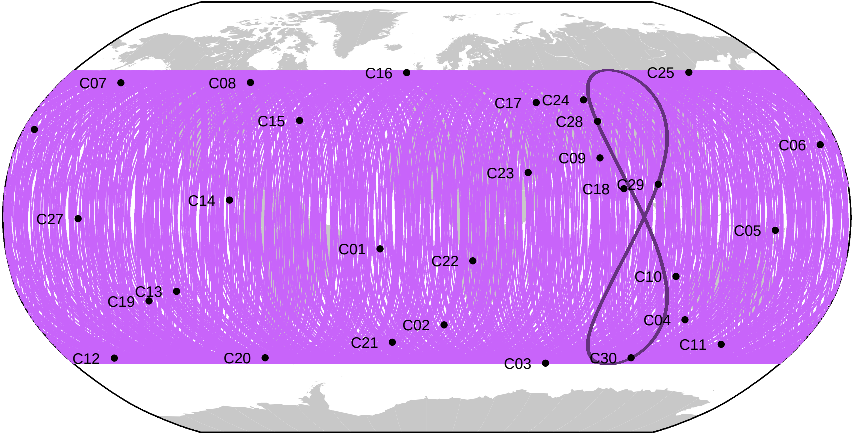
Fictive C01 to C09 for 10 days

BeiDou Constellation



Fictitive BeiDou constellation for 10 days

BeiDou Constellation



Fictitive BeiDou constellation for 10 days

BeiDou Constellation

Fact sheet (part 2)

Orbital elements for BeiDou satellites

a:	42 000 km	
e:	0	(circular orbit)
i:	55° (IGSO)	0° (GEO)

Distribution of orbital planes for IGSO–satellites

Number	3	separated by $\Omega_i = \Omega_0 + n \cdot 120^\circ$
Satellites	1	distributed in a way that all satellites follow the same ground track
	$=$ 3 nominal constellation	

Fact sheet (part 2)

Orbital elements for BeiDou satellites

a:	42 000 km	
e:	0	(circular orbit)
i:	55° (IGSO)	0° (GEO)

Distribution of orbital planes for IGSO–satellites

Number	3	separated by $\Omega_i = \Omega_0 + n \cdot 120^\circ$
Satellites	1	distributed in a way that all satellites follow the same ground track
	$=$ 3 nominal constellation	

Repetition rates

Revolution period	23 h 56 min
Constellation	after one revolutions within 1 sidereal day

Fact sheet

Orbital elements for QZSS satellites

a:	42 000 km		
e:	0.075	ω:	270°
i:	43°		

Distribution of orbital planes

Number	3	separated by $\Omega_i = \Omega_0 + n \cdot 120^\circ$
Satellites	1	distributed in a way that all satellites follow the same ground track
	= 3 nominal constellation	

QZSS Constellation

Fact sheet

Orbital elements for QZSS satellites

a:	42 000 km		
e:	0.075	ω:	270°
i:	43°		

Distribution of orbital planes

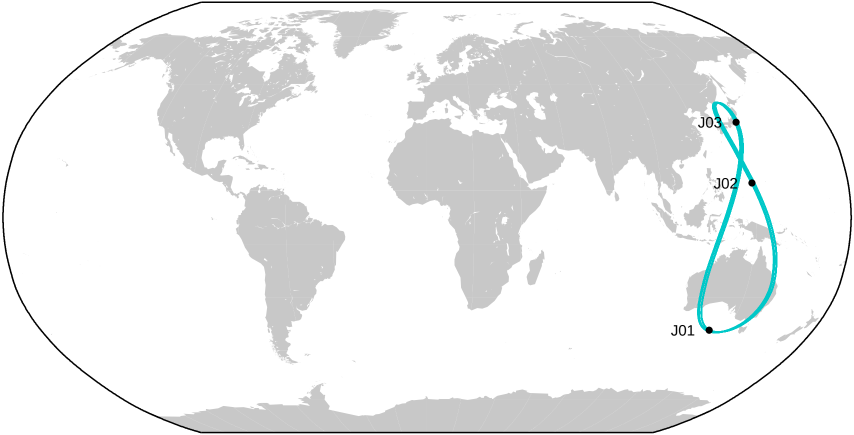
Number	3	separated by $\Omega_i = \Omega_0 + n \cdot 120°$
Satellites	1	distributed in a way that all satellites follow the same ground track

= 3 nominal constellation

Repetition rates

Revolution period	23 h 56 min
Constellation	after one revolutions within 1 sidereal day

QZSS Constellation



Fictive QZSS constellation for 10 days

QZSS Constellation

QZSS Satellites

Approximate dimensions:

bus: $3 \times 3 \times 6\,\mathrm{m}$

solar panels: $2.9 \times 3.1 \times 6.2\,\mathrm{m}$

width tip-to-tip $25\,\mathrm{m}$

Mass at launch: $\approx 4\,\mathrm{t}$

Pictures from JAXA .

GNSS Constellation Summary

Global Navigation Systems

GPS

GLONASS

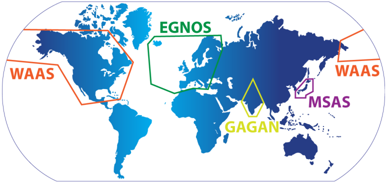
Galileo

BeiDou

Regional and Augmentation Systems

QZSS

NAVIC



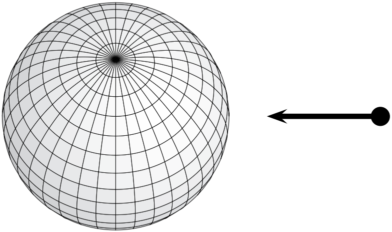
SBAS

Effects Acting on Satellites and Related Models

Gravitational Forces

Gravitational Forces

Gravitational Forces



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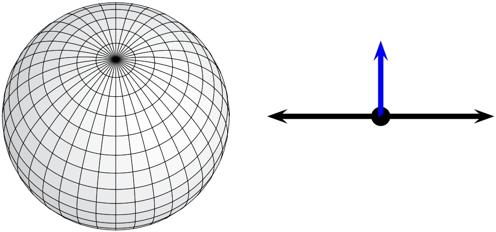

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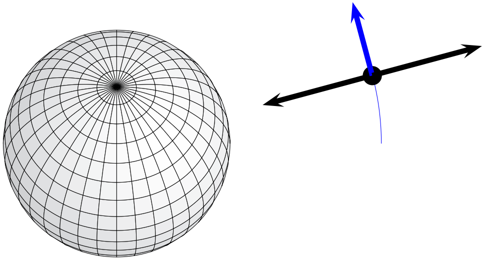

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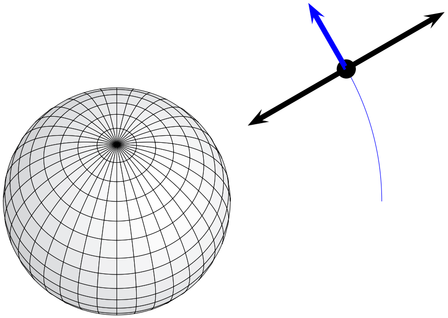

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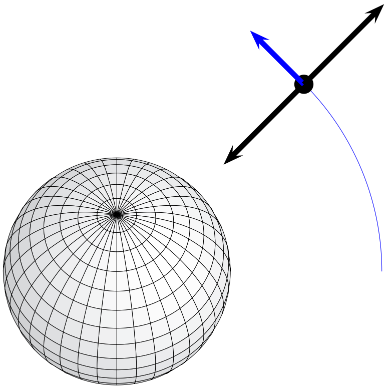

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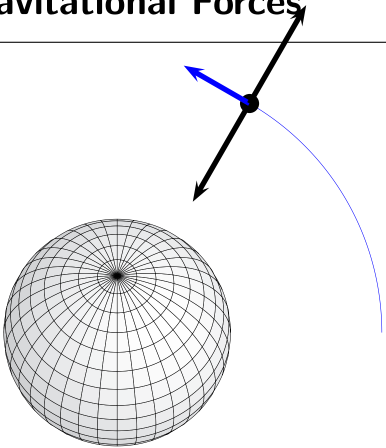

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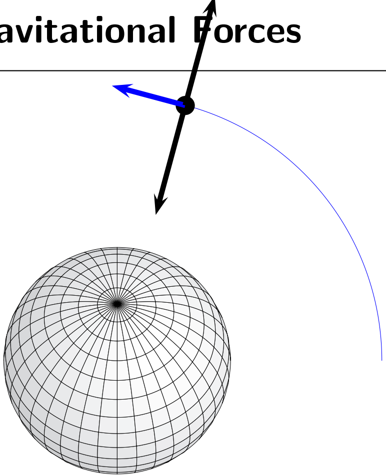

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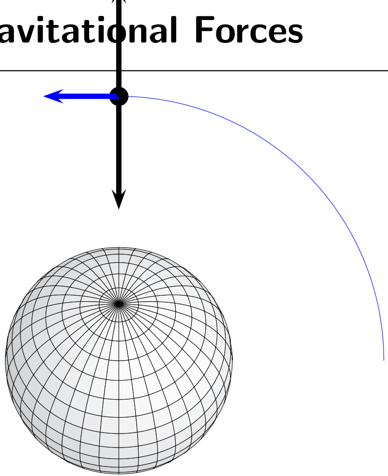

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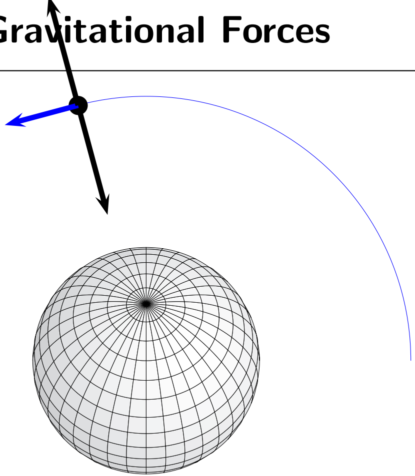

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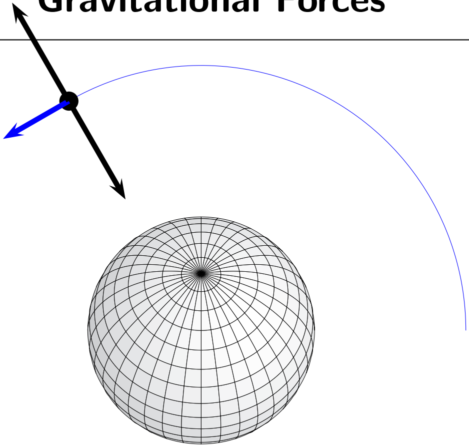

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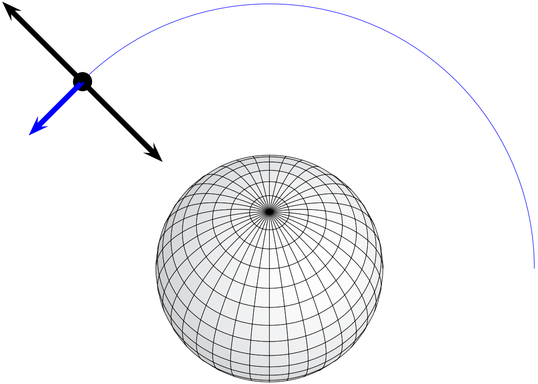

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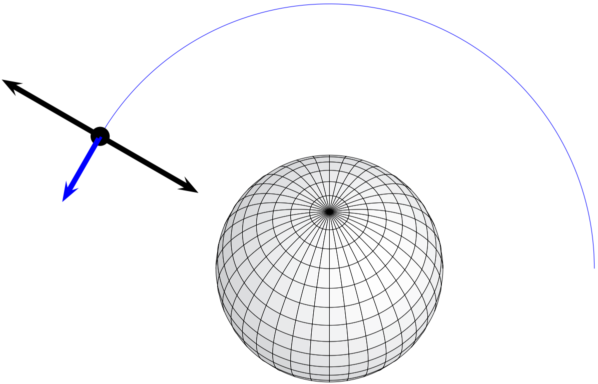

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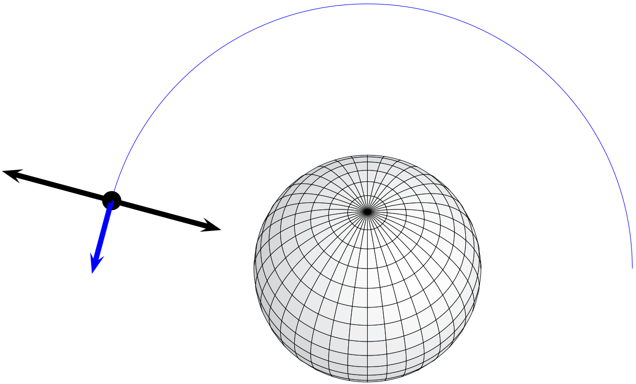

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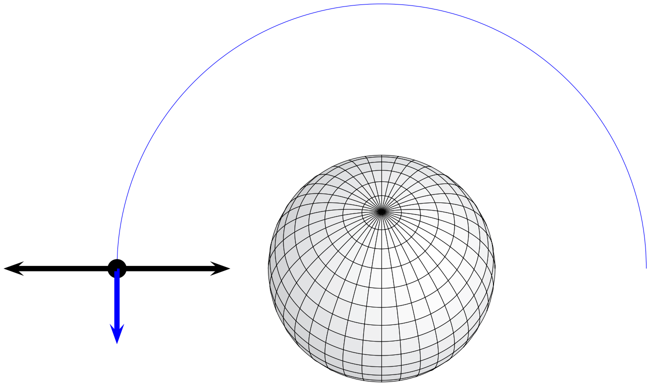

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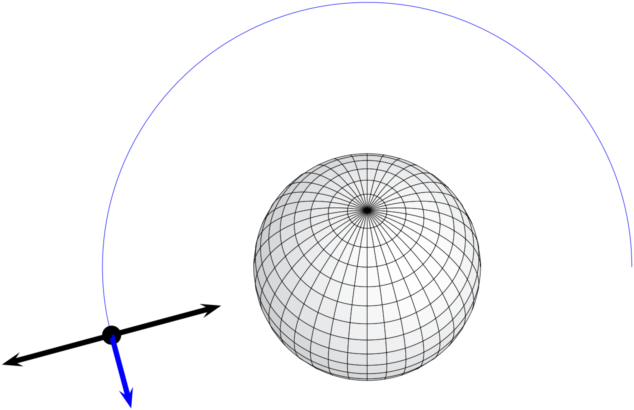

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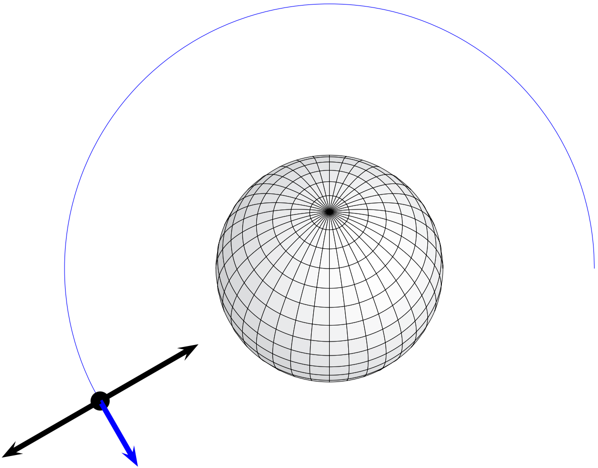

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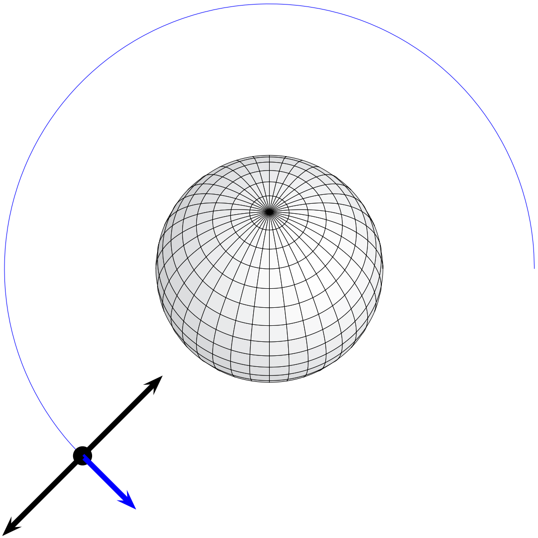

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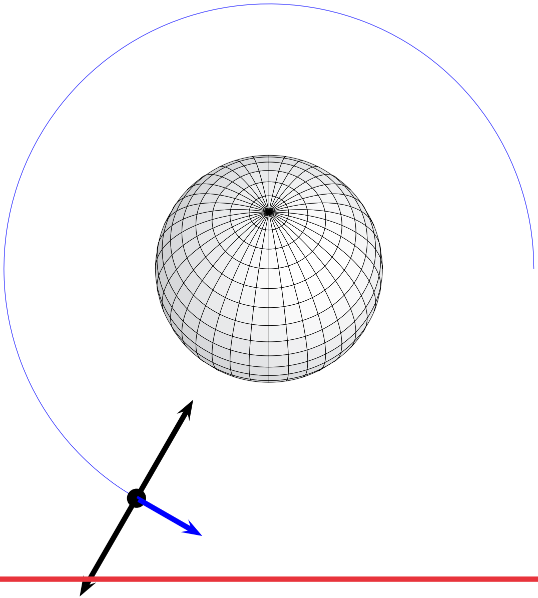

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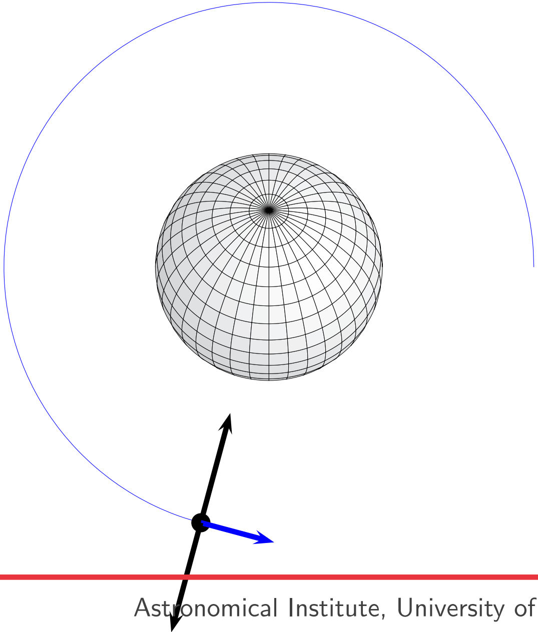

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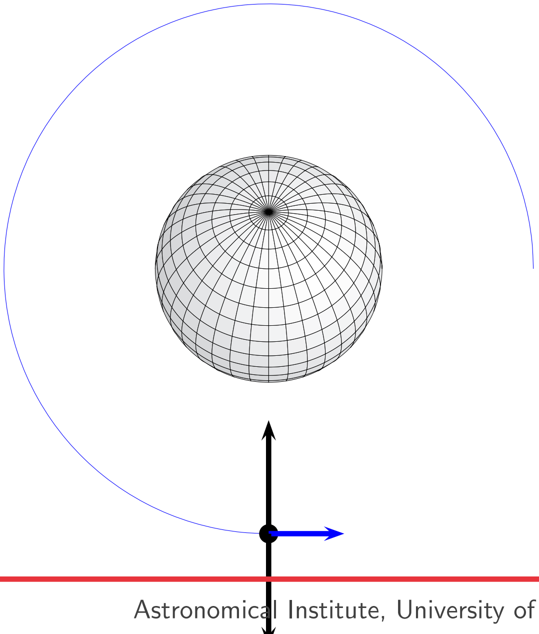

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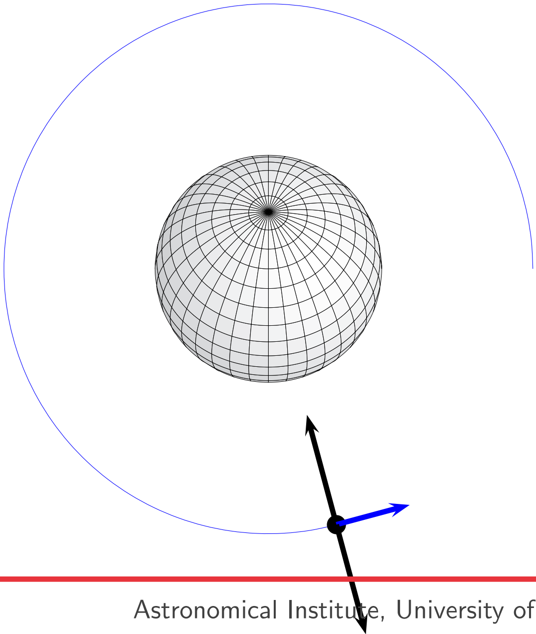

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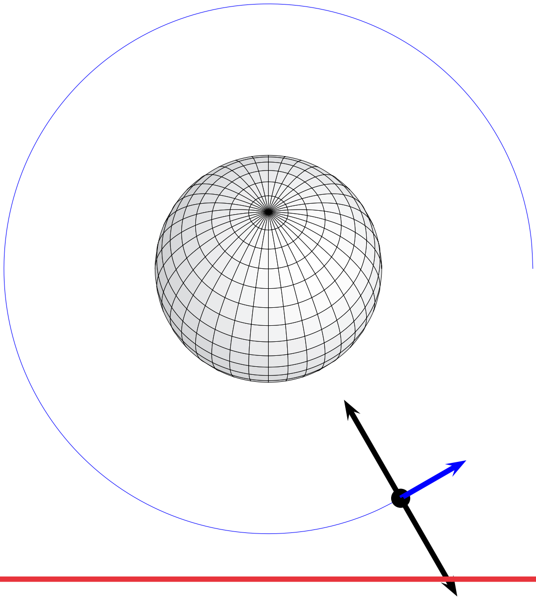

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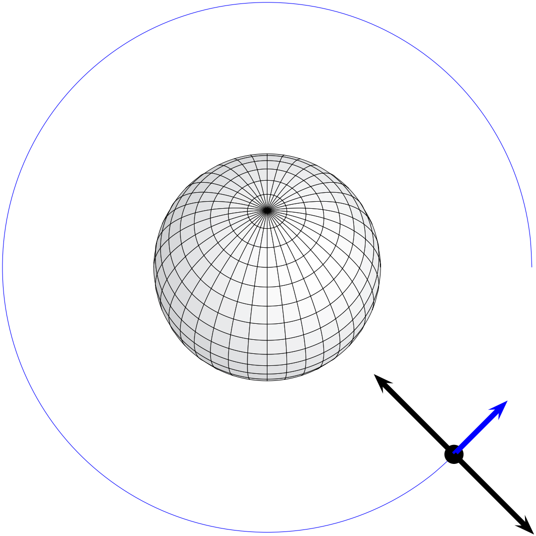

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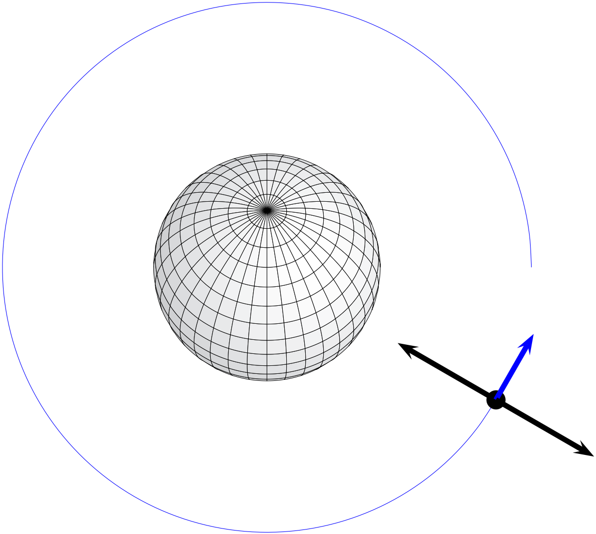

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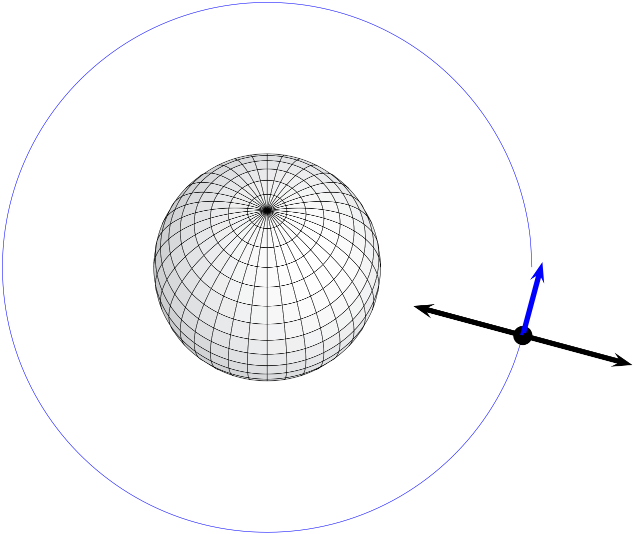

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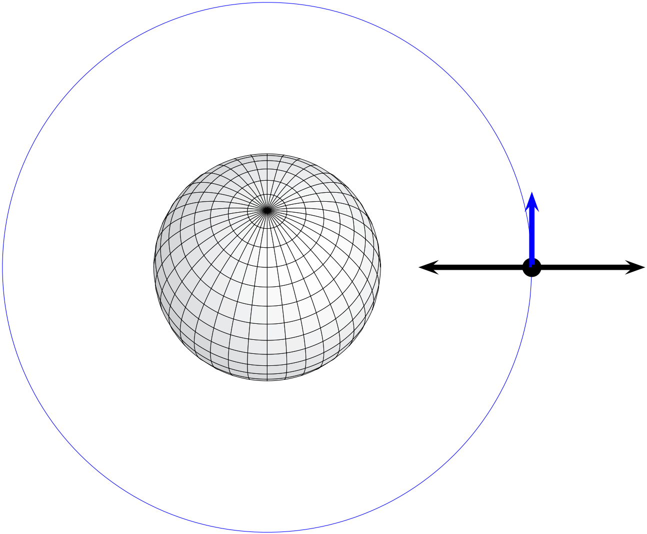

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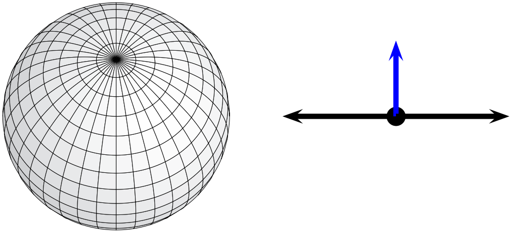

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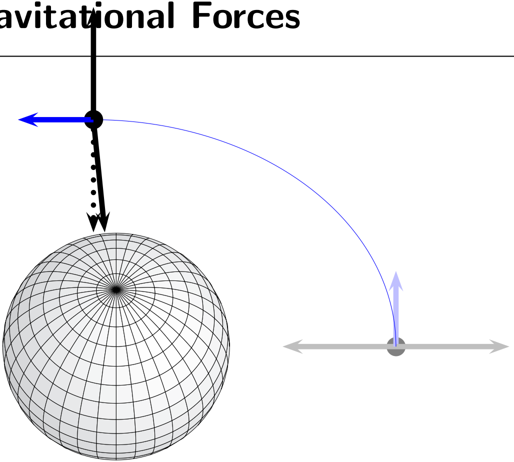

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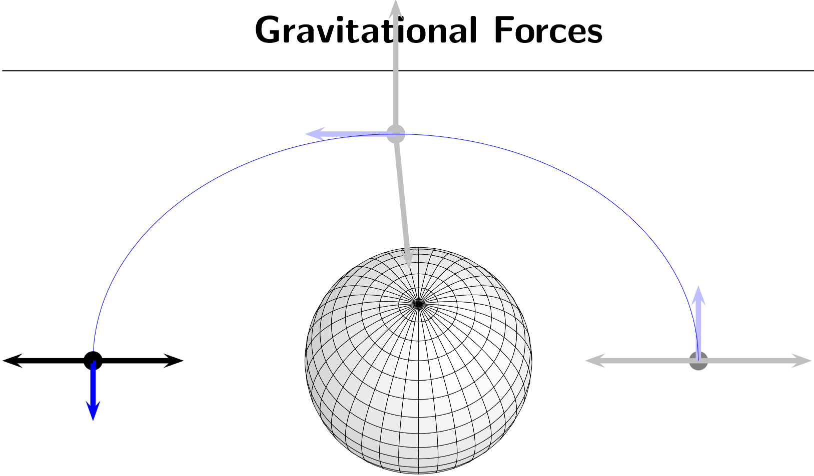

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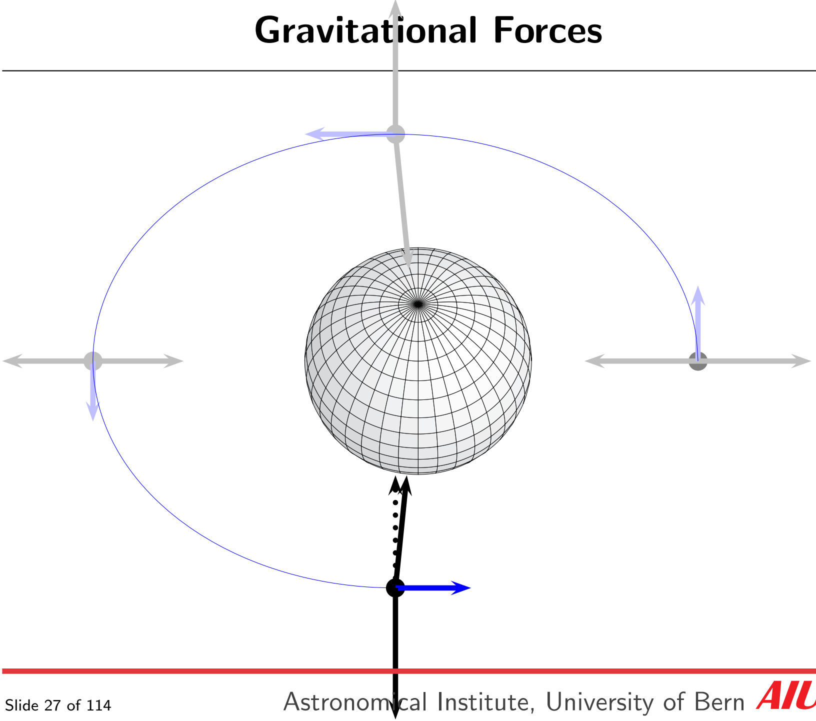

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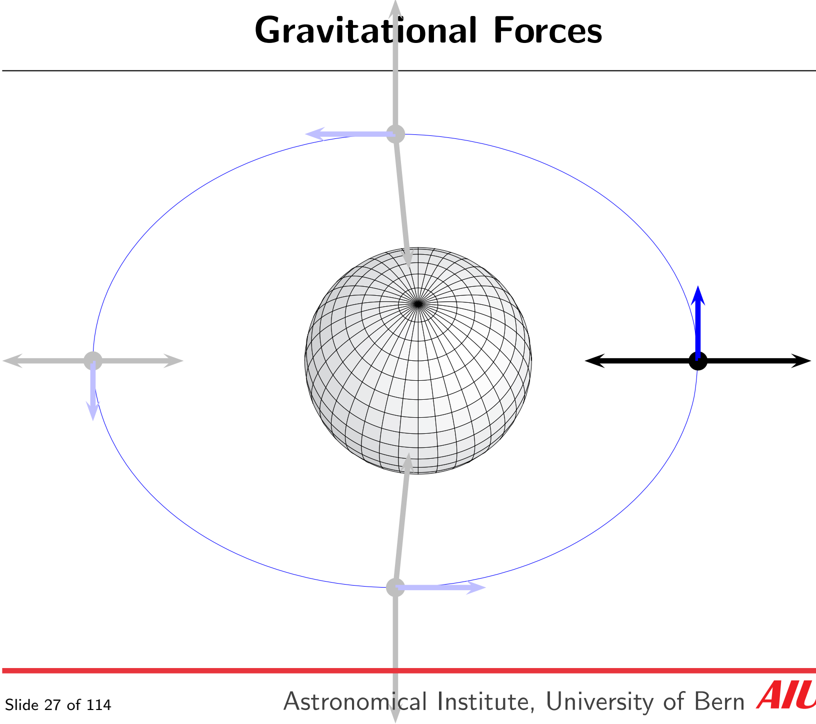

R. Dach: GNSS Satellite Orbit Modelling
NGK Summer School, 29.Aug.–01. Sep. 2016, Båstad

Gravitational Forces

Acceleration due to centrifugal force:

$$\ddot{\vec{r}} = \frac{|\dot{\vec{r}}|^2}{|\vec{r}|} \cdot \frac{\vec{r}}{|\vec{r}|} \qquad (1)$$

Acceleration due to gravitational force:

$$\ddot{\vec{r}} = -GM_E \cdot \frac{\vec{r}}{|\vec{r}|^3} \qquad (2)$$

GM_E	product of the constant of gravity and the mass of the Earth
$\vec{r}$	geocentric vector to the satellite
$\dot{\vec{r}}$	the related first time derivative (velocity vector)
$\ddot{\vec{r}}$	the related second time derivative (acceleration vector)

Gravitational Forces

Velocities of selected GNSS satellites:

- Starting with the radius of the satellite orbit, the gravitational acceleration can be computed according to equation (2), with $GM_E = 398.6004415 \cdot 10^{12}\,\frac{\mathrm{m}^3}{\mathrm{s}^2}$.
- To compensate the gravitational acceleration a velocity of the satellite according to equation (1) is needed.

Satellite	$\lvert\vec{r}\rvert$ in km	$\lvert\ddot{\vec{r}}\rvert$ in $\frac{\mathrm{m}}{\mathrm{s}^2}$	$\lvert\dot{\vec{r}}\rvert$ in $\frac{\mathrm{km}}{\mathrm{s}}$
GLONASS	25 500	0.613	3.95
GPS	26 560	0.565	3.87
Galileo	30 000	0.443	3.65
BeiDou, IGSO	42 000	0.226	3.08

Keplerian Orbit

The Equation of Motion:

$$\ddot{\vec{r}} = -GM_E \frac{\vec{r}}{|\vec{r}|^3} \tag{3}$$

- describes the motion of a satellite around a spherically symmetric Earth.

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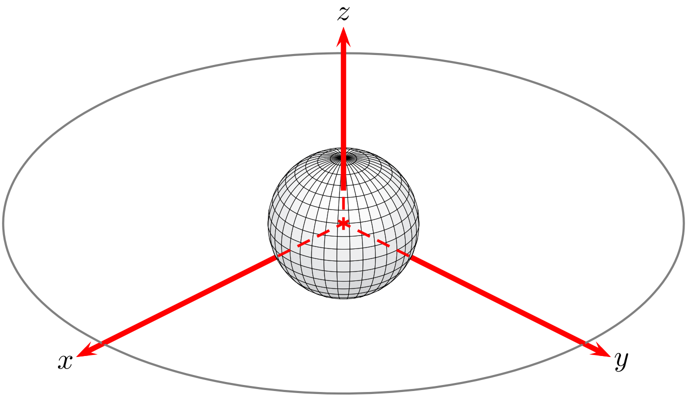
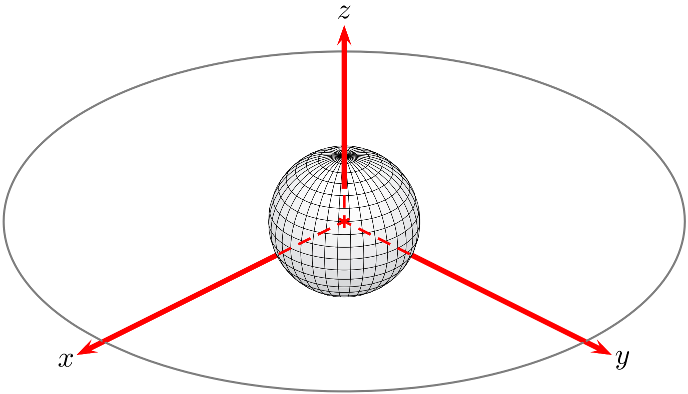
  It describes the trajectory of the satellite along a so called Keplerian orbit ellipse.

Quasi-Inertial Coordinate System



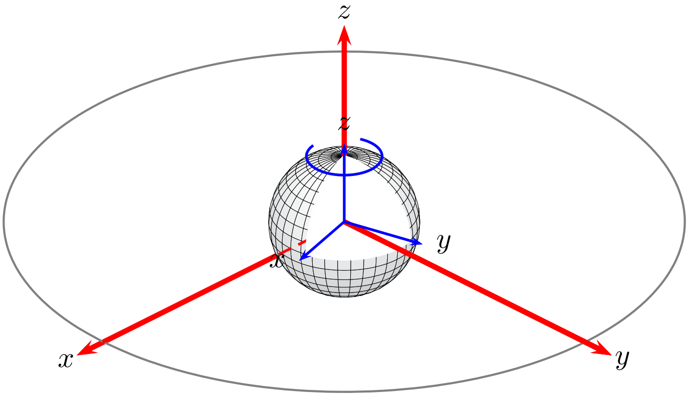
- Origin is located in the center of mass of the Earth.

Quasi-Inertial Coordinate System



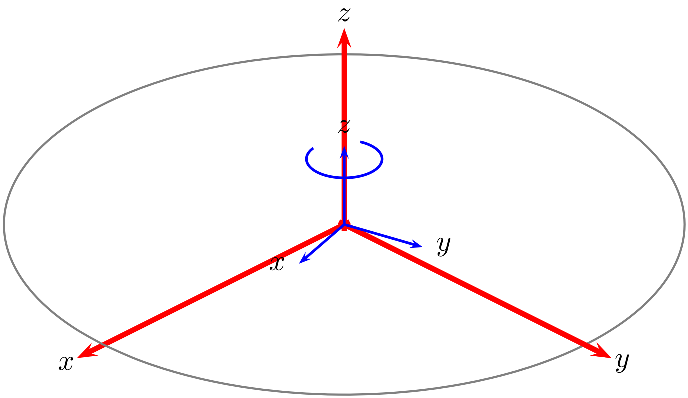
- Z-axis corresponds to the mean rotation axis of the Earth.
- X-axis points to the vernal equinox (intersection with the ecliptic).

Quasi-Inertial Coordinate System



- The coordinate system does not follow the rotation of the Earth but follows the motion of the Earth around the Sun.

Quasi-Inertial Coordinate System



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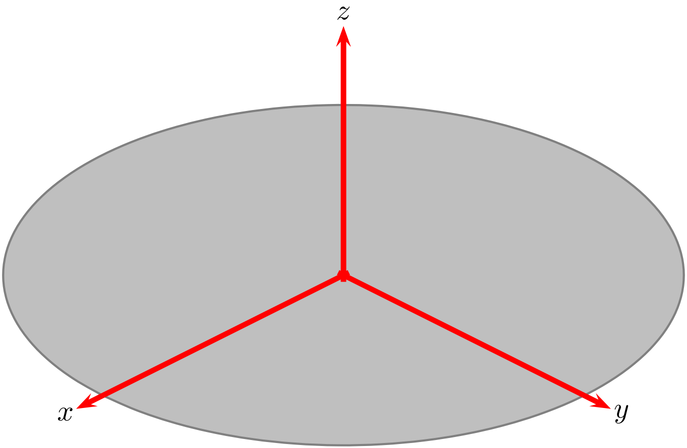
Keplerian Orbit Ellipse



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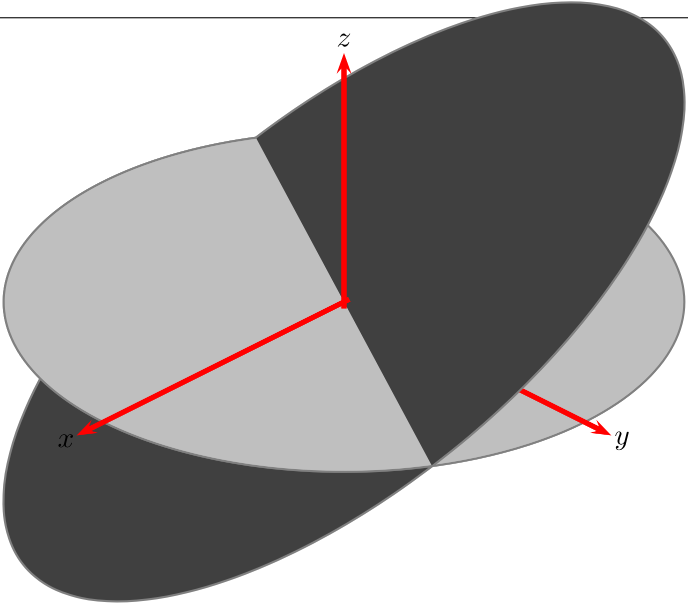

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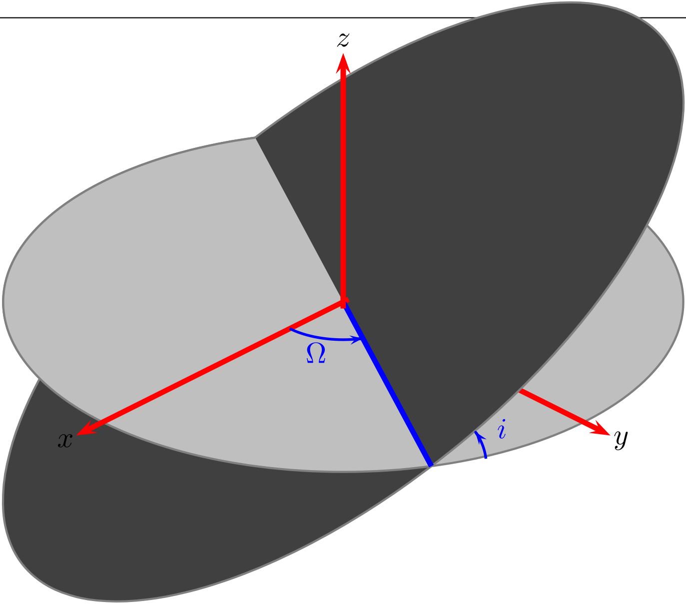

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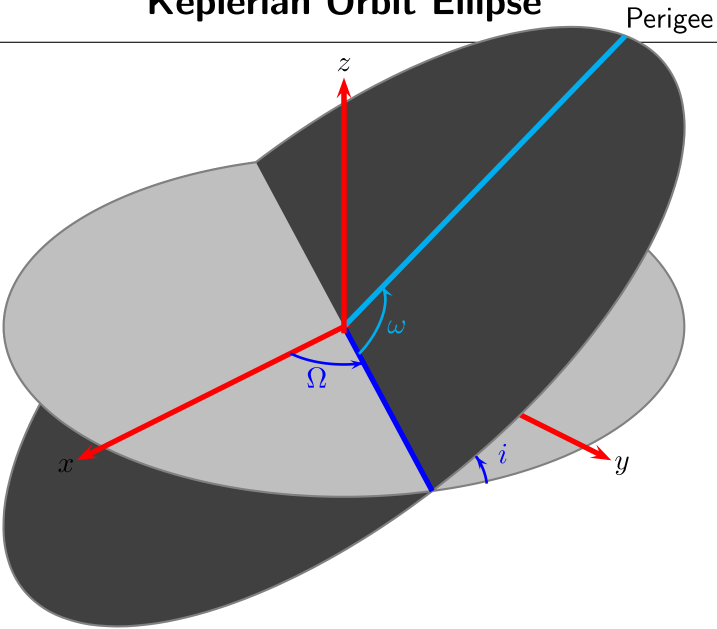

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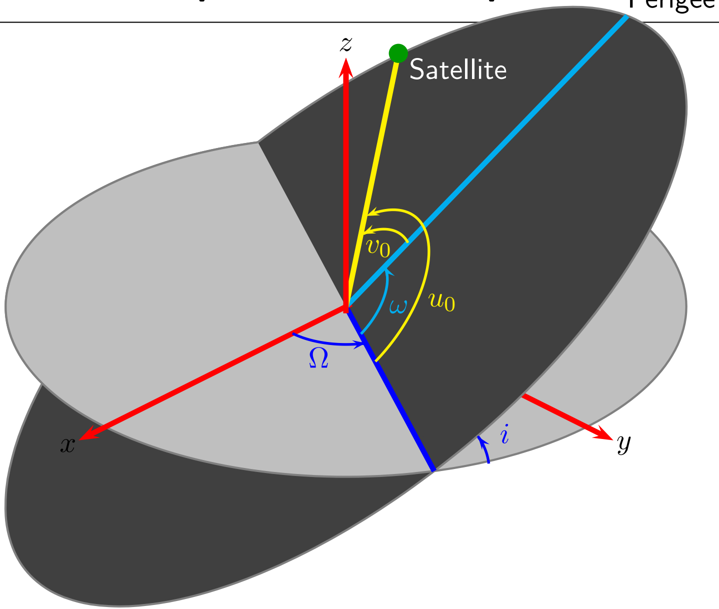


Keplerian Orbit Ellipse

Description of the orbit ellipse

a	semimajor axis
e	numerical eccentricity

Location of the orbit ellipse

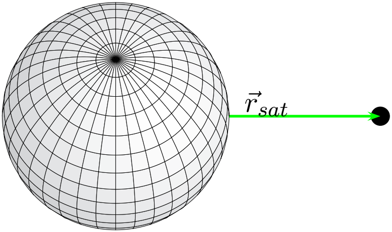
i	inclination of the orbital plane
Ω	right ascension of the ascending node
ω	argument of perigee

Location of the satellite within the orbit ellipse

$u_0(t_0)$	argument of latitude of the satellite at t_0
$v_0(t_0)$	true anomaly at epoch t_0
	with $u_0(t_0) = \omega + v_0(t_0)$

Advancing the Keplerian Orbit Theory

Gravitational effect of other celestial bodies:



Advancing the Keplerian Orbit Theory

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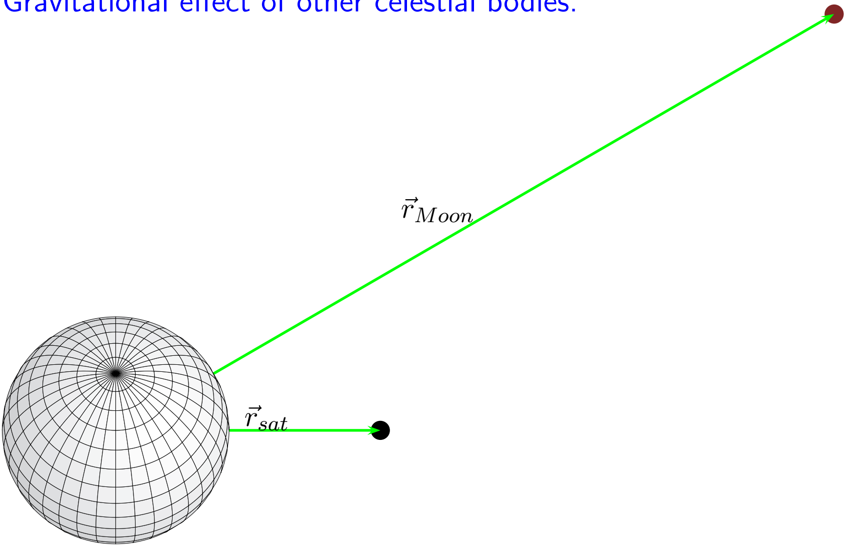

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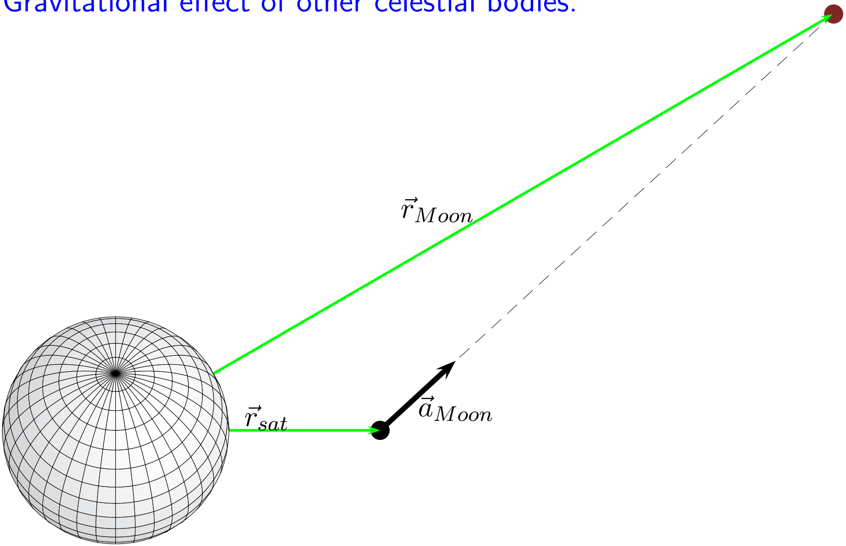

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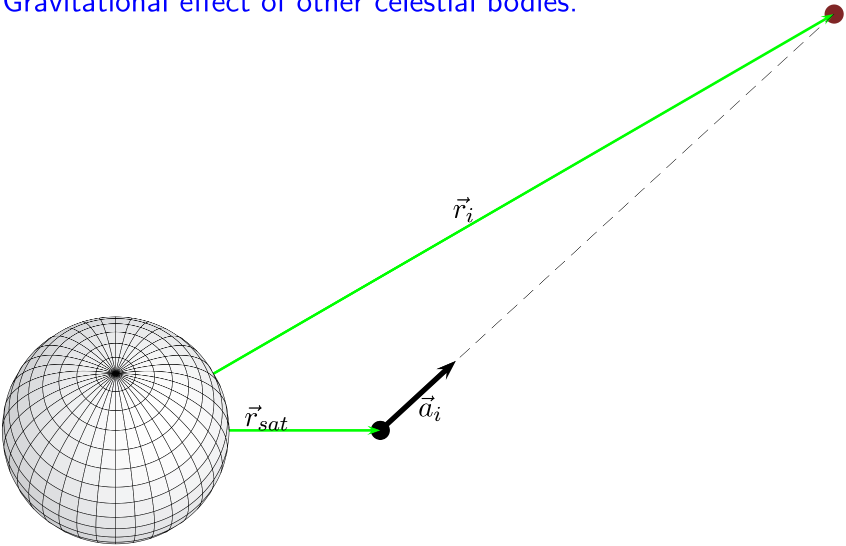

Advancing the Keplerian Orbit Theory

Gravitational effect of other celestial bodies:

$$\ddot{\vec{r}}_{sat} = -GM_E \frac{\vec{r}_{sat}}{|\vec{r}_{sat}|^3} - G \cdot \sum_{i=1}^{n} M_i \frac{\vec{r}_i - \vec{r}_{sat}}{|\vec{r}_i - \vec{r}_{sat}|^3}$$

Advancing the Keplerian Orbit Theory

Gravitational effect of other celestial bodies:

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- Further gravitational effects act on the satellite as well.

  The resulting satellite motion is described by a perturbed Keplerian motion.

Advancing the Keplerian Orbit Theory

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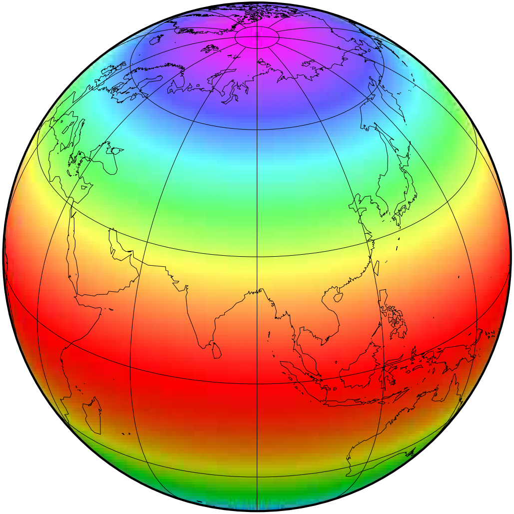
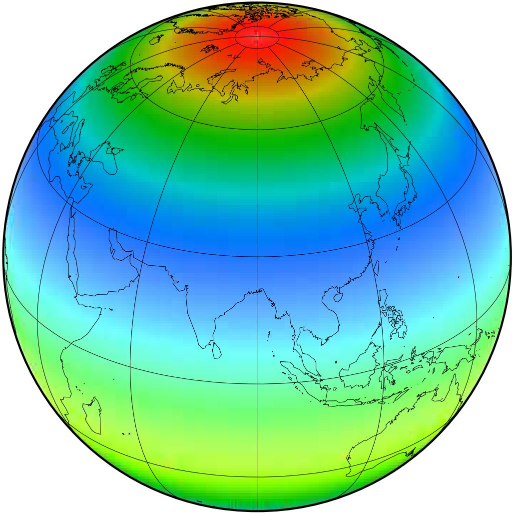
- The elements of the orbit ellipse change continuously due to the perturbing forces – "osculating elements".

The Earth is not Spherically Symmetric



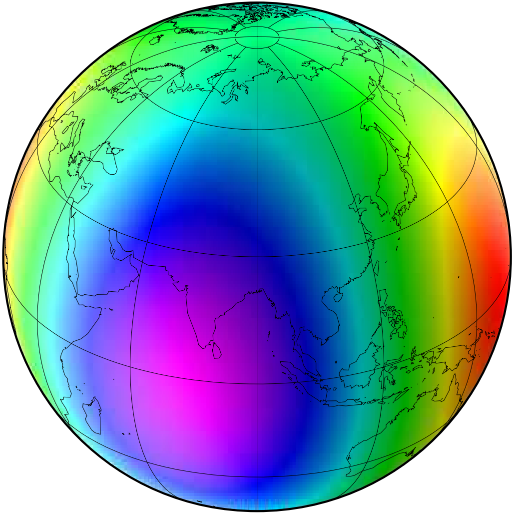
The dominate structure is oblateness of the Earth

The Earth is not Spherically Symmetric



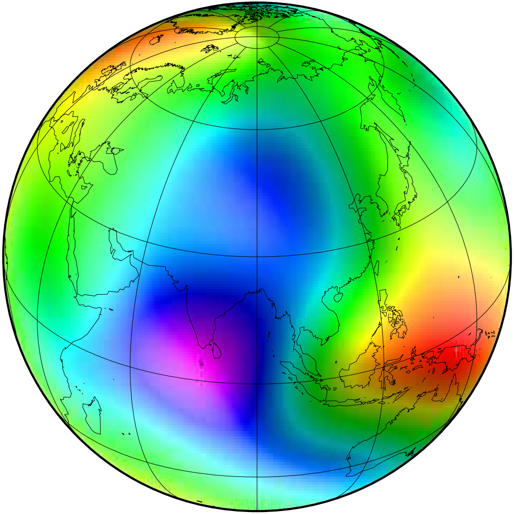
In the next order it has a shape of a pear

The Earth is not Spherically Symmetric



There are also relevant longitude-dependent structures

The Earth is not Spherically Symmetric



Many more details of the gravity field are well known today. . .

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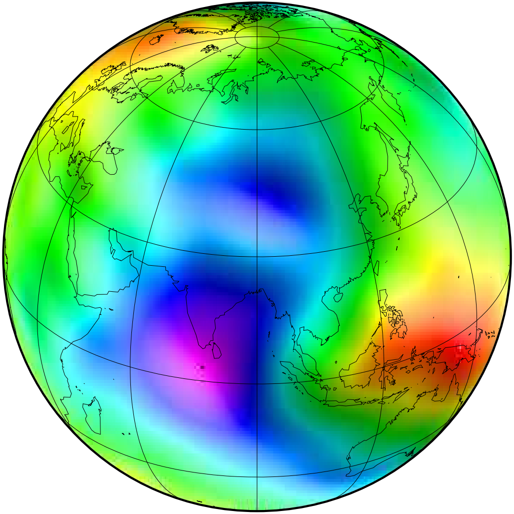

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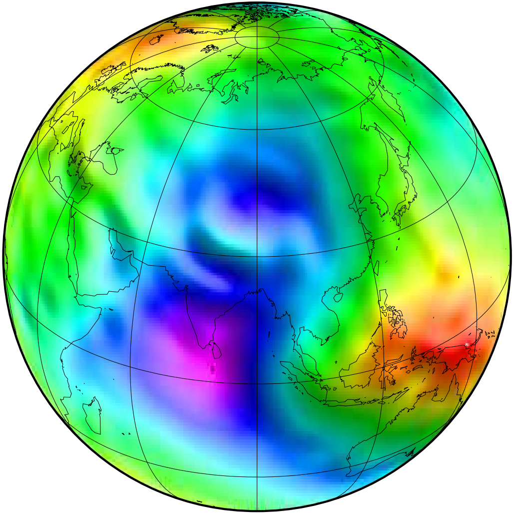

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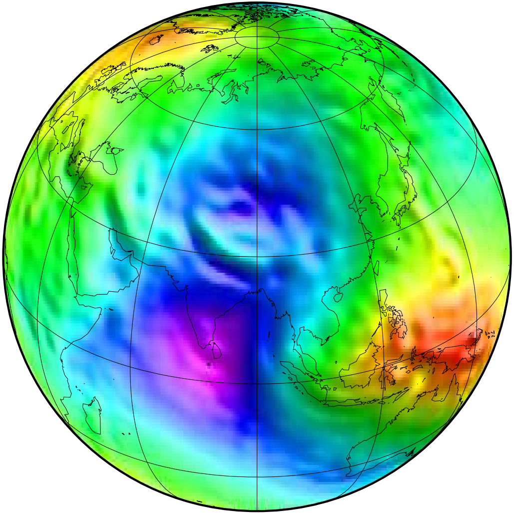

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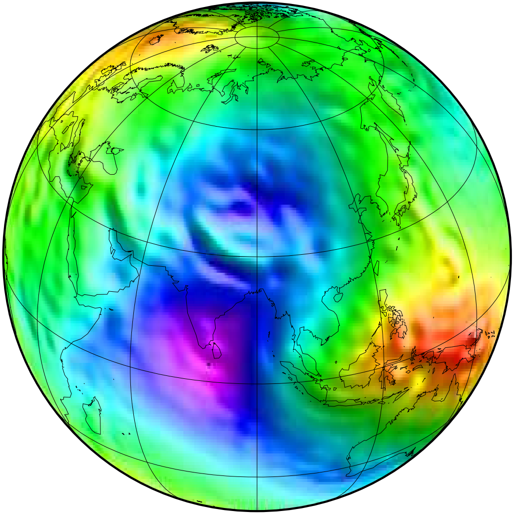
The Earth is not Spherically Symmetric

Considering the gravity field for GNSS orbit determination



Many more details of the gravity field are well known today. . .

The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

$$\ddot{\vec{r}}_{sat} = -GM_E \int_{V_E} \varrho'(\vec{r}_P) \frac{\vec{r}_{sat} - \vec{r}_P}{|\vec{r}_{sat} - \vec{r}_P|^3} dV_E - G \cdot \sum_{i=1}^{n} M_i \frac{\vec{r}_i - \vec{r}_{sat}}{|\vec{r}_i - \vec{r}_{sat}|^3}$$

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- The first term represents the gravitational attraction by the Earth, where $\varrho'(\vec{r}_P)$ is the density at $\vec{r}_P$ in the Earth's interior.

The Earth is not Spherically Symmetric

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- The density function of the Earth is given in an Earth-fixed system.

$$-GM_E \int_{V_E} \varrho'(\vec{r}_P) \ldots dV_E \quad \Longrightarrow \quad -GM_E \mathbf{T} \int_{V_E} \varrho(\vec{r}_P) \ldots dV_E$$

where $\mathbf{T}$ is the transformation matrix from the Earth-fixed into the quasi-inertial frame.

The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

- The related gravity field of the Earth is considered as a conservative vector field

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The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

- The related gravity field of the Earth is considered as a conservative vector field that gradients may be represented by a spherical harmonic expansion of the potential:

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$$= \frac{GM}{|\vec{r}_{sat}|} \sum_{i=0}^{\infty} \left(\frac{a_e}{|\vec{r}_{sat}|} \right)^i \cdot \sum_{k=0}^{i} P_i^k(\sin\phi) \{ C_{ik} \cos k\lambda + S_{ik} \sin k\lambda \}$$

with

ϕ, λ	the spherical latitude and longitude of the satellite,
$P_i^k(\sin\phi)$	the associated Legendre functions of degree i and order k,
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The Earth is not Spherically Symmetric

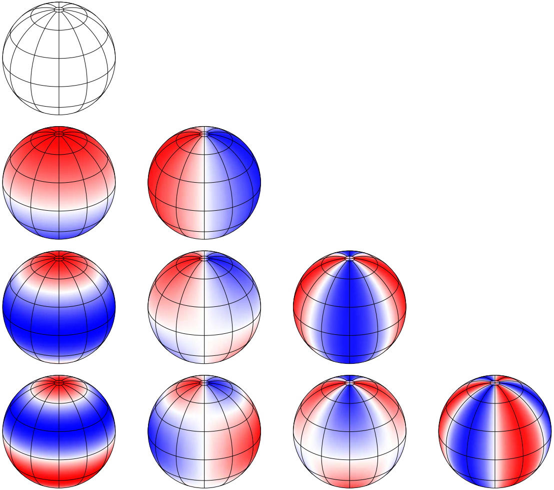
Considering the mass distribution of the Earth:

$k = 0 \quad l = 0$

$k = 1 \quad l = 0, 1$

$k = 2 \quad l = 0, \ldots, 2$

$k = 3 \quad l = 0, \ldots, 3$



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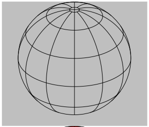
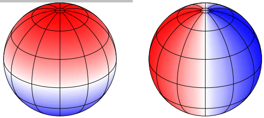
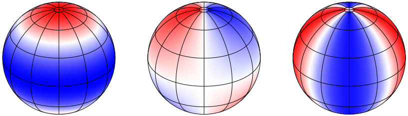


The first term C_{00} is a constant.

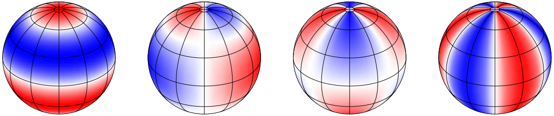
$k = 1 \quad l = 0, 1$



$k = 2 \quad l = 0, \ldots, 2$



$k = 3 \quad l = 0, \ldots, 3$



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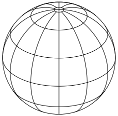

The terms C_{10}, C_{11}, and S_{11} are related to the center of mass of the Earth.

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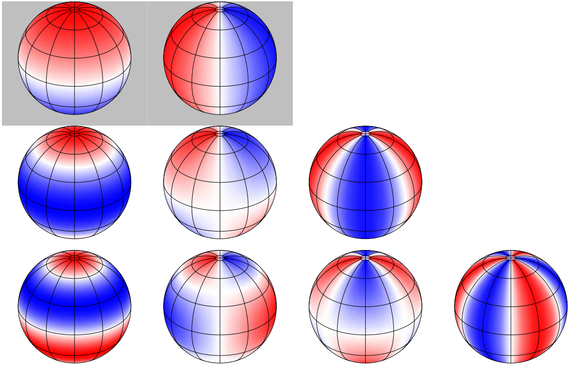

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Considering the mass distribution of the Earth:

The C_{20} term represents the flattening of the Earth.

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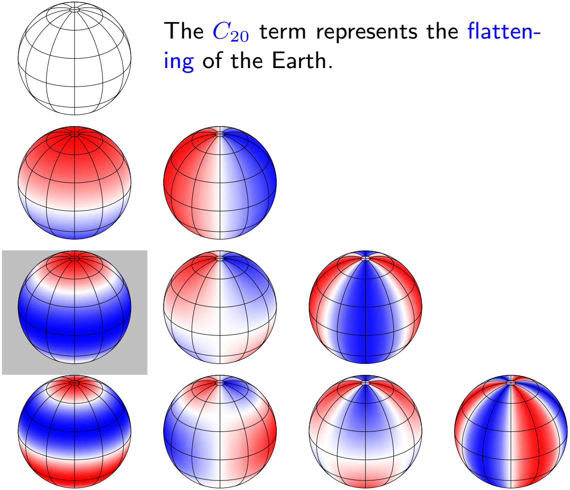

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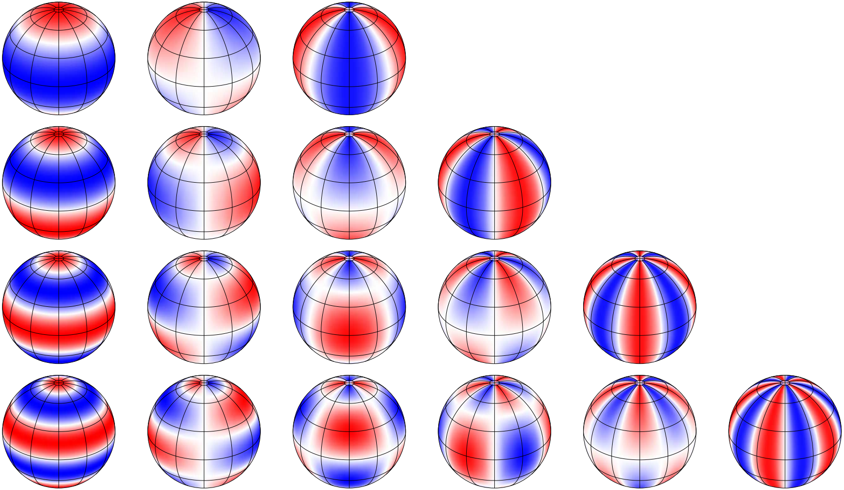

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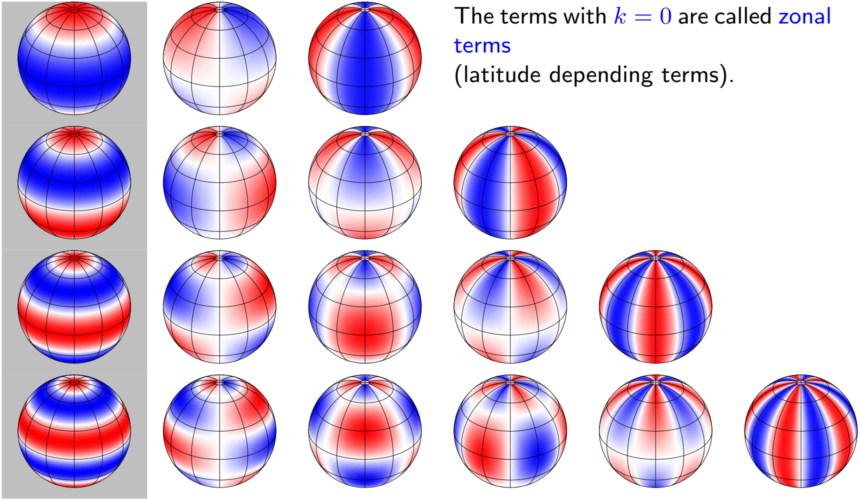

The terms with $k = 0$ are called zonal terms
(latitude depending terms).

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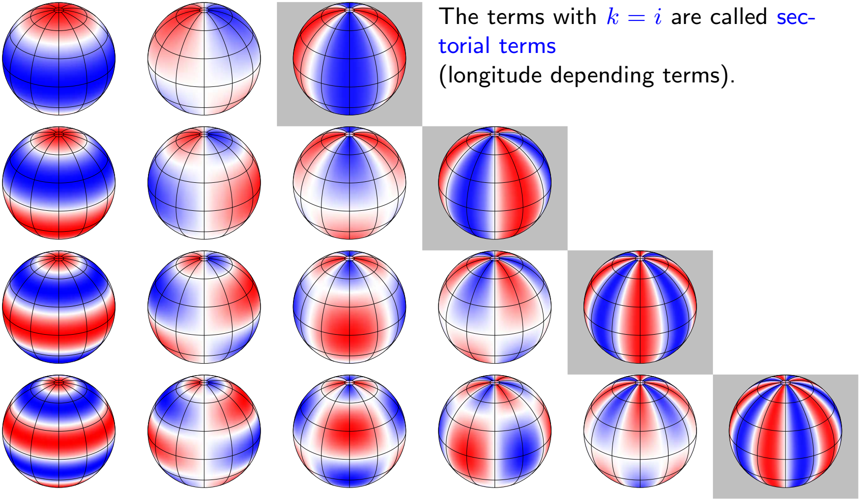

The terms with $k = i$ are called sectorial terms
(longitude depending terms).

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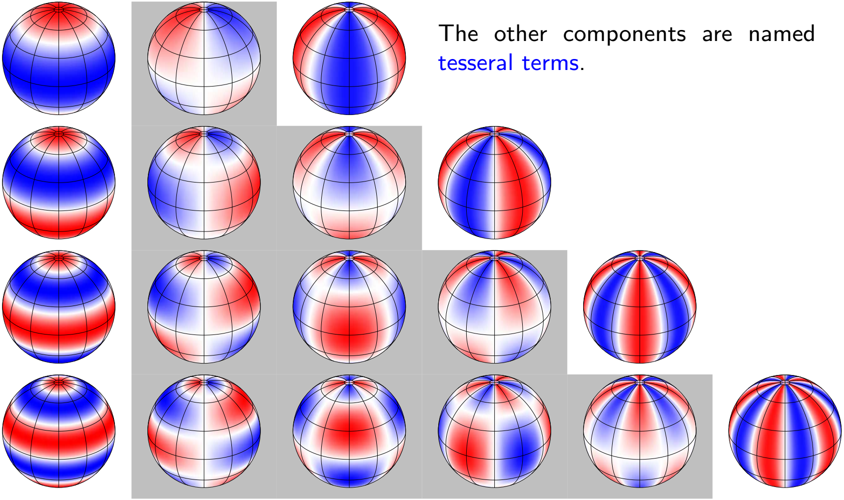

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GLONASS	$\approx 5\,\mathrm{m}$	$\approx 0.5\,\mathrm{m}$	$\approx 8\,\mathrm{cm}$	$\approx 1.5\,\mathrm{cm}$	$\approx 5\,\mathrm{mm}$
GPS	$\approx 5\,\mathrm{m}$	$\approx 0.5\,\mathrm{m}$	$\approx 8\,\mathrm{cm}$	$\approx 1.5\,\mathrm{cm}$	$< 5\,\mathrm{mm}$
Galileo	$\approx 2\,\mathrm{m}$	$\approx 0.2\,\mathrm{m}$	$\approx 3\,\mathrm{cm}$	$\approx 5\,\mathrm{mm}$	$\approx 1\,\mathrm{mm}$
IGSO/GEO	$< 1\,\mathrm{m}$	$\approx 5\,\mathrm{cm}$	$< 5\,\mathrm{mm}$	$\approx 1\,\mathrm{mm}$	—

3D-RMS of the orbit differences w.r.t. an orbit based on a gravity field expanded up to degree and order 20.

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3D-RMS of the orbit differences w.r.t. an orbit based on a gravity field expanded up to degree and order 20.

- for MEO satellites the gravity field needs to be considered up to degree and order 7,
- whereas for satellites in the higher IGSO or GEO a expansion up to degree and order 5 is sufficient.

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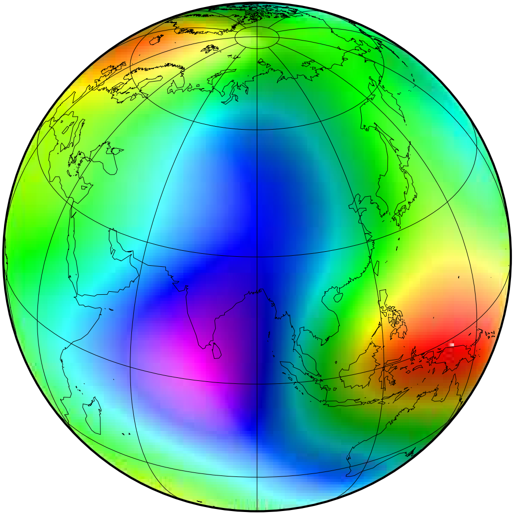

Resolution of the Earth gravity field relevant for modelling the orbits of GNSS satellites in MEO orbits.

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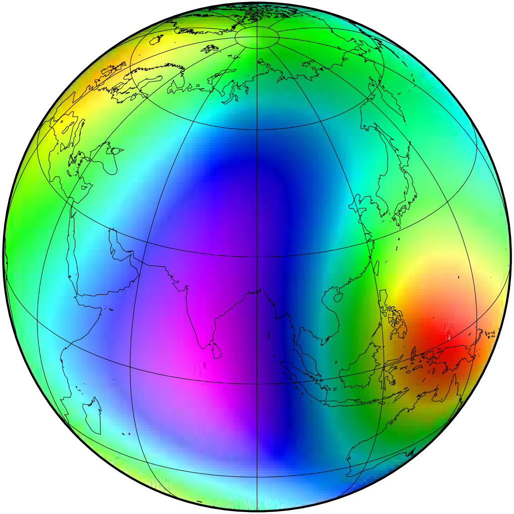

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Summary:

- All relevant masses acting on a GNSS satellite must be known (including their location).

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- Regarding the body Earth even a more detailed distribution of the masses need to be considered.

Gravitational Forces

The most relevant gravitational effects for GNSS orbit modelling:

- **Oblateness of the Earth**
  GPS: $\approx 40\,\mathrm{km}$ Galileo: $\approx 27\,\mathrm{km}$ QZSS: $\approx 15\,\mathrm{km}$

Maximal influence of the effect on the orbit after one day of orbit integration.

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- **Gravitational effect due to ocean tides**
  GPS: $< 1\,\mathrm{cm}$ Galileo: $< 5\,\mathrm{mm}$ QZSS: $\approx 1\,\mathrm{mm}$

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Effects Acting on Satellites and Related Models

- Gravitational Forces
- Radiation Pressure Effects
- Emmission Effects

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An interaction of radiation with a surface causes an exchange of momentum and therefore a force.

Direct Radiation Pressure from the Sun

For the orbit modelling we need the resulting acceleration:

$$\vec{a}_{SRP} = \vec{C} \cdot \frac{(1\,\mathrm{AU})^2}{|\vec{r}_{sat} - \vec{r}_{Sun}|^2} \cdot \frac{\Phi}{c} \cdot \frac{A_{sat}}{m_{sat}}$$

where

$\vec{C}$	is the vectorial radiation pressure coefficient (on the optical properties of the surface),
A_{sat}	is the area of the surface,
m_{sat}	is the mass of the satellite,
$\Phi \approx 1367\,\frac{\mathrm{W}}{\mathrm{m}^2}$	is the solar flux (the energy passing through a unit area in a unit time) at the distance of $1\,\mathrm{AU}$, and
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The direction of the resulting acceleration depends on the kind of interaction of the radiation with the surface.

Specular reflection:

As the photon is specularly reflected from the surface,

Interaction of a Photon with a Surface

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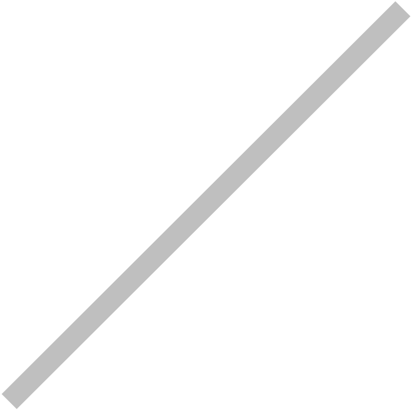
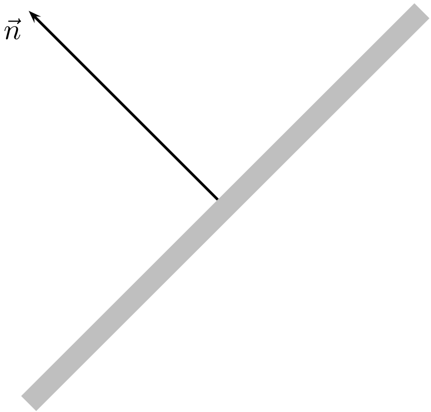


Interaction of a Photon with a Surface



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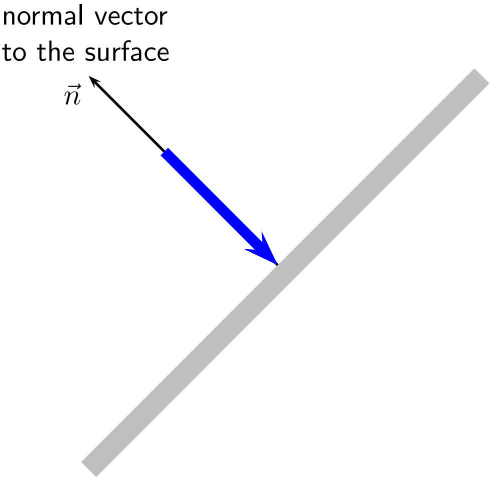

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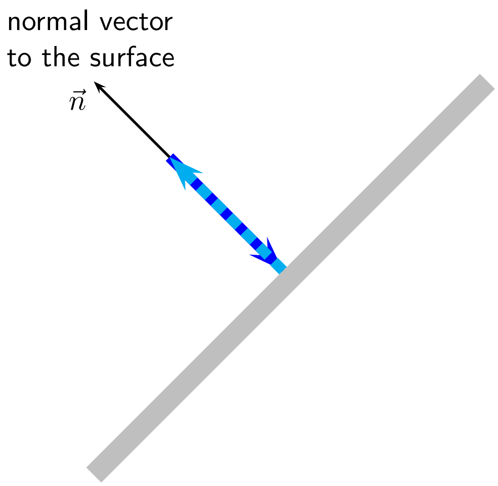

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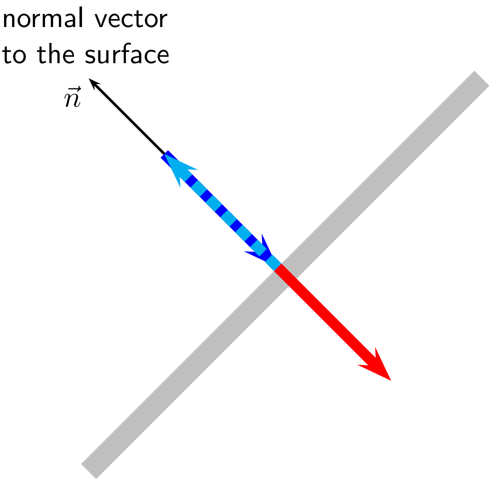

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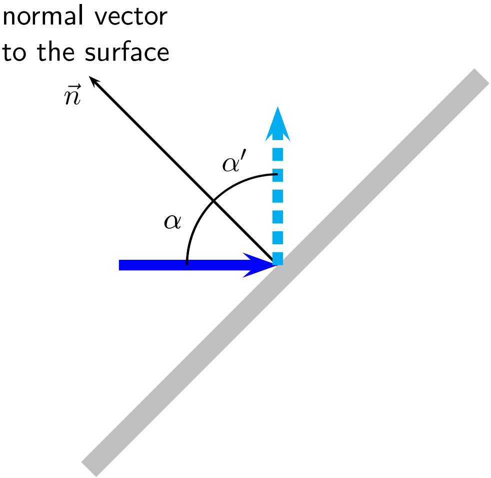

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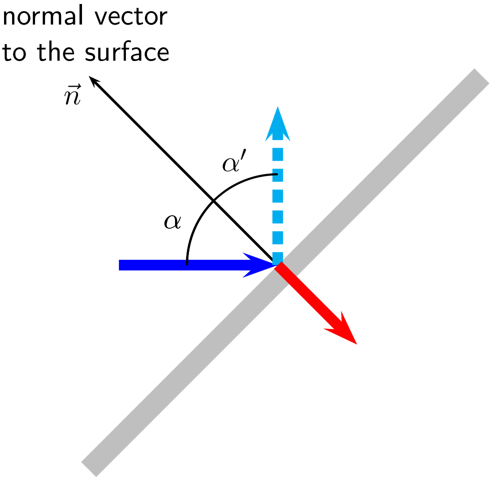


Specular reflection:
As the photon is specularly reflected from the surface, only a normal force is produced:

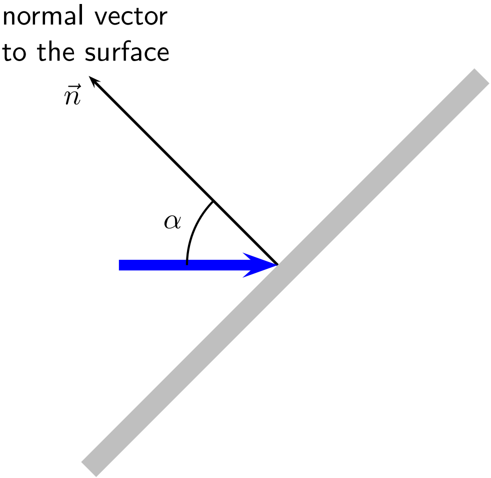
$$\vec{C}_s = -2 \cdot \cos^2 \alpha \cdot \vec{n}$$

Interaction of a Photon with a Surface

Diffuse reflection:

Interaction of a Photon with a Surface

Diffuse reflection:



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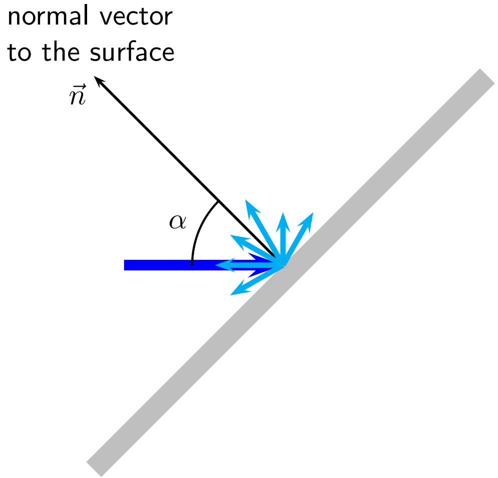

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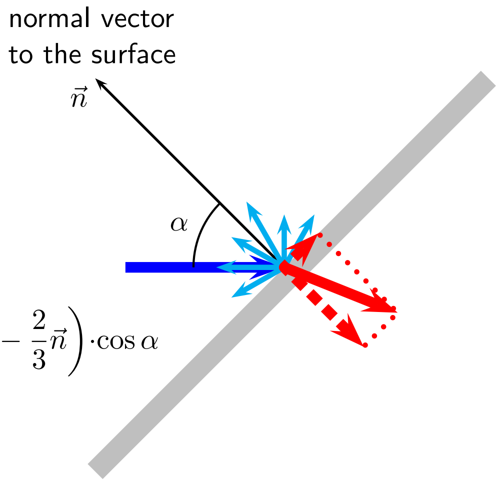

Diffuse reflection:
This kind of reflection produces both normal and tangential forces:

$$\vec{C}_d = \left(\frac{\vec{r}_{sat} - \vec{r}_{Sun}}{|\vec{r}_{sat} - \vec{r}_{Sun}|} - \frac{2}{3}\vec{n} \right) \cdot \cos\alpha$$

(assuming diffuse reflection according to Lambert's cosine law)

Interaction of a Photon with a Surface

Absorption:

The photon is fully absorbed by the surface.

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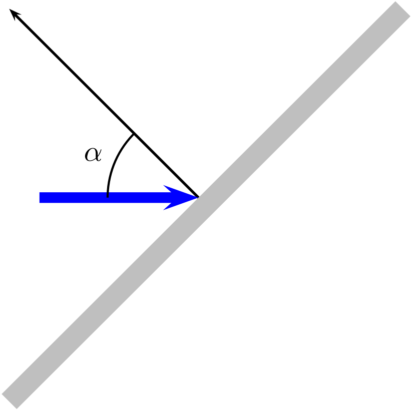

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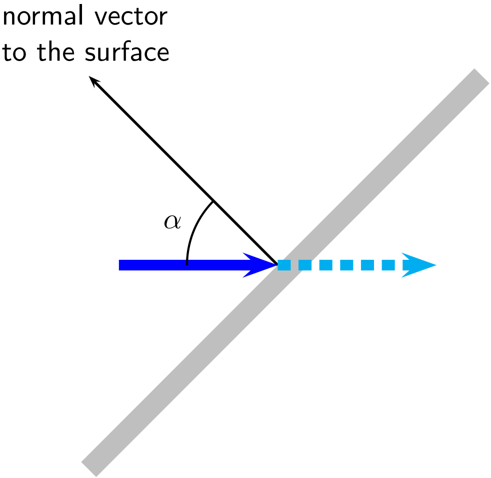

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Interaction of a Photon with a Surface

Absorption:
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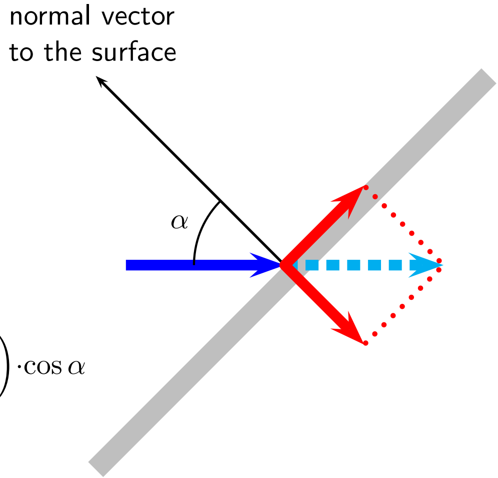
$$\vec{C}_a = \left(\frac{\vec{r}_{sat} - \vec{r}_{Sun}}{|\vec{r}_{sat} - \vec{r}_{Sun}|} \right) \cdot \cos\alpha$$



Interaction of a Photon with a Surface

In general, realistic satellite surfaces show a mixture of the three optical properties. If

p_s	is the portion of specularly reflected photons,
p_d	is the portion of diffusely reflected photons, and
$(1 - p_d - p_s)$	is the portion of absorbed photons,

the resulting radiation coefficient is

$$\vec{C}_r = p_s \cdot \vec{C}_s + p_d \cdot \vec{C}_d + (1 - p_d - p_s) \cdot \vec{C}_a \, .$$

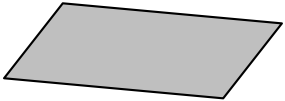
Thermal Re-radiation Effect

The heat generated by the absorption (or any other thermal emission of the satellite) produces an additional force as a Lambert diffuser:

$$d\vec{F}_{therm} = -\frac{2}{3} \cdot \frac{\epsilon\, \sigma\, {T_A}^4}{c}\, dA \cdot \vec{e}_A$$

with

ϵ	is the emissivity,
σ	the Stephan-Boltzmann constant,
c	the speed of light,
T_A	the temperature of the surface,
A	the surface area, and
$\vec{e}_A$	the unit vector normal to the emitting surface.



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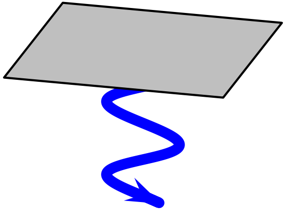

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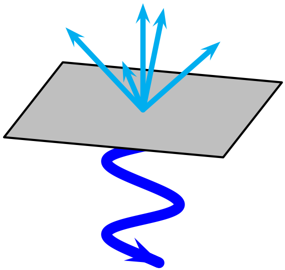

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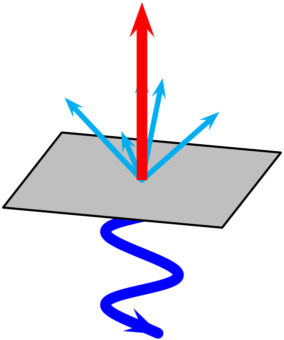

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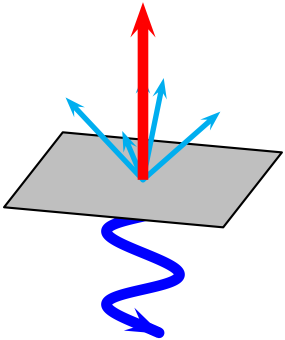

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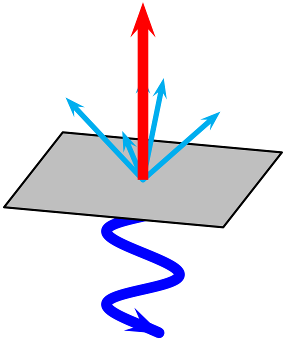

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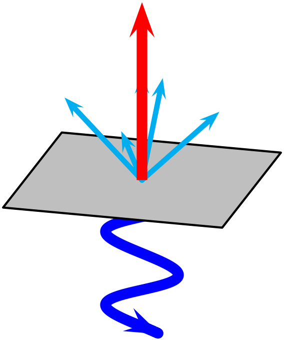

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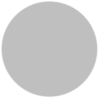

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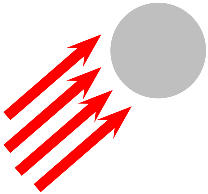

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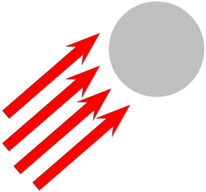

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With a ray tracing the resulting acceleration can be computed but this needs a big computational effort.

Semi-Analytical Modelling

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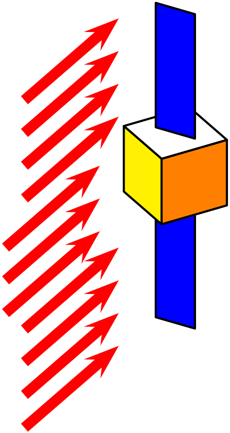

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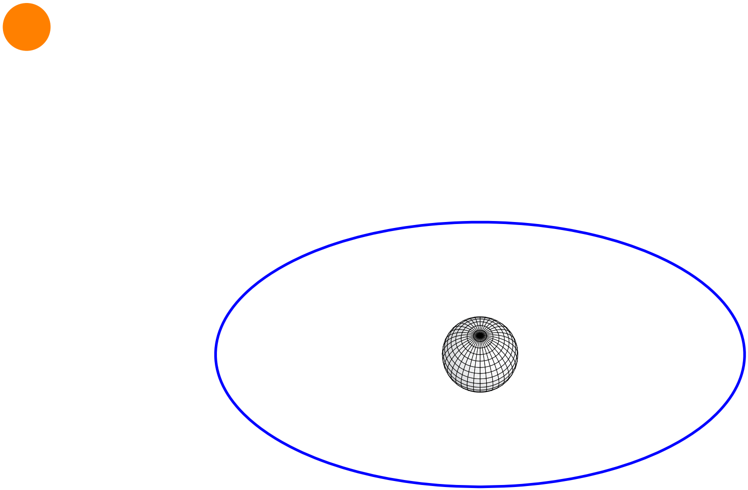

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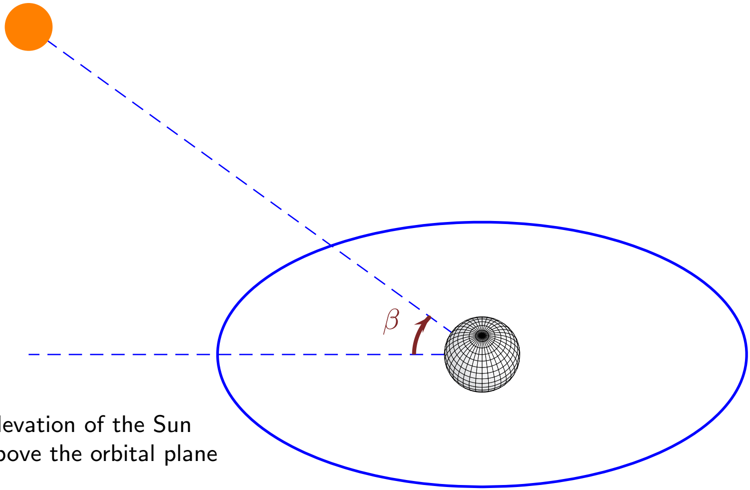
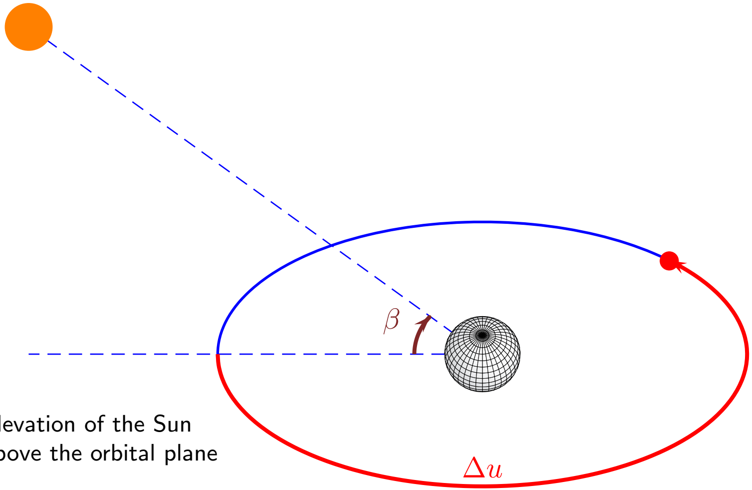


β Elevation of the Sun above the orbital plane

Semi-Analytical Modelling



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Δu Difference $u_{sat} - u_{Sun}$

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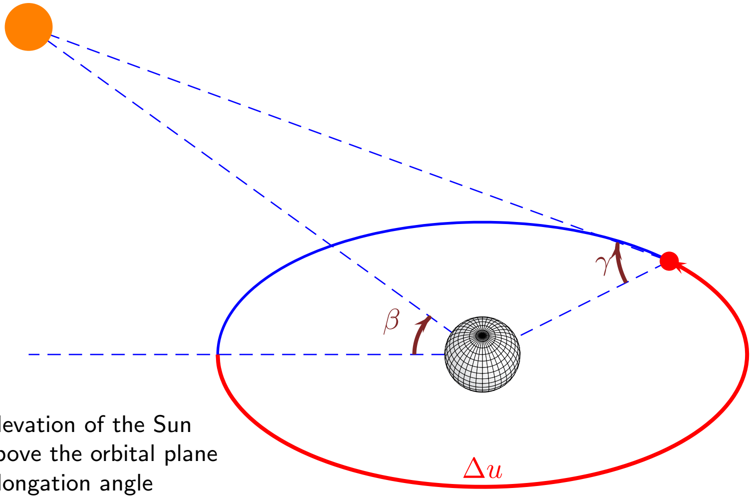
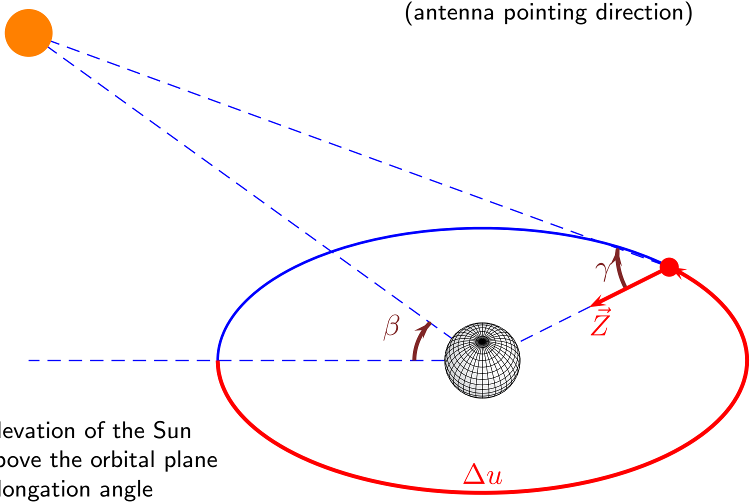


β Elevation of the Sun above the orbital plane
γ Elongation angle
Δu Difference $u_{sat} - u_{Sun}$

Semi-Analytical Modelling

$\vec{Z}$ Direction satellite → Earth (antenna pointing direction)



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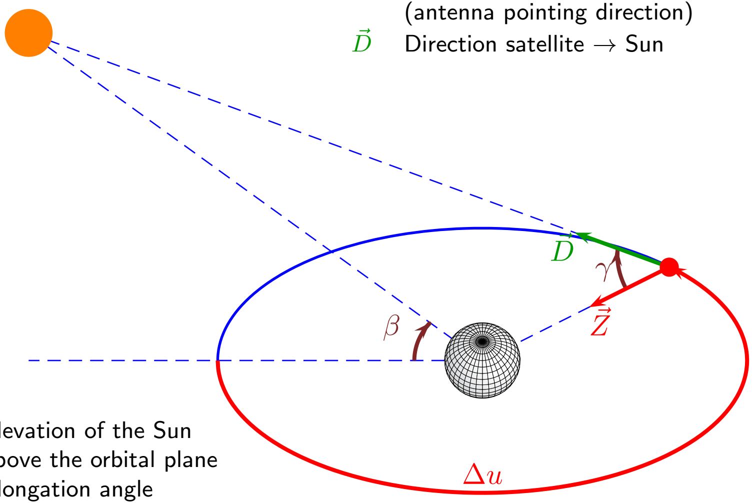

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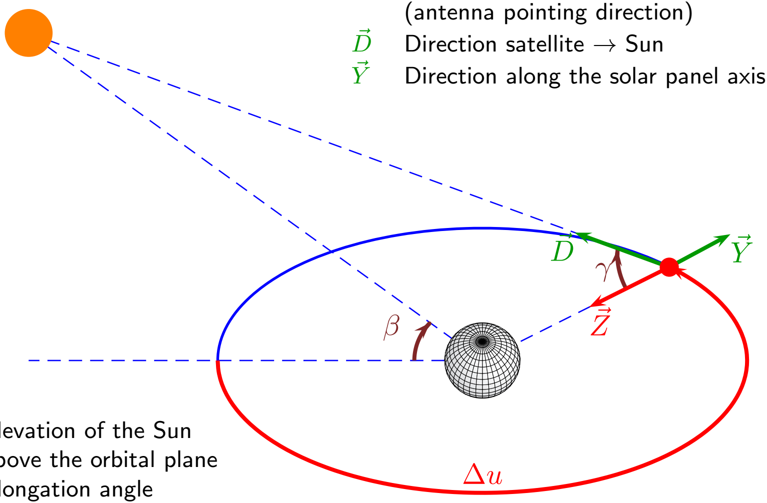

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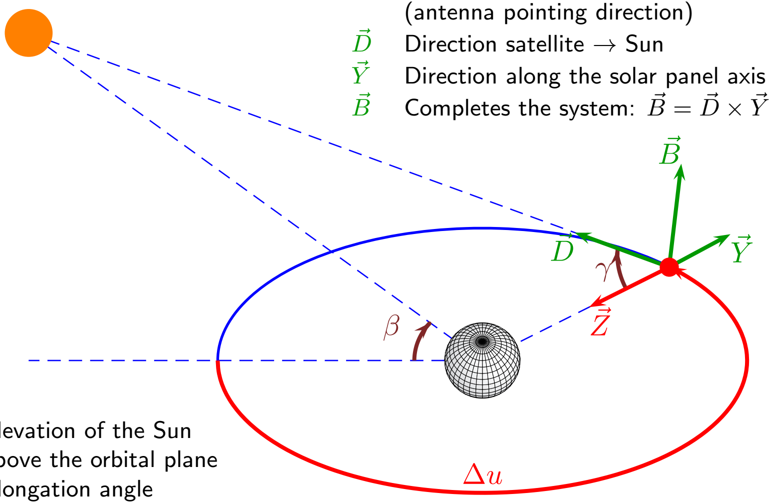
$\vec{B}$ Completes the system: $\vec{B} = \vec{D} \times \vec{Y}$



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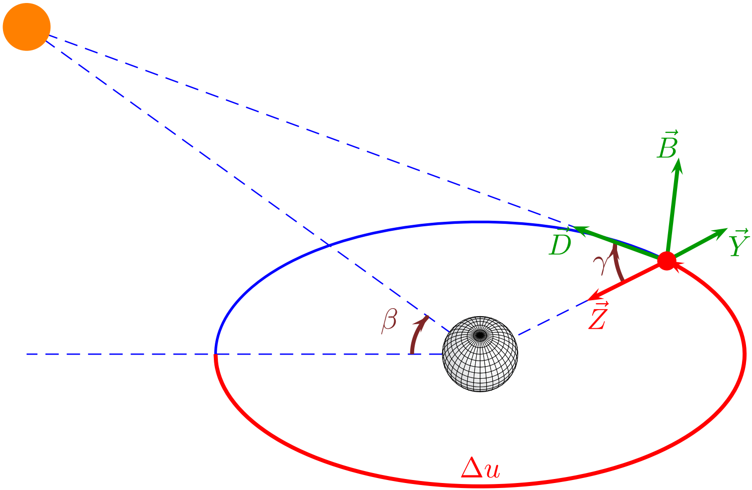

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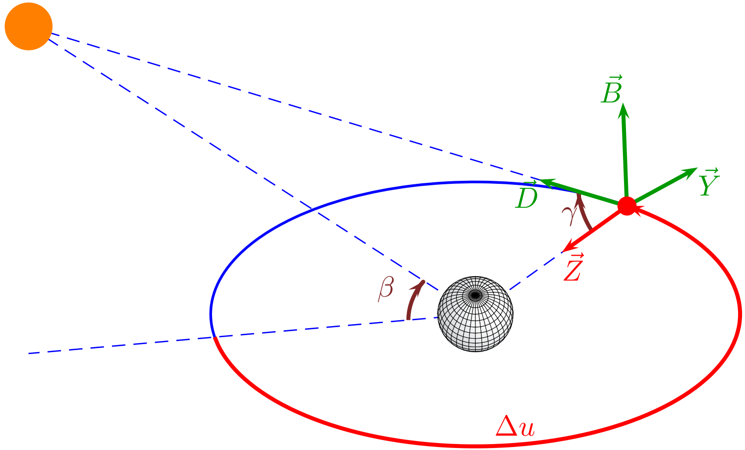

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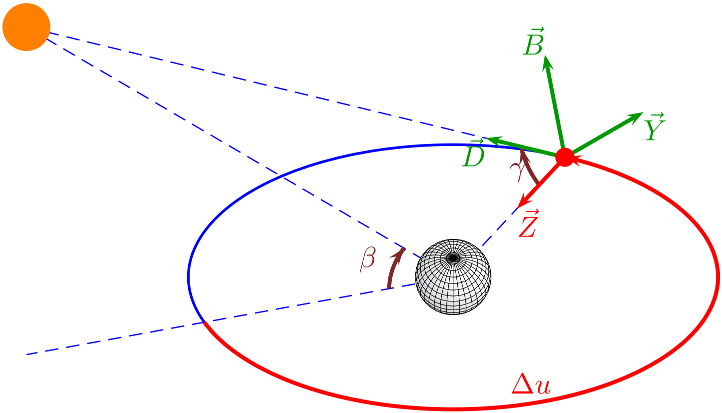

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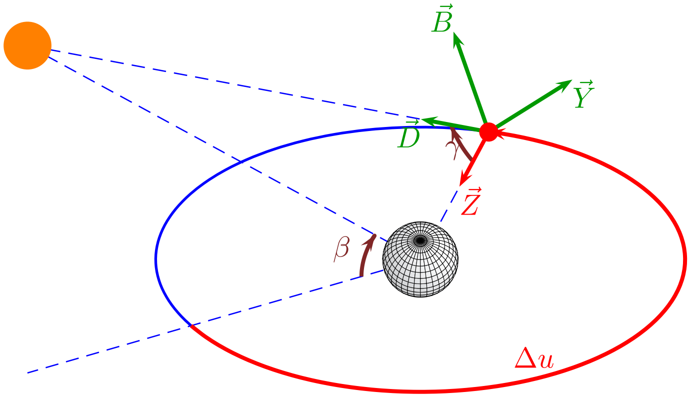

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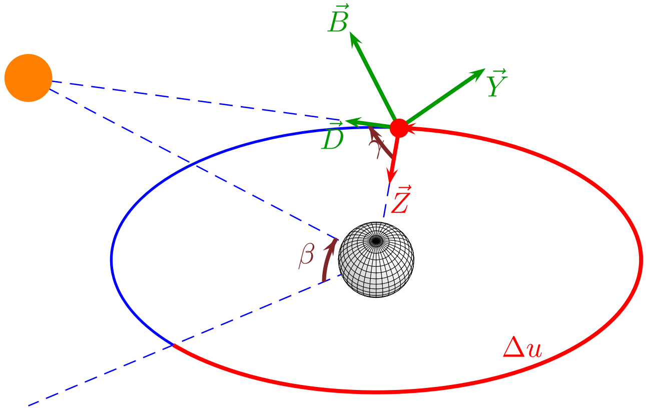

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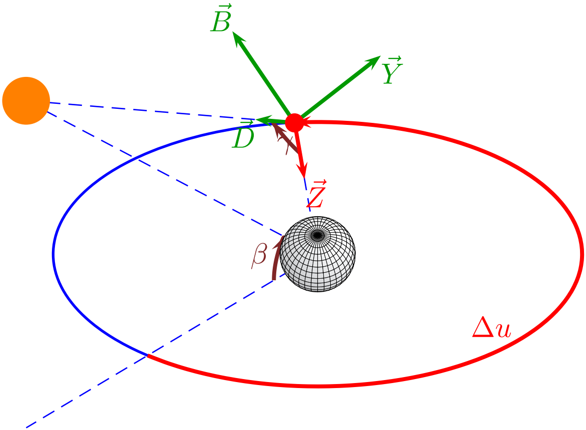
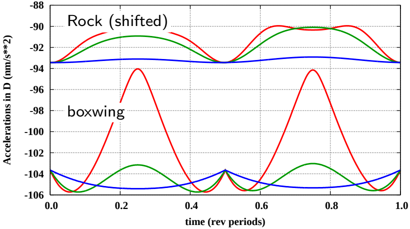


Semi-Analytical Modelling

Accelerations derived for GPS (Block IIA) satellites from a boxwing[1] and Rock-S[2] model

D component



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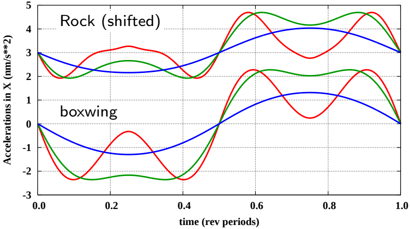

Computed for $\beta = 10°$ $\beta = 45°$ $\beta = 78°$

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C. Rodriguez-Solano has proposed to directly adjust the effect acting on the solar panels and the body of the satellite in the parameter adjustment.
These parameters are highly correlated and need a sophisticated system of constraints to become solvable.

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Box-wing a priori model:
A (more or less detailed) radiation pressure model is introduced in the orbit modelling process. Empirical parameters are estimated during the parameter adjustment process as well.

Empirical Modelling

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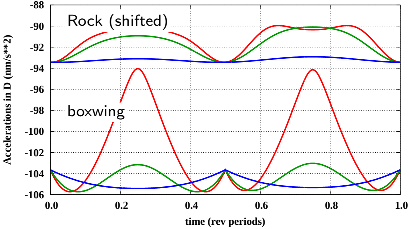

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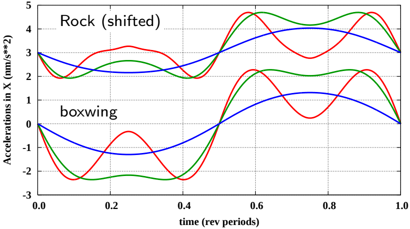

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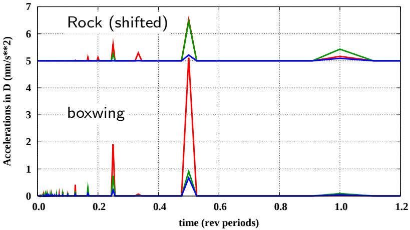

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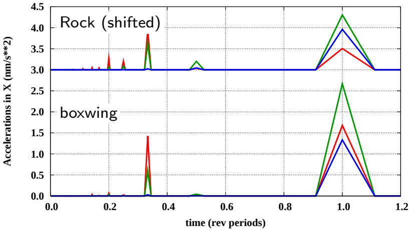

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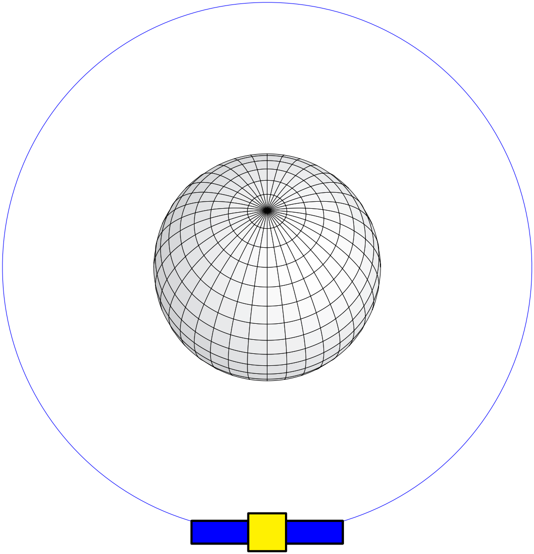

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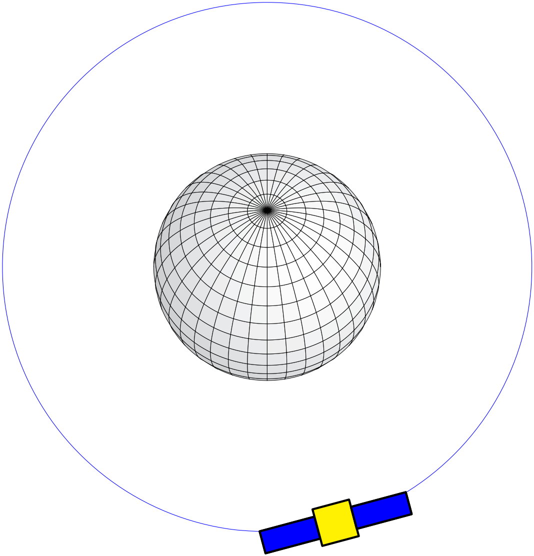

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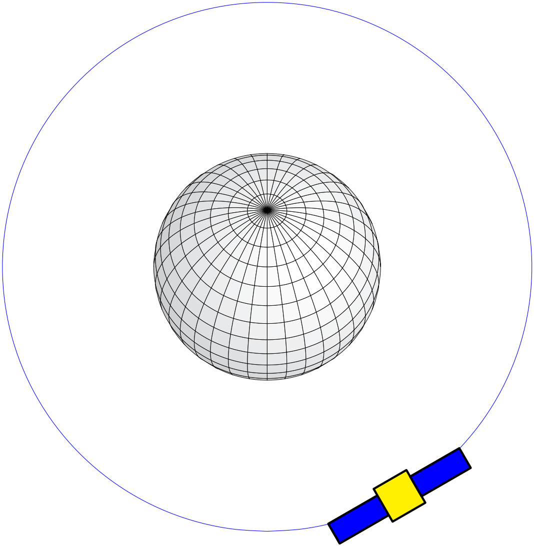

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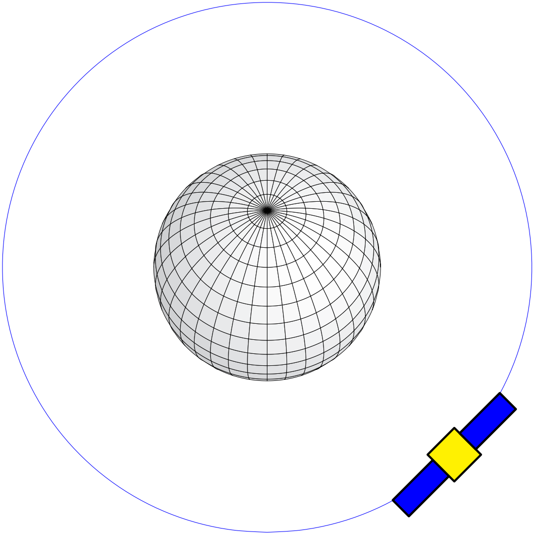

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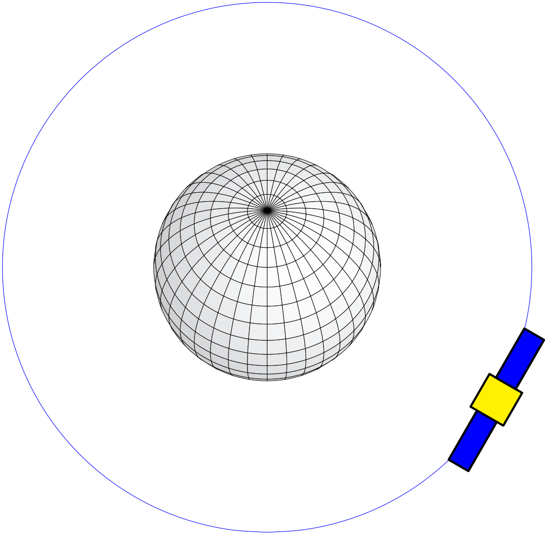

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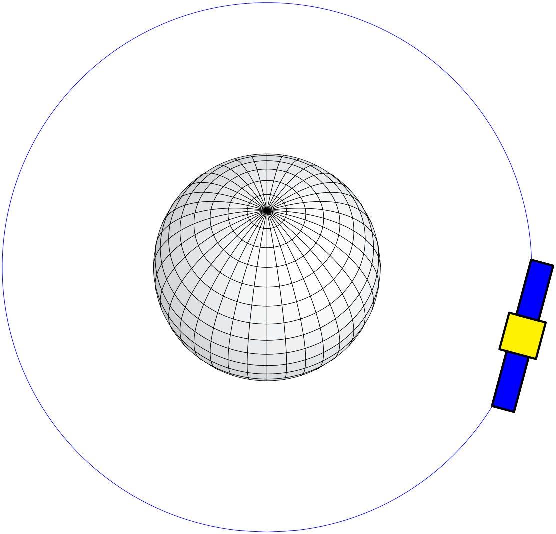

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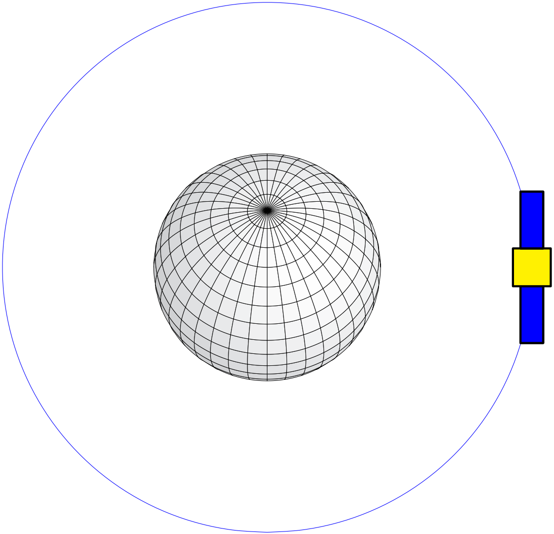

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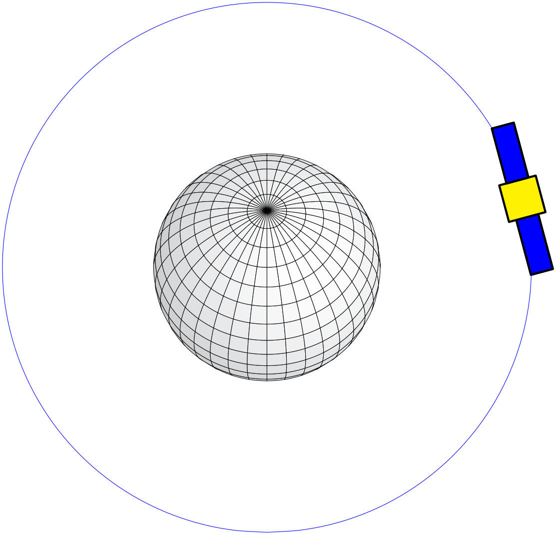

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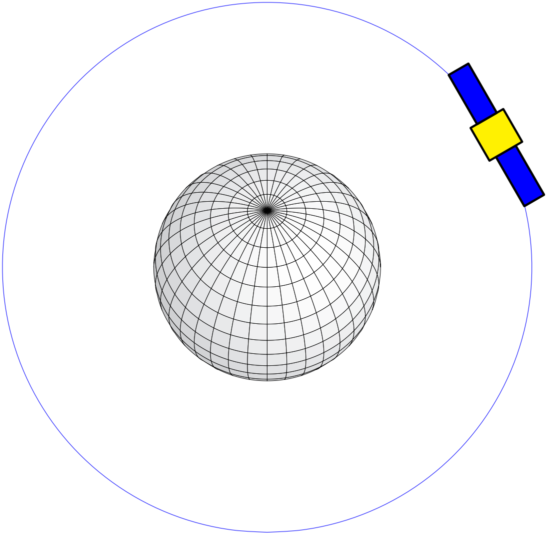

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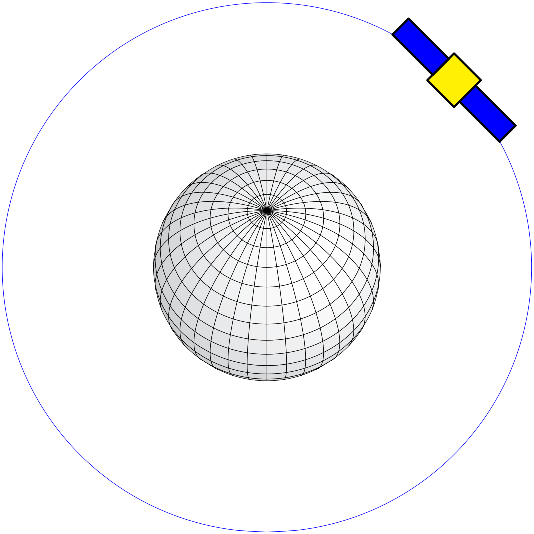

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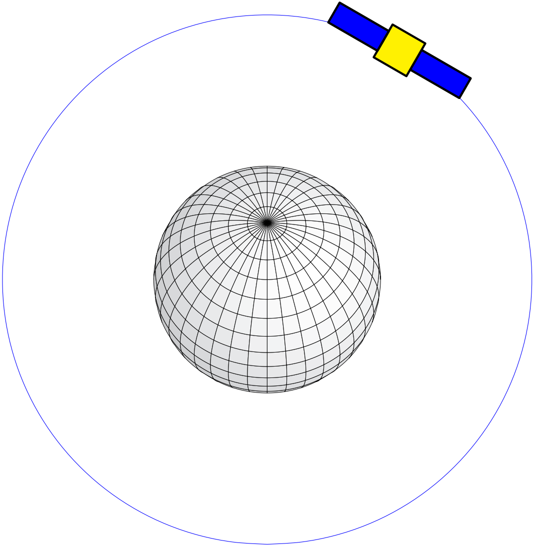

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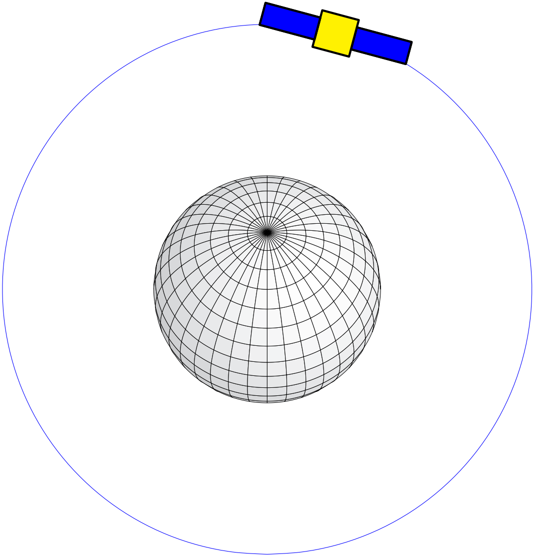

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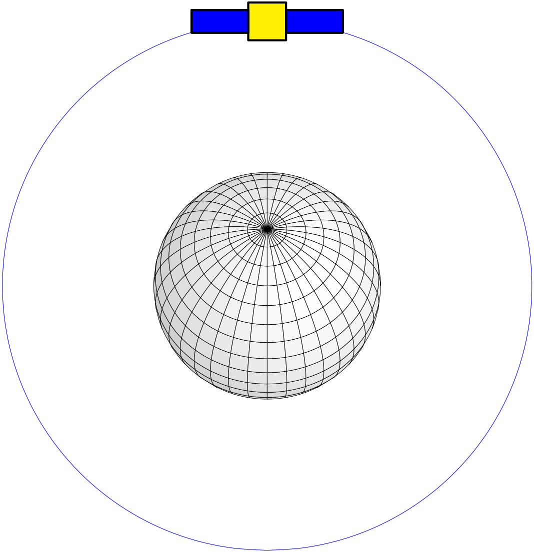

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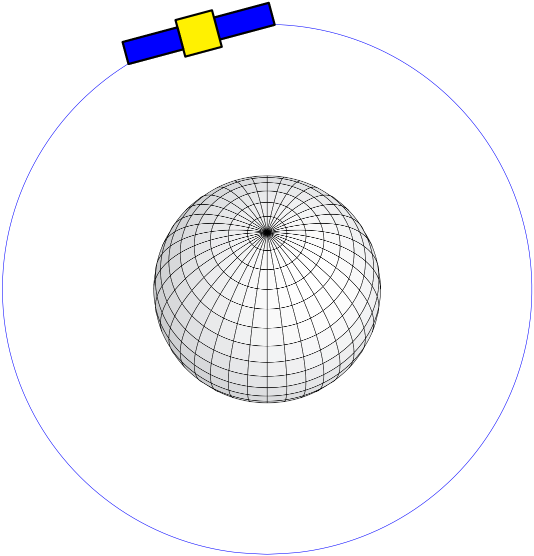

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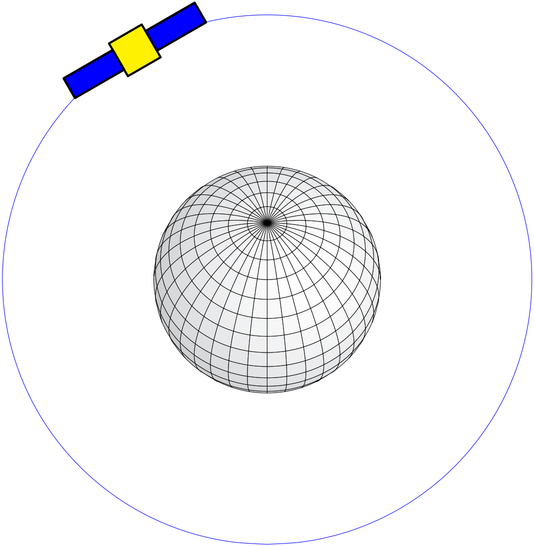

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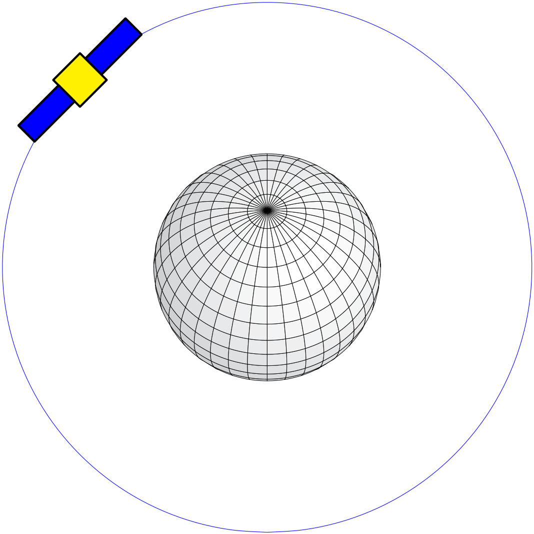

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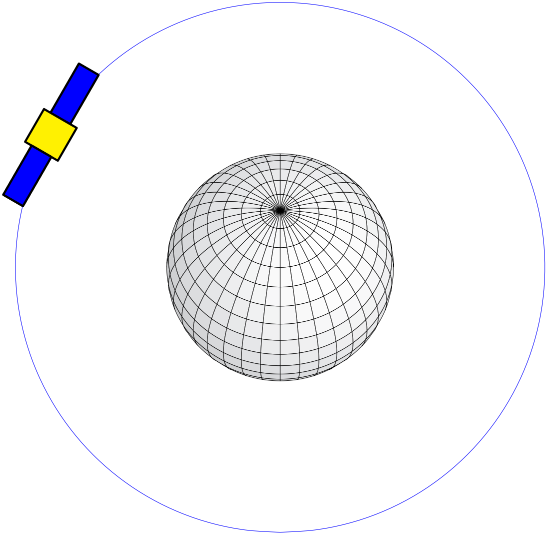

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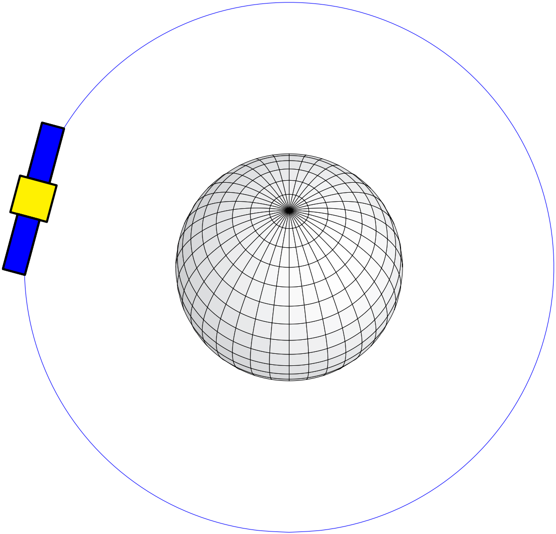

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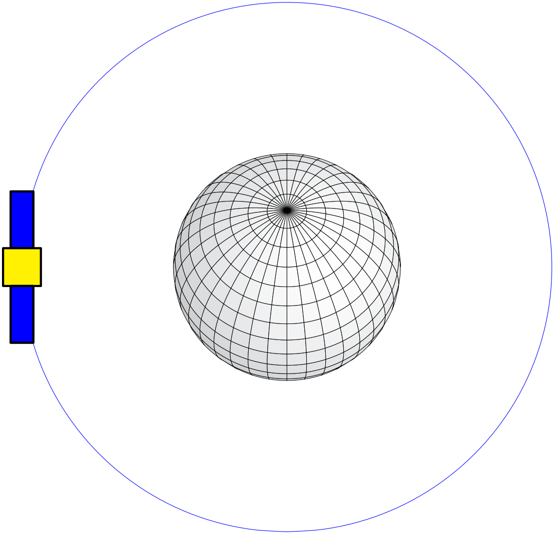

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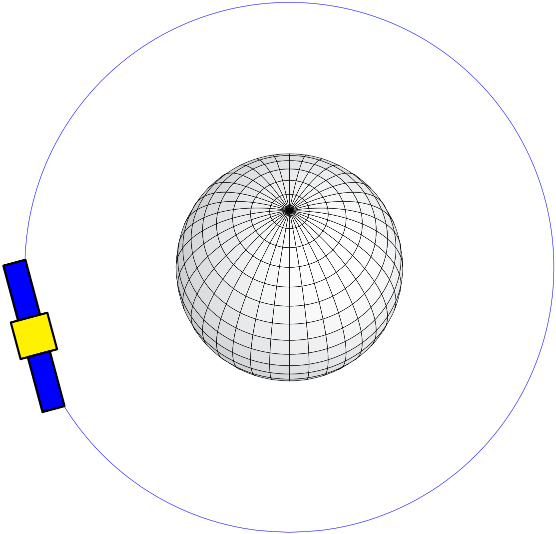

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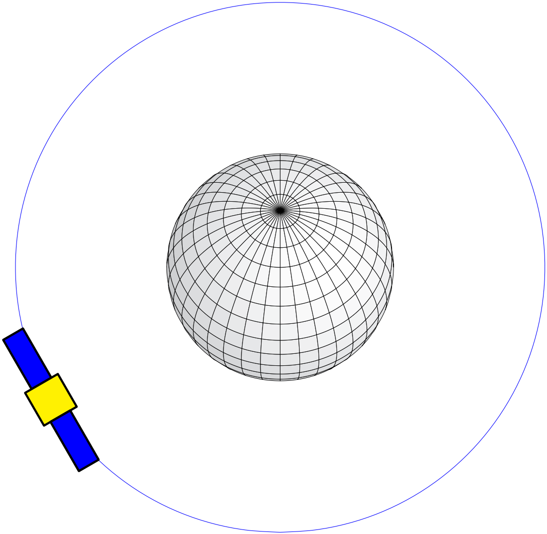

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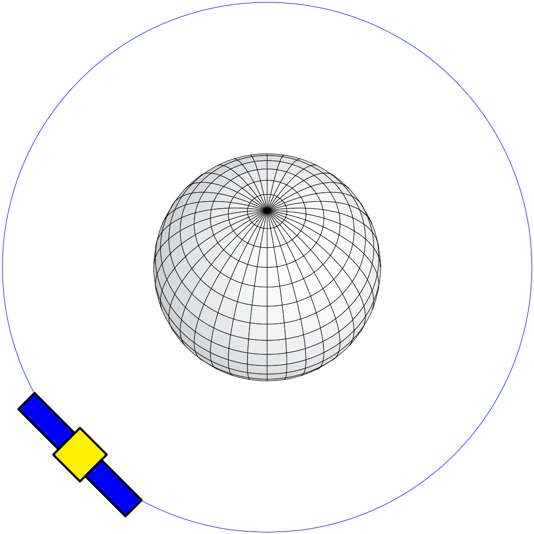

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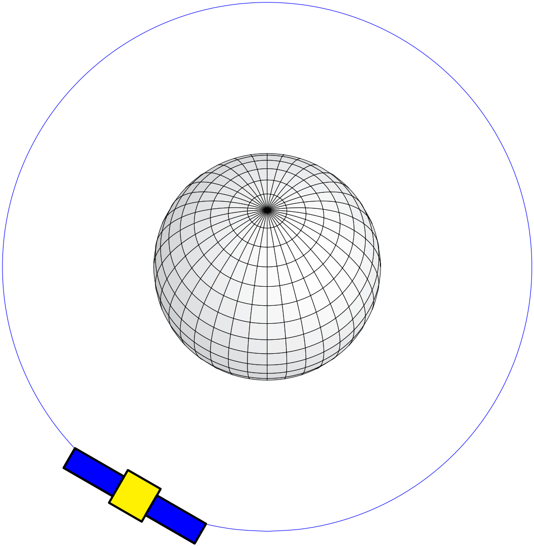

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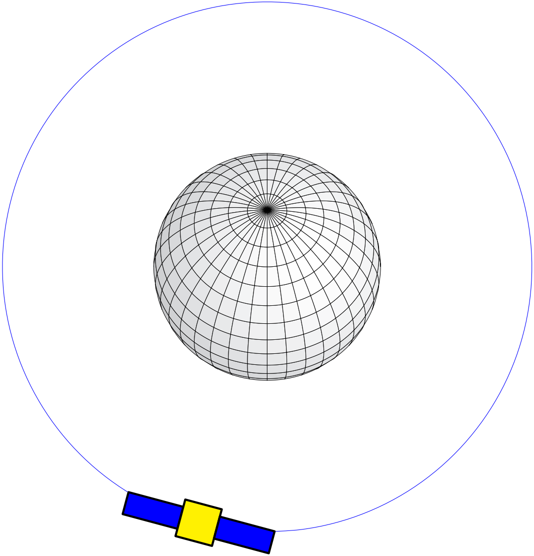

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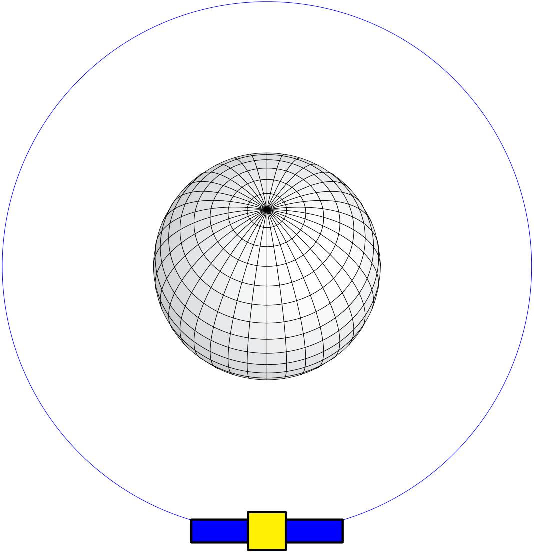

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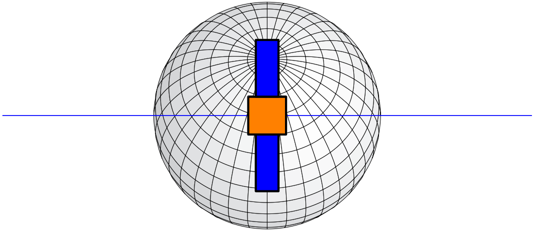

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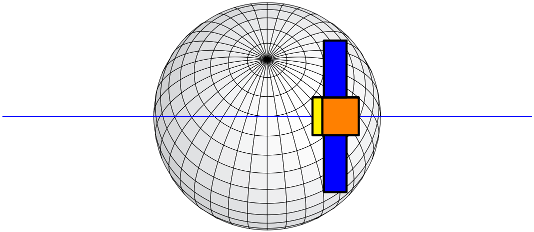

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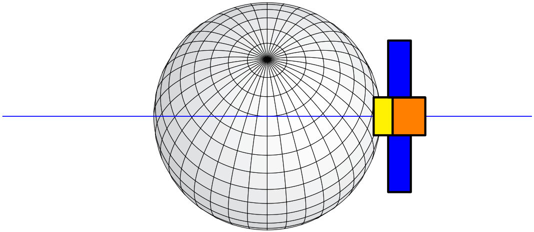

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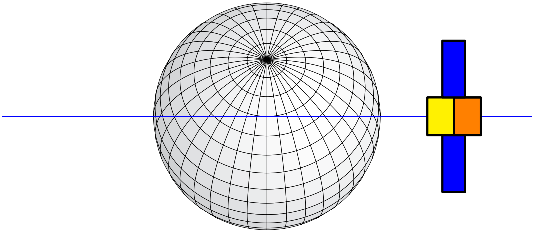

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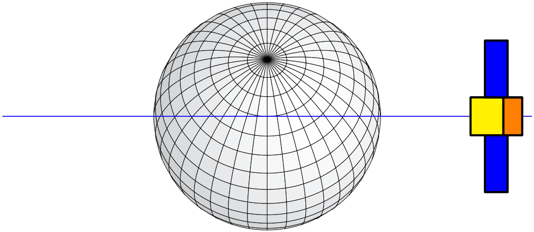

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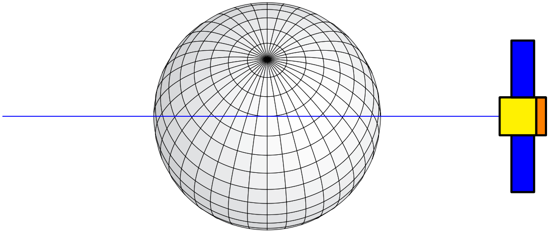

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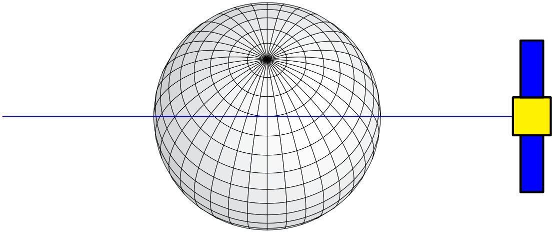

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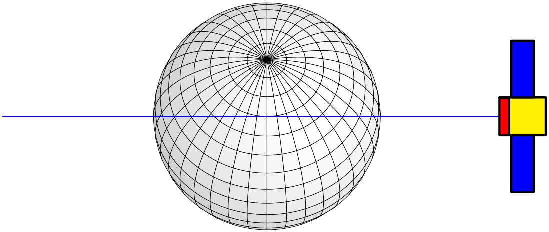

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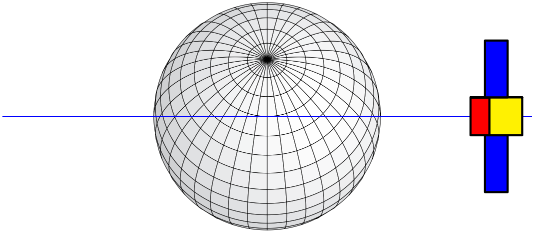

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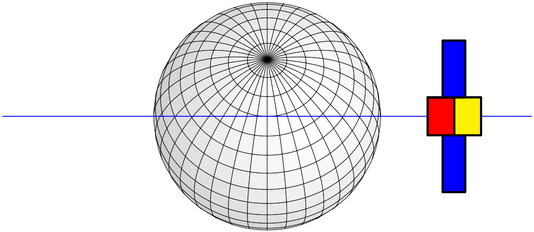

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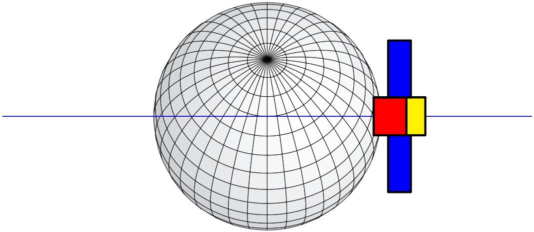

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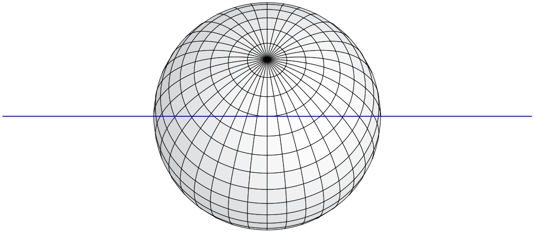

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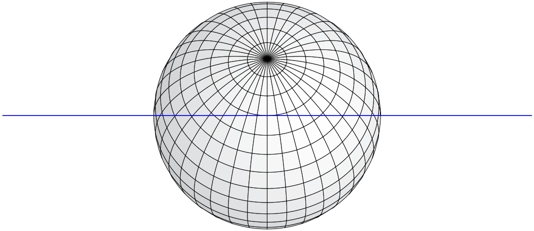

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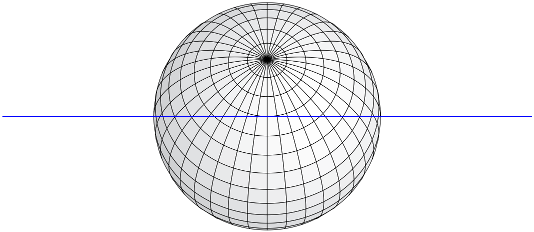

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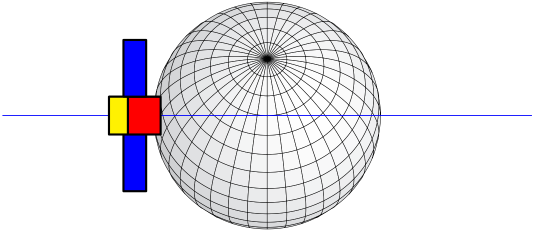

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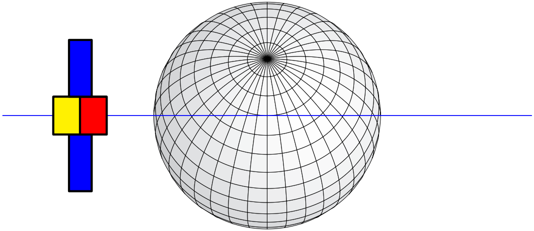

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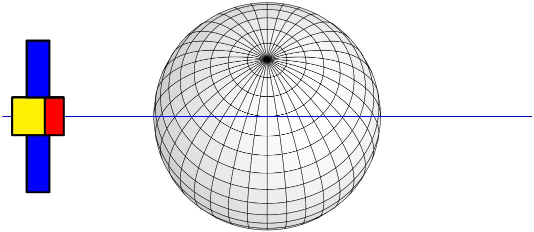

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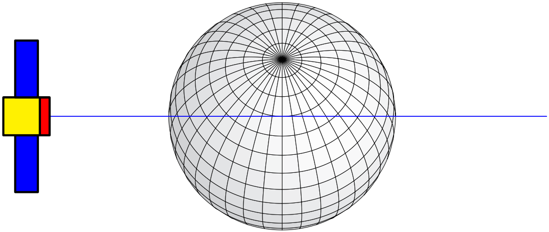

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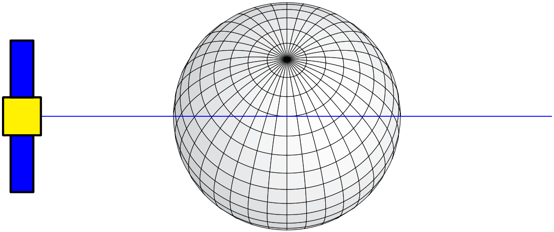

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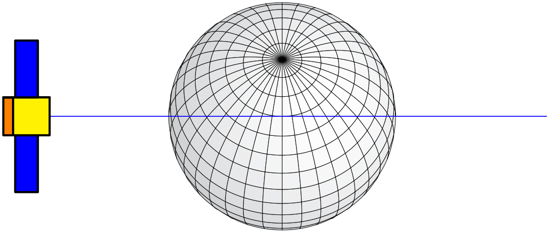

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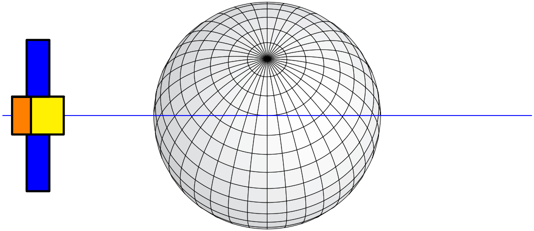

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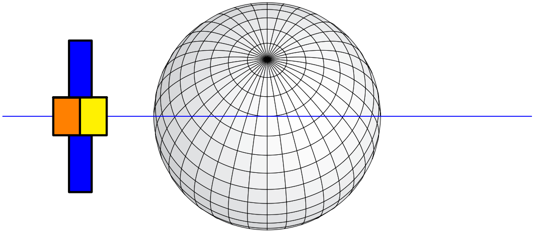

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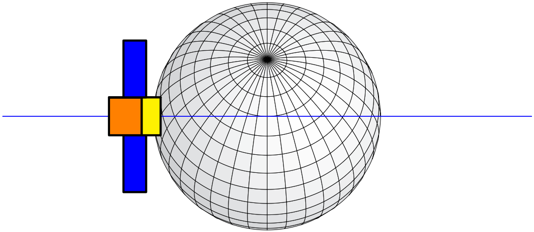

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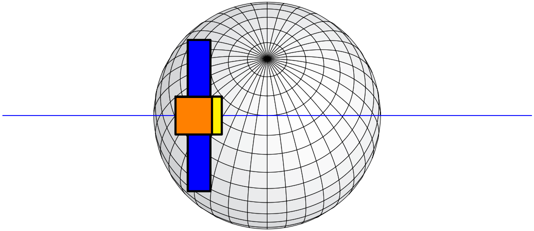

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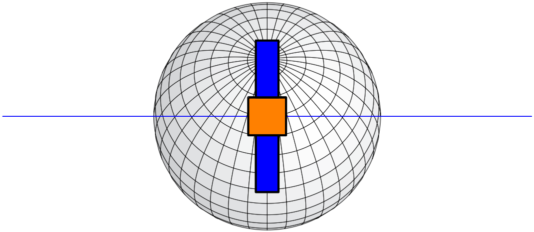

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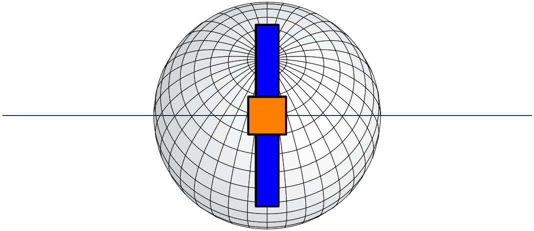

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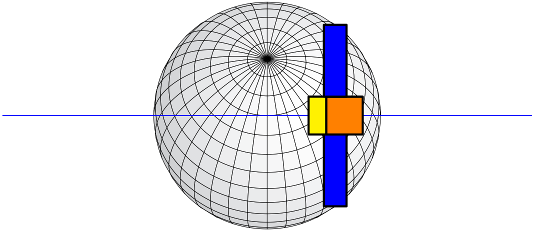

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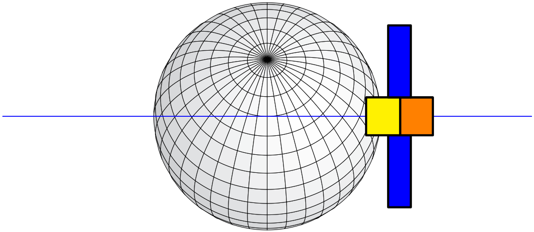

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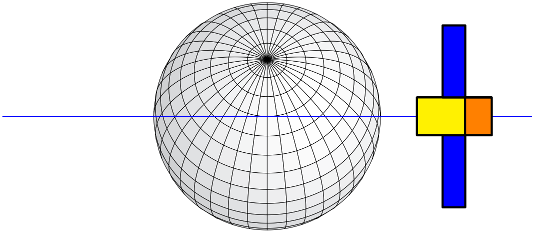

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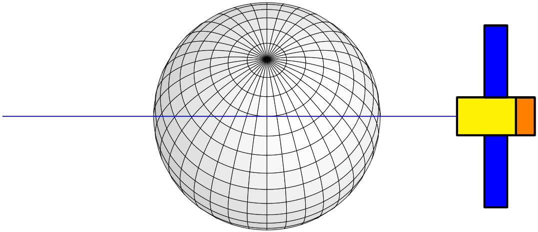

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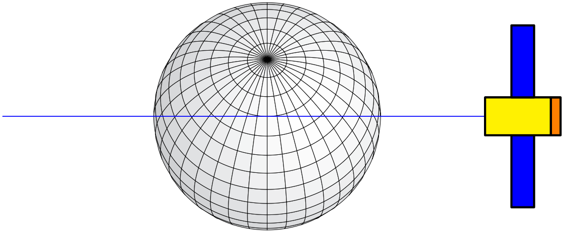

R. Dach: GNSS Satellite Orbit Modelling
NGK Summer School, 29.Aug.–01. Sep. 2016, Båstad

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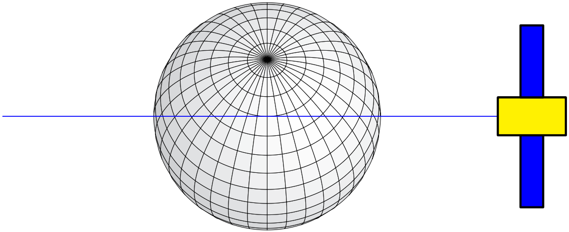

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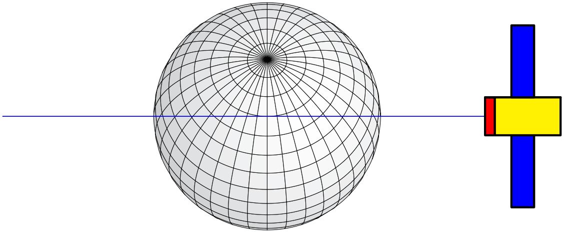

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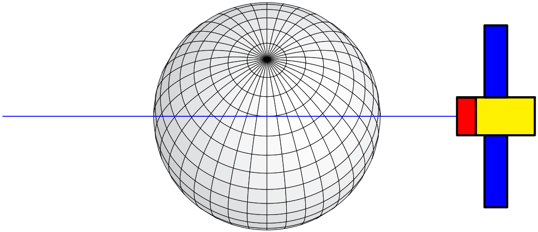

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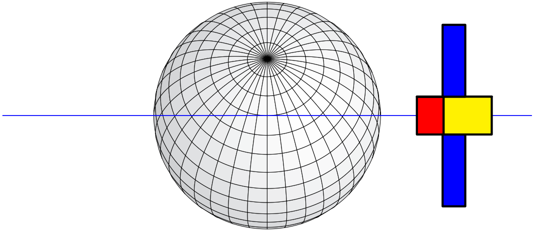

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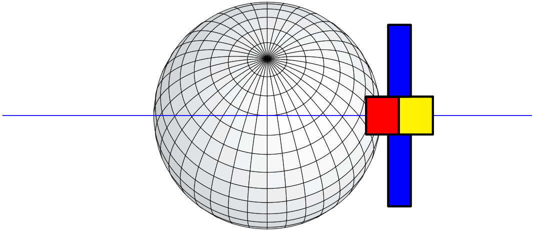

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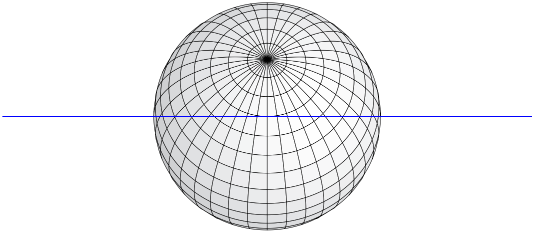

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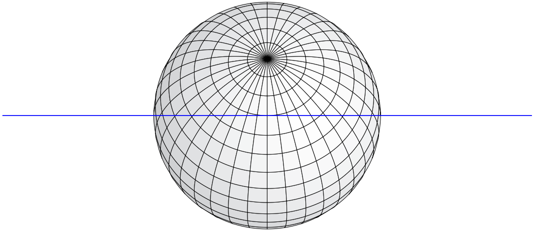

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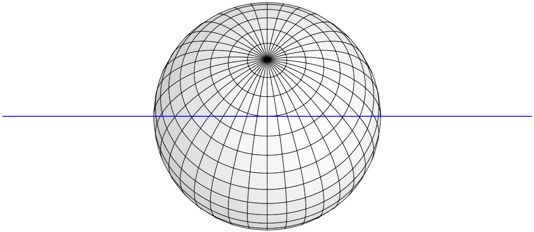

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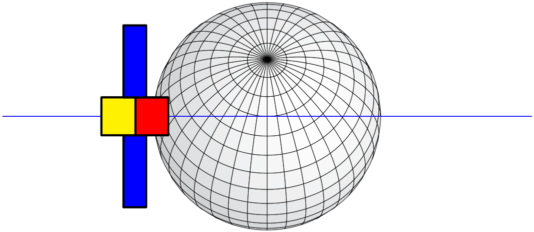

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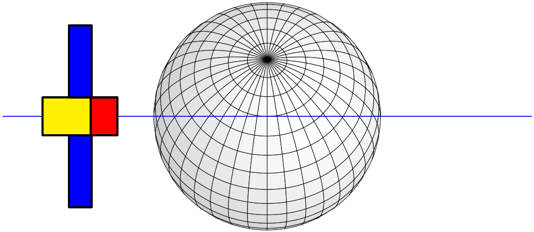

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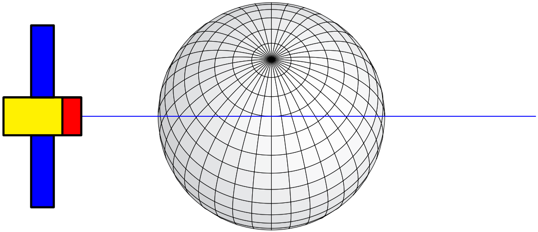

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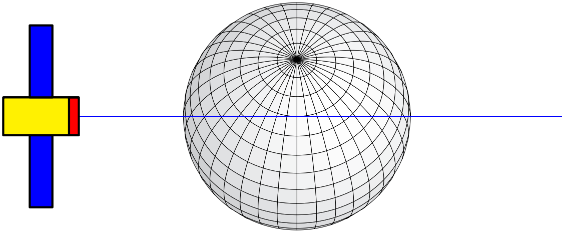

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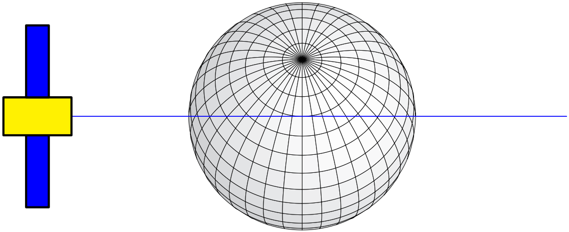

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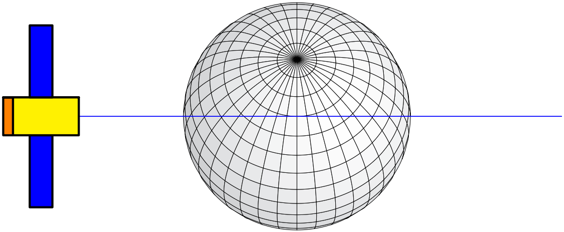

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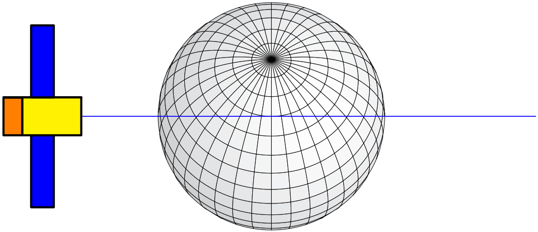

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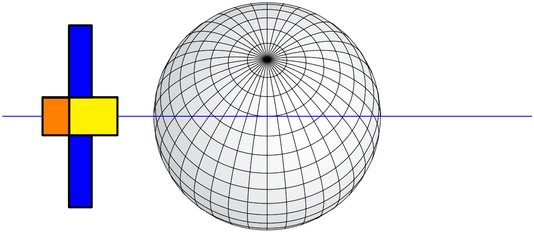

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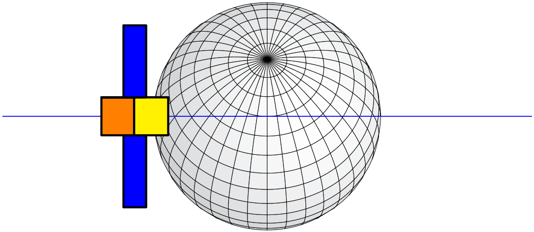

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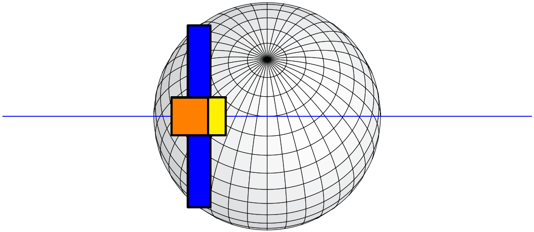

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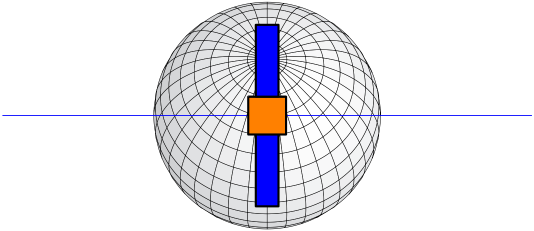


Conclusions

- A Sun–fixed argument for the periodic terms is helpful to obtain interpretable series of these parameters:

$$\Delta u = u_{sat} - u_{Sun}$$

The Empirical CODE Orbit Model

Conclusions

- A Sun–fixed argument for the periodic terms is helpful to obtain interpretable series of these parameters:

$$\Delta u = u_{sat} - u_{Sun}$$

- Solar radiation pressure for satellites flying according to the previously mentioned models can be represented by:

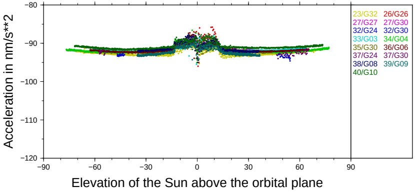
$$\begin{aligned} D &= D_0 + D_2 \cos(2\Delta u) + D_4 \cos(4\Delta u) + \ldots \\ Y &= Y_0 \\ B &= B_1 \cos(1\Delta u) + B_3 \cos(3\Delta u) + \ldots \end{aligned}$$

$Y_0 \neq 0$ if the satellite is flying "misaligned" with a Y–bias (e.g., GPS, except for Block IIF).

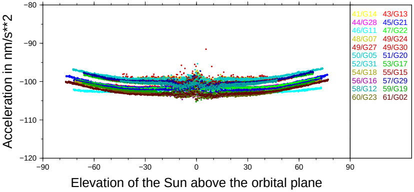
Estimated Solar Radiation Pressure

Component: D_0

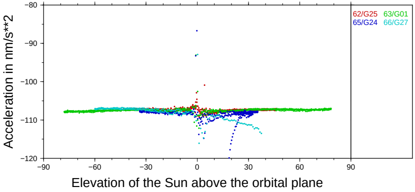
GPS Block IIA



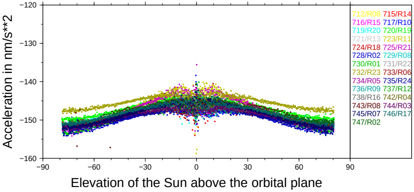
GPS Block IIR



GPS Block IIF

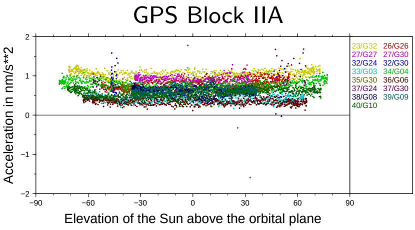


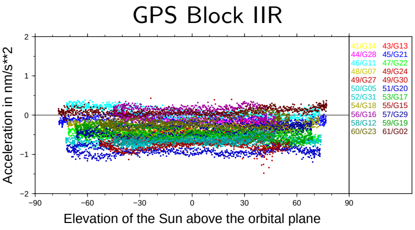
GLONASS–M

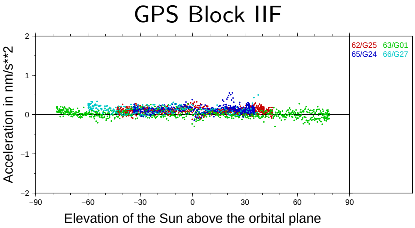


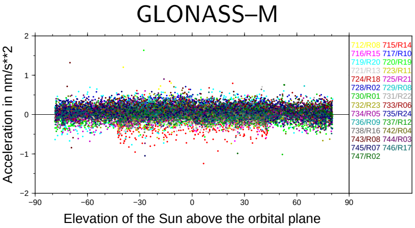
Estimated Solar Radiation Pressure

Component: Y_0 (small scale)





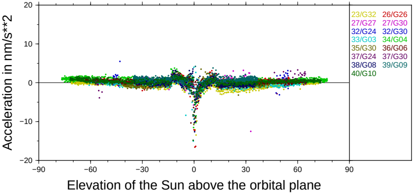




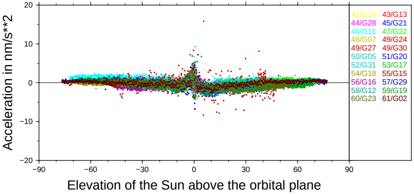
Estimated Solar Radiation Pressure

Component: $B_1 \cdot \cos(1\Delta u)$

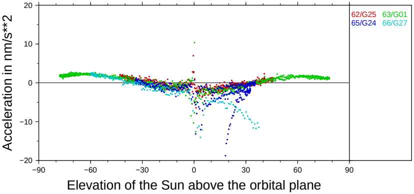
GPS Block IIA



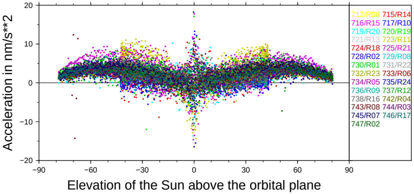
GPS Block IIR



GPS Block IIF



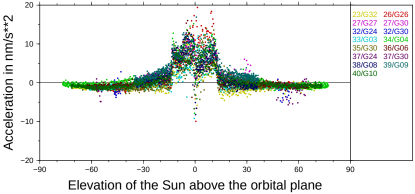
GLONASS–M



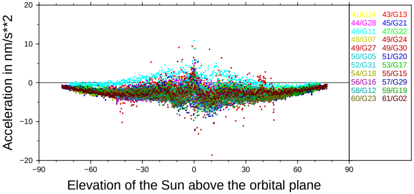
Estimated Solar Radiation Pressure

Component: $D_2 \cdot \cos(2\Delta u)$

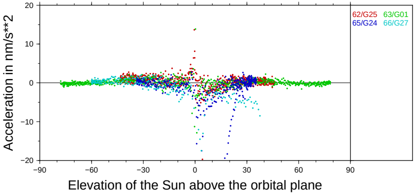
GPS Block IIA



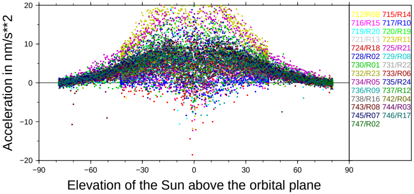
GPS Block IIR



GPS Block IIF



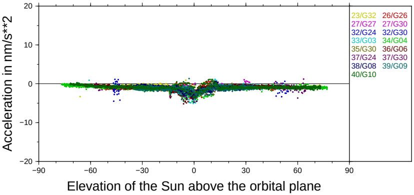
GLONASS–M



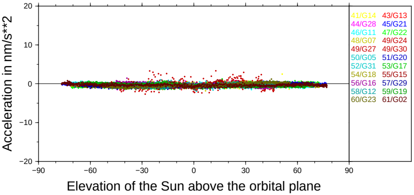
Estimated Solar Radiation Pressure

Component: $B_1 \cdot \sin(1\Delta u)$

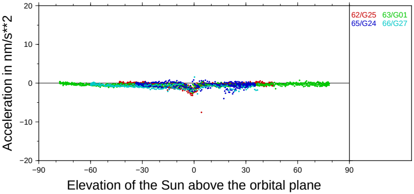
GPS Block IIA



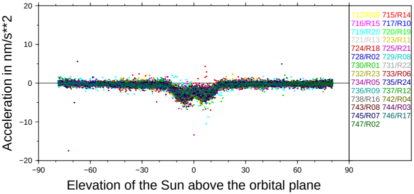
GPS Block IIR



GPS Block IIF



GLONASS–M



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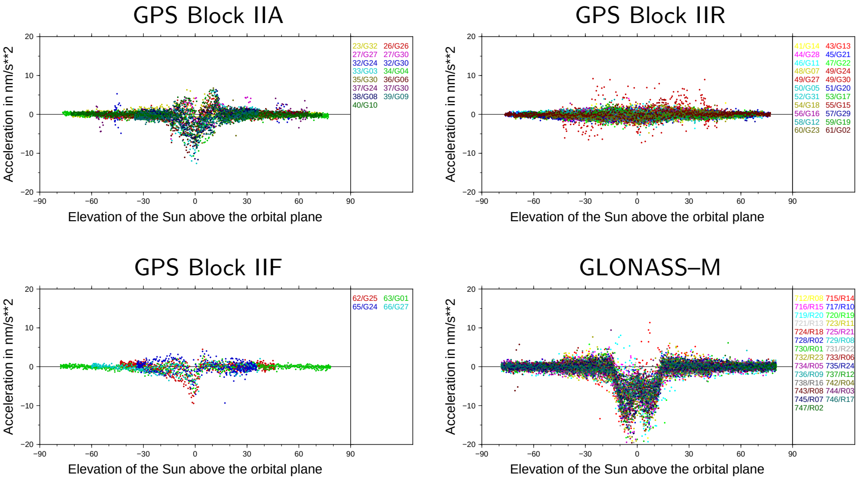
Estimated Solar Radiation Pressure

Component: $D_2 \cdot \sin(2\Delta u)$



Estimated Solar Radiation Pressure

Conclusions

- The definition of the angular argument $(\Delta u = u_{sat} - u_{Sun}$ instead of $u_{sat})$ allows a better interpretion of estimated parameter series, e.g., w.r.t. the elevation of the Sun above the orbital plane.

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Estimated Solar Radiation Pressure

Conclusions

- The definition of the angular argument ($\Delta u = u_{sat} - u_{Sun}$ instead of u_{sat}) allows a better interpretion of estimated parameter series, e.g., w.r.t. the elevation of the Sun above the orbital plane.

- Adding twice-per-revolution terms in D-component improves the orbit solution, in particular for satellites with stretched bodies.

- Even if the sin-terms are not necessary according to theory they are needed for representing real satellite trajectories.

The Empirical CODE Orbit Model

$$D = D_0 + \sum_{i=1}^{n_D} D_{2i,c} \cos(2i \cdot \Delta u) + D_{2i,s} \sin(2i \cdot \Delta u) \qquad (4)$$

$$Y = Y_0$$

$$B = B_0 + \sum_{i=1}^{n_B} B_{2i-1,c} \cos((2i-1) \cdot \Delta u) + B_{2i-1,s} \sin((2i-1) \cdot \Delta u)$$

- In practice the expansion is only used up to $n_D = n_B = 1$.

The Empirical CODE Orbit Model

- The empirical CODE Orbit Mode (ECOM) as shown in Equation 4 on slide 65 was developed in Arnold et al., 2015 .

- The extension considers in particular the effect on satellites with stretched bodies (e.g., GLONASS, Galileo, QZSS).

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- It is an extension of the classical ECOM as introduced by Beutler et al., 1994 . $(n_D = 0$ and $n_B = 1)$

- The ECOM is widely used within the IGS.

  In the semi-analytical approach the ECOM is also often in use to compensate for the deficiencies of the introduced a priori models.

Shadow Effects

- If the elevation of the Sun β becomes smaller than a certain angle β_0, so called eclipse phases occur where the satellite is not illuminated by the Sun.
  During eclipse, the force caused by the solar radiation needs to be switched off in the orbit model during the eclipse phase.

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  (with $a_{Earth} = 6\,380\,\text{km}$):

GLONASS	$a = 25\,500\,\text{km}$	$\beta_0 = 14.5°$
GPS	$a = 26\,560\,\text{km}$	$\beta_0 = 13.9°$
Galileo	$a = 30\,000\,\text{km}$	$\beta_0 = 12.3°$
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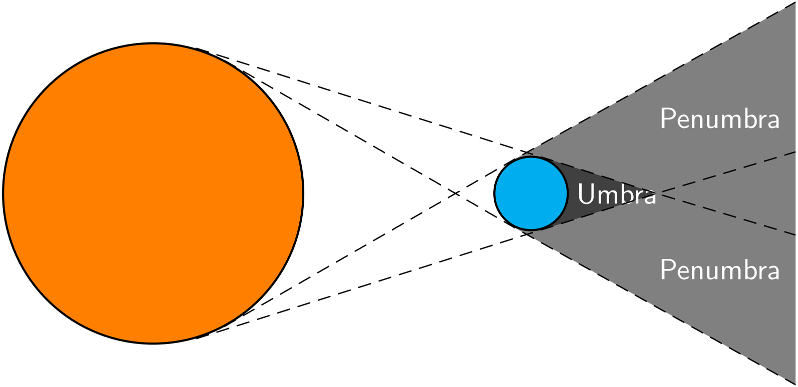
- The period where the satellite crosses the shadow of the Earth takes about one hour for a GNSS satellite in a MEO orbit.

Shadow Effects



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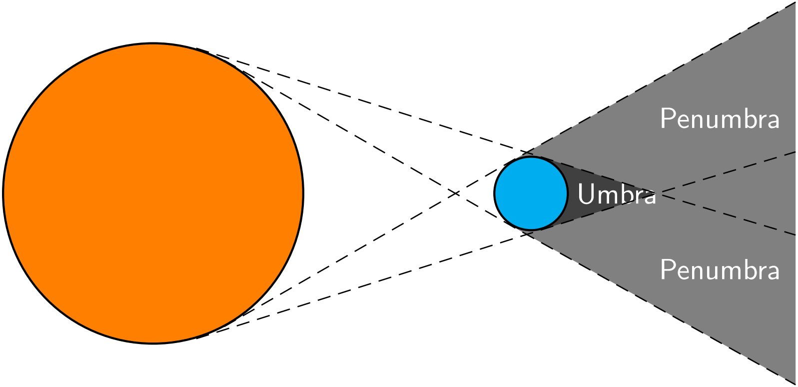

- A satellite flying into the shadow area behind the Earth crosses the penumbra in such a short interval that is can be neglected.

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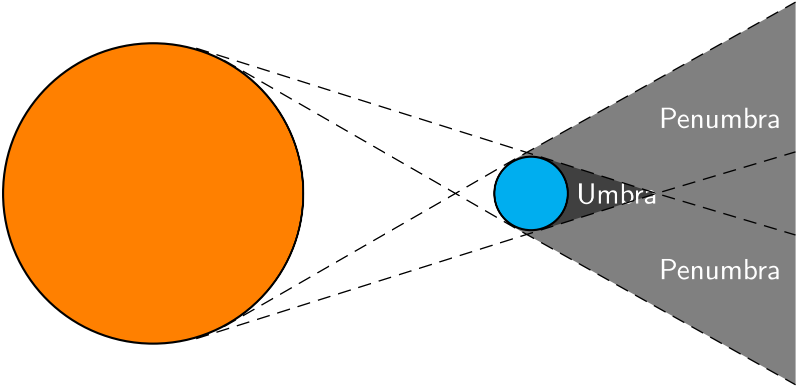

- A satellite flying into the shadow area behind the Earth crosses the penumbra in such a short interval that is can be neglected.
- The penumbra is on the other hand essential for the shadow generated by the Moon.

Other Radiation Pressure Effects

The biggest contribution comes from the

- **solar (or direct) radiation pressure**
  GPS: $\approx 250\,\mathrm{m}$ Galileo: $\approx 350\,\mathrm{m}$ QZSS: $\approx 700\,\mathrm{m}$

Maximal influence of the effect on the orbit after one day of orbit integration.

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Indirect radiation pressure effects due to solar radiation are

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Maximal influence of the effect on the orbit after one day of orbit integration.

Precise Orbit Determination for GNSS Satellites

Equation of Motion

In order to consider the gravitational and non-gravitational perturbations described before we have to extend the initial version of the equation of motion, see eqn. (3), by a function f:

$$\ddot{\vec{r}} = -GM_E \frac{\vec{r}}{|\vec{r}|^3} + f(t, \vec{r}, \dot{\vec{r}}, Q_1, \ldots, Q_n)\,,$$

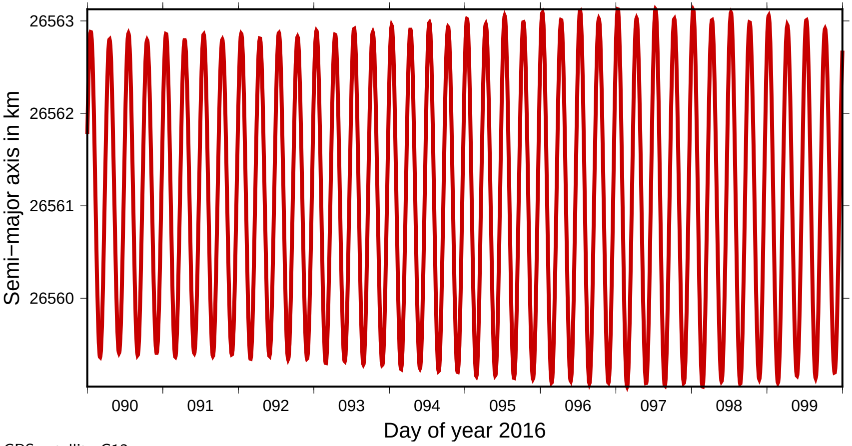
with initial conditions

$$\vec{r}(t_0) = \vec{r}(a, e, i, \Omega, \omega, u_0; t_0) \quad \text{and}$$
$$\dot{\vec{r}}(t_0) = \dot{\vec{r}}(a, e, i, \Omega, \omega, u_0; t_0)\,,$$

as well as $Q_1, \ldots, Q_n$ shall represent all known and unknown parameters of the force model (e.g., for the Earth's gravity field or the solar radiation pressure).

Osculating Elements

The perturbations described by the function f cause a permanent change of the orbital elements, the so call osculating elements:



GPS satellite G12.

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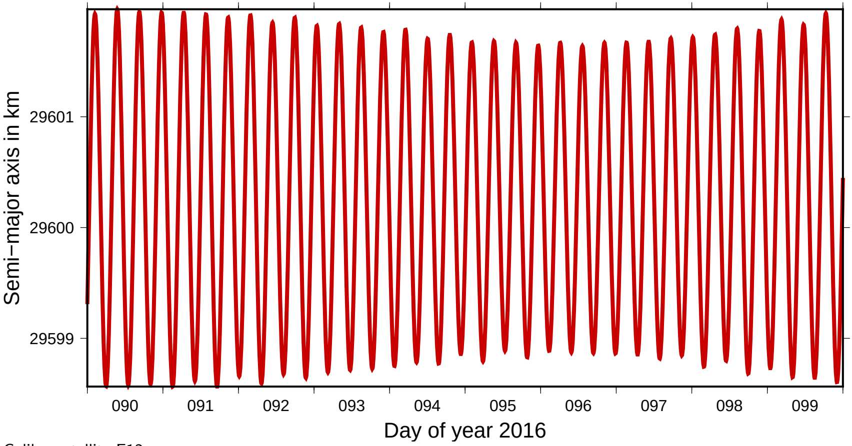

Galileo satellite E12.

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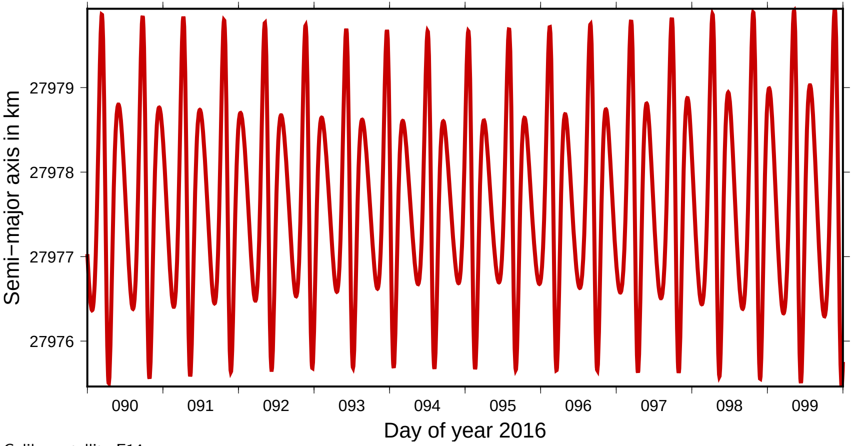

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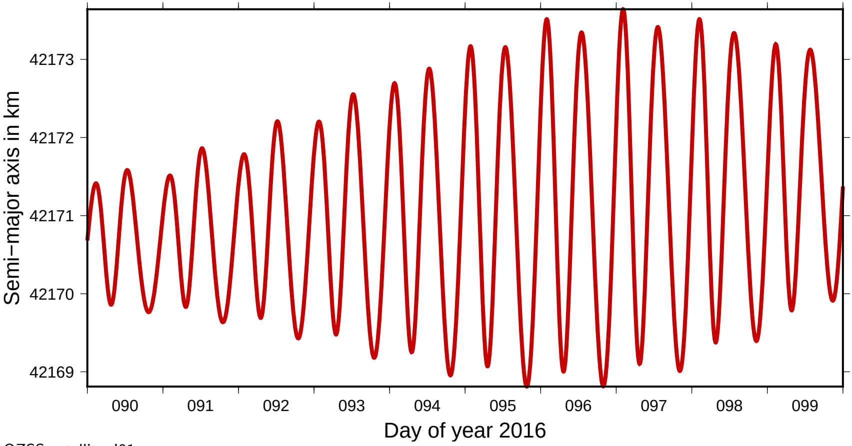

QZSS satellite J01.

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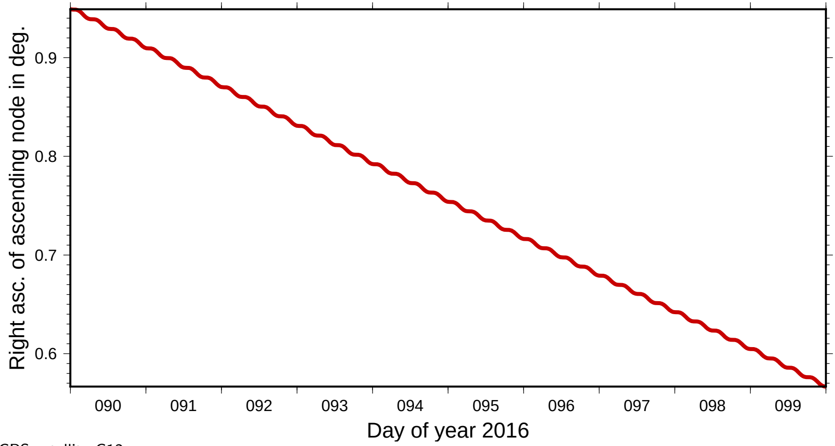

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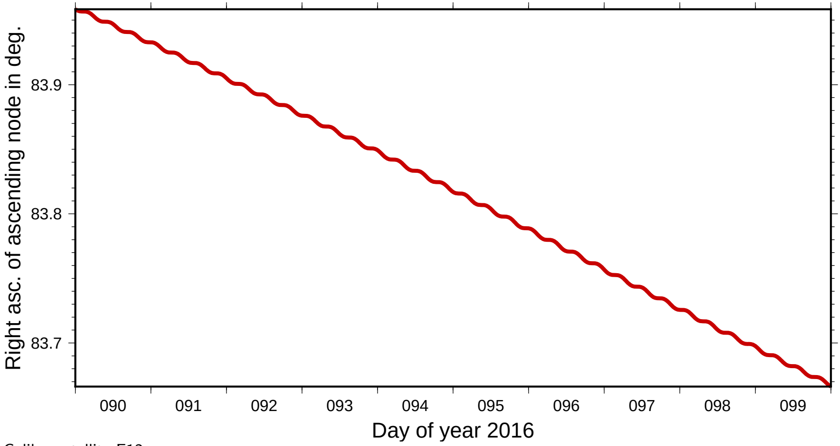

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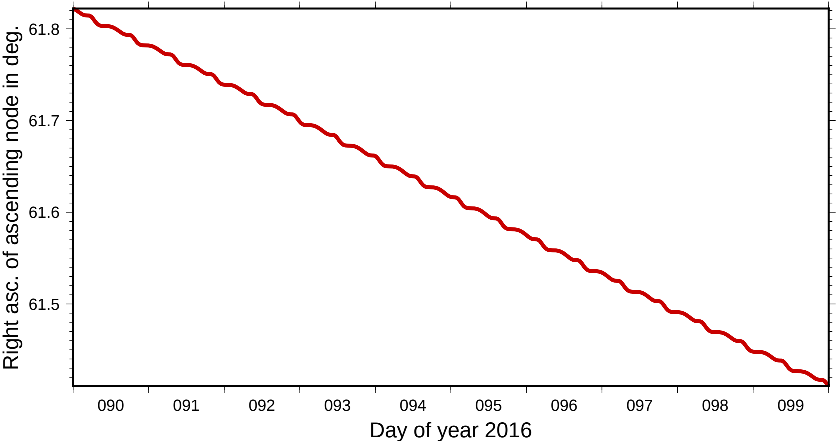

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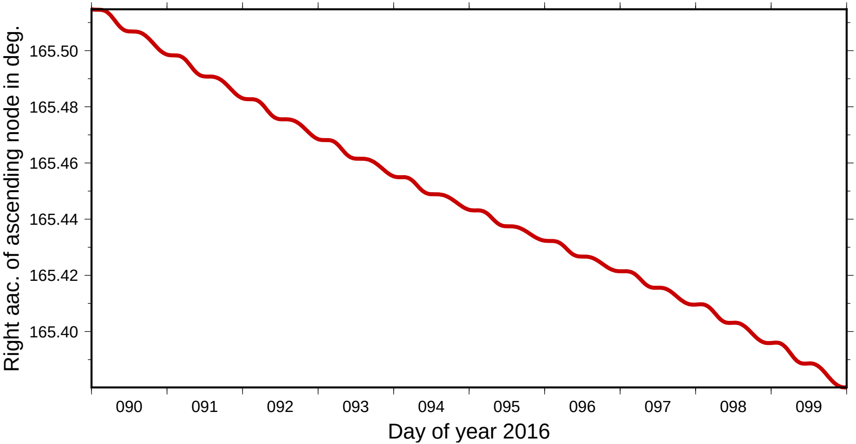

QZSS satellite J01.

Principle of Orbit Determination

The actual orbit $\vec{r}(t)$ is expressed as a truncated Taylor series:

$$\vec{r}(t) = \vec{r}_0(t) + \sum_{i=1}^{m} \frac{\partial \vec{r}_0}{\partial P_i}(t) \cdot (P_i - P_{0,i})$$

with

$\vec{r}_0(t)$	the a priori orbit,
$\frac{\partial \vec{r}_0}{\partial P_i}(t)$	the partial derivative of the a priori orbit $\vec{r}_0(t)$ w.r.t. parameter P_i,
$P_{0,i}$	the a priori parameter values of the a priori orbit $\vec{r}_0(t)$, and
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A least-squares adjustment of GNSS tracking data $L_{1,\ldots,n}$ yields corrections to the a priori parameter values $P_{0,i}$. Using the above equation, the improved (linearized) orbit $\vec{r}(t)$ may be computed.

Partial Derivatives

The partial derivative of the observation L_j w.r.t. orbit parameter P_i may be expressed as

$$\frac{\partial L_j}{\partial P_i}(t) = (\nabla(L_j))^T \cdot \frac{\partial \vec{r}_0}{\partial P_i}(t)$$

with the gradient given by

$$(\nabla(L_j))^T = \begin{pmatrix} \frac{\partial L_j}{\partial r_{0,1}} & \frac{\partial L_j}{\partial r_{0,2}} & \frac{\partial L_j}{\partial r_{0,3}} \end{pmatrix}$$

if the observations only depend on the geocentric position vector and are referring to only one epoch.

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if the observations only depend on the geocentric position vector and are referring to only one epoch. The gradient only depends on the type of observations used, whereas the second term is independent of the observation type and is related to the variational equations. This separates the observation-specific (geometric) part from the dynamic part.

Variational Equations

For each orbit parameter P_i the corresponding variational equation reads as

$$\ddot{\vec{r}}_{P_i} = A_0 \cdot \vec{r}_{P_i} + A_1 \cdot \dot{\vec{r}}_{P_i} + \frac{\partial f_i}{\partial P_i}$$

with the 3×3 matrices defined by

$$A_{0[i,k]} \doteq \frac{\partial f_i}{\partial r_{0,k}} \qquad \text{and} \qquad A_{1[i,k]} \doteq \frac{\partial f_i}{\partial \dot{r}_{0,k}}$$

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f_i	i-th component of the total acceleration function
$\vec{r}_0$, $\dot{\vec{r}}_0$	positions and velocities from the a priori orbit
$r_{0,k}$	k-th component of the geocentric position $\vec{r}_0$

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$$\ddot{\vec{r}}_{P_i} = A_0 \cdot \vec{r}_{P_i} + A_1 \cdot \dot{\vec{r}}_{P_i} + \frac{\partial f_i}{\partial P_i}$$

with the 3×3 matrices defined by

$$A_{0[i,k]} \doteq \frac{\partial f_i}{\partial r_{0,k}} \qquad \text{and} \qquad A_{1[i,k]} \doteq \frac{\partial f_i}{\partial \dot{r}_{0,k}}$$

f_i	i-th component of the total acceleration function
$\vec{r}_0, \dot{\vec{r}}_0$	positions and velocities from the a priori orbit
$r_{0,k}$	k-th component of the geocentric position $\vec{r}_0$

For each orbit parameter P_i the variational equation is a linear differential equation system of second order in time. Their solutions are all needed for orbit determination.

Variational Equations

The variational equation is a linear, homogeneous system with initial values

$\vec{r}_{P_i}(t_0) \neq 0$ and $\dot{\vec{r}}_{P_i}(t_0) \neq 0$ for $P_i \in a, e, i, \Omega, \omega, u_0$

Variational Equations

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$$\vec{r}_{P_i}(t_0) \neq 0 \qquad \text{and} \qquad \dot{\vec{r}}_{P_i}(t_0) \neq 0 \qquad \text{for} \qquad P_i \in a, e, i, \Omega, \omega, u_0$$

and a linear, inhomogeneous system with initial values

$$\vec{r}_{P_i}(t_0) = 0 \qquad \text{and} \qquad \dot{\vec{r}}_{P_i}(t_0) = 0 \qquad \text{for} \qquad P_i \in Q_1, \ldots, Q_n$$

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Let us assume that the functions $\vec{r}_{O_j}(t),\ j = 1, \ldots, 6$ are the partials w.r.t. the six parameters $O_j,\ j = 1, \ldots, 6$ defining the initial conditions at time t_0.

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Let us assume that the functions $\vec{r}_{O_j}(t),\ j = 1, \ldots, 6$ are the partials w.r.t. the six parameters $O_j,\ j = 1, \ldots, 6$ defining the initial conditions at time t_0. The ensemble of these six functions forms one complete system of solutions of the homogeneous part of the variational equation, which allows to obtain the solution of the inhomogeneous system by the method of "variation of constants".

Variational Equations

The solution and its first time derivative may be written as

$$z_{P_i}^{(k)}(t) = \sum_{j=1}^{6} \alpha_{O_j,P_i}(t) \cdot z_{O_j}^{(k)}(t); \qquad k = 0, 1$$

with the coefficient functions defined by

$$\alpha_{P_i}(t) \doteq \int_{t_0}^{t} Z^{-1}(t') \cdot h_{P_i}(t') dt'$$

α_{P_i} column array defined by $(\alpha_{O_1.P_i}, \ldots, \alpha_{O_6.P_i})^T$
Z 6×6 matrix defined by $Z_{[1,\ldots,3;j]} \doteq z_{O_j}$, $Z_{[4,\ldots,6;j]} \doteq \dot{z}_{O_j}$
h_{P_i} column array defined by $(O^T, \frac{\partial f^T}{\partial P_i})^T$

Variational Equations

Note that the solutions $z_{P_i}(t)$ of the variational equation and its time derivative may be expressed with the same functions α_{O_j,P_i} as a linear combination with the homogeneous solutions $z_{O_j}(t)$ and $\dot{z}_{O_j}(t)$, respectively. Therefore, only the six initial value problems associated with the initial conditions have to be actually treated as differential equation systems. Their solutions have to be either obtained approximately, or by numerical integration techniques.

All variational equations related to dynamical orbit parameters may be reduced to definite integrals. They can be efficiently solved numerically, e.g., by a Gaussian quadrature technique.

It must be emphasized that each additional orbit parameter requires an additional numerical solution of a definite integral.

Numerical Integration

Collocation algorithms (one particular class of numerical integration techniques) are subsequently used to briefly illustrate the principles of numerical integration:

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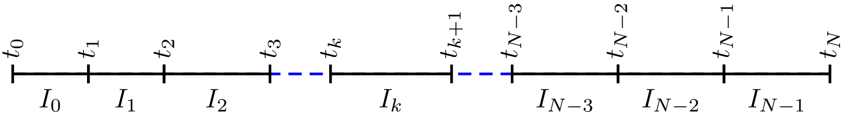

The original interval is divided into N integration intervals.

Numerical Integration

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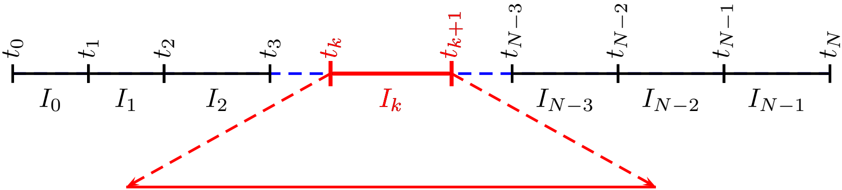

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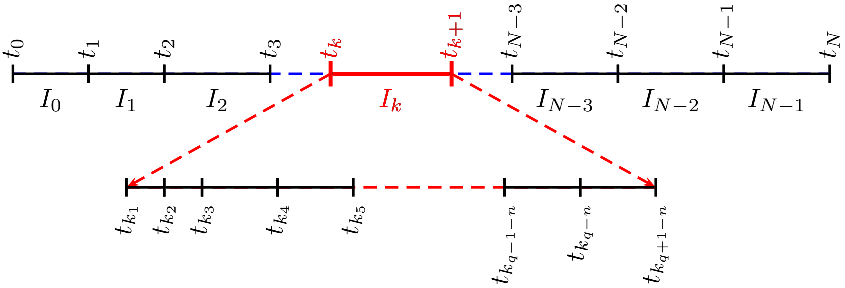


The original interval is divided into N integration intervals. For each interval I_k a further subdivision is performed according to the order q of the adopted method. At these points t_{k_j} the numerical solution is requested to solve the differential equation system of order n.

Numerical Integration

Initial value problem in the interval t_k is given by:

$$\ddot{\vec{r}}_k = f(t, \vec{r}_k, \dot{\vec{r}}_k)$$

with initial conditions

$$\vec{r}_k(t_k) \doteq \vec{r}_{k_0} \qquad \text{and} \qquad \dot{\vec{r}}_k(t_k) \doteq \dot{\vec{r}}_{k_0}$$

where the initial values are defined as

$$\vec{r}^{(i)}_{k_0} = \begin{cases} \vec{r}^{(i)}_0 & k = 0 \\ \vec{r}^{(i)}_{k-1} & k > 0 \end{cases}$$

Numerical Integration

The collocation method approximates the solution in the interval I_k by:

$$\vec{r}_k(t) = \sum_{l=0}^{q} \frac{1}{l!}(t - t_k)^l \vec{r}_{k0}^{(l)}$$

The coefficients $\vec{r}_{k0}^{(l)}, l = 0, \ldots, q$ are obtained by requesting that the numerical solution assumes the initial values and solves the differential equation system at $q-1$ different epochs $t_{k_j}, j = 1, \ldots, q-1$. This leads to the conditions

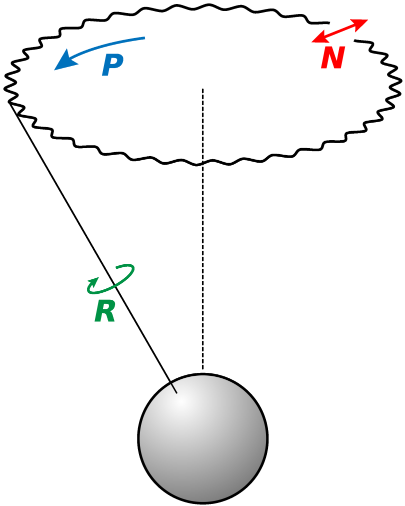
$$\sum_{i=2}^{q} \frac{(t_{k_j} - t_k)^{l-2}}{(l-2)!} \cdot \vec{r}_{k0}^{(l)} = f(t_{k_j}, \vec{r}_k(t_{k_j}), \dot{\vec{r}}_k(t_{k_j})) \quad j = 1, \ldots, q-1$$

They are non-linear but can be solved efficiently by an iterative procedure. See Beutler, 2005.

Transition Quasi-Inertial to Earth-fixed System

Contribution $Q(t)$:

Precession and nutation are caused by Moon and Sun and can be assumed to be known from their ephemeris.

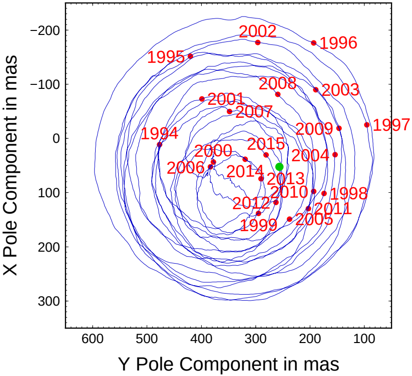


Transition Quasi-Inertial to Earth-fixed System

Contributions $W(t)$ and $R(t)$:

The location of the rotation axis of the Earth is moving with respect to the Earth surface: polar motion.

The rotation velocity of the Earth also varies: Excess length of day.



These variations are caused by mass redistributions in the Earth body, of the water on the surface of the Earth as well as within the Earth's atmosphere.

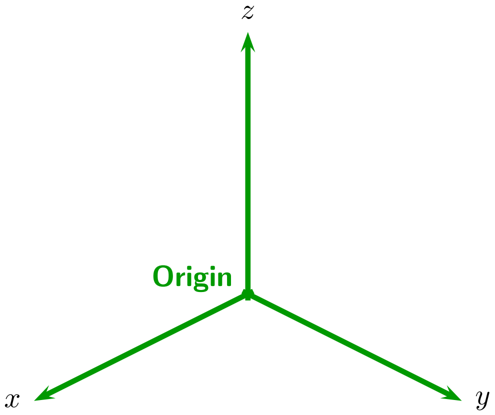
Transition Quasi-Inertial to Earth-fixed System

The transition from the Earth-fixed $(x_E\ y_E\ z_E)^T$ into the quasi-inertial $(x_R\ y_R\ z_R)^T$ coordinate system is based on the following rotations:

1. $W(t)$: polar motion
   (location of the rotation axis of the Earth)
2. $R(t)$: rotation of the Earth
3. $Q(t)$: nutation and precession
   (rotation of the celestial pole)

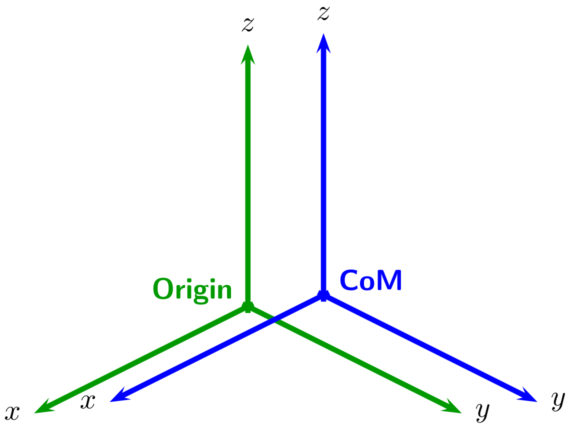
$$\begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix} = Q(t) \cdot R(t) \cdot W(t) \cdot \begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix}$$

Transition Quasi-Inertial to Earth-fixed System



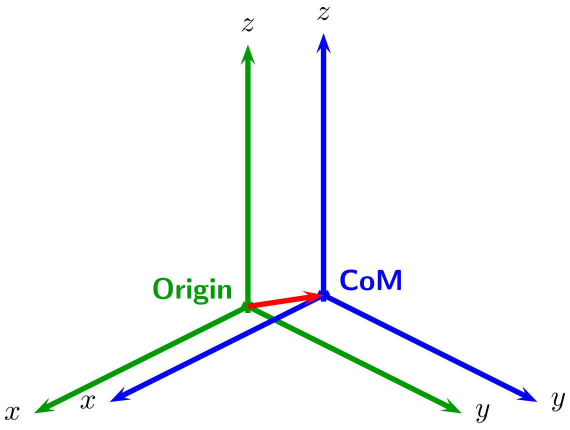
- Origin of the terrestrial reference system

Transition Quasi-Inertial to Earth-fixed System



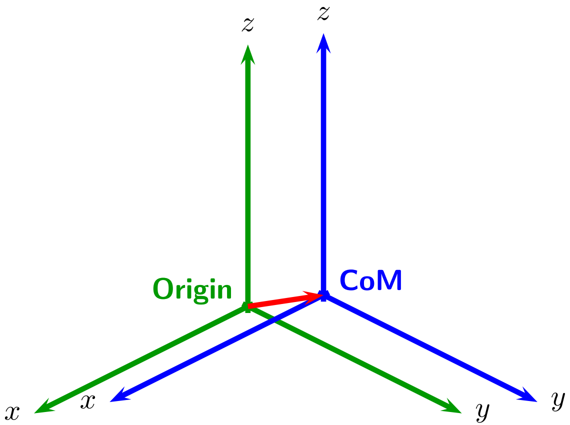
- Origin of the terrestrial reference system
- Center of mass of the Earth

Transition Quasi-Inertial to Earth-fixed System



- Origin of the terrestrial reference system
- Center of mass of the Earth
- Geocenter vector

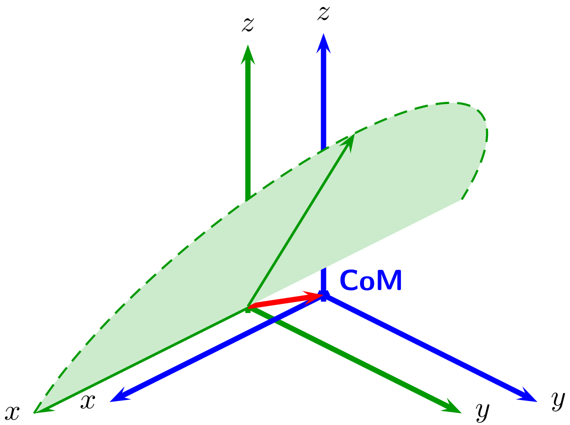
Transition Quasi-Inertial to Earth-fixed System



- Origin of the terrestrial reference system
- Center of mass of the Earth
- Geocenter vector

The origin of the terrestrial reference frame is located in the long-term averaged position of the center of mass of the Earth. The geocenter vector points to the instantaneous center of mass.

Transition Quasi-Inertial to Earth-fixed System



- Origin of the terrestrial reference system
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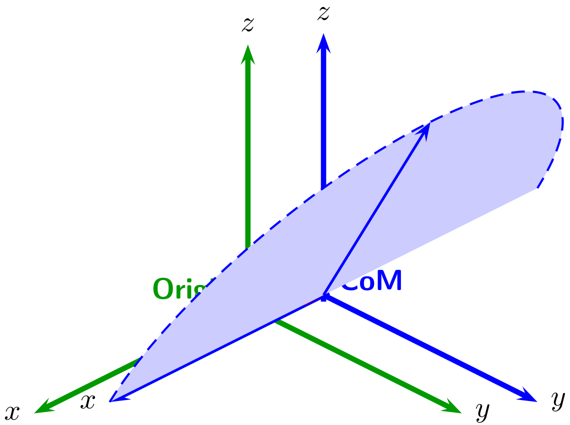
The satellite orbit refers to the origin of the terrestrial reference system if the transformation from the terrestrial into the quasi–inertial system contains only rotations (Earth rotation parameters).

Transition Quasi-Inertial to Earth-fixed System



- Origin of the terrestrial reference system
- Center of mass of the Earth
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The satellite orbit need to refer to the center of mass of the Earth because the physics of celestial mechanics is based on the principle of gravitation.

Conclusion – the correct way is:

Transition Quasi-Inertial to Earth-fixed System

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Satellite positions in the terrestrial system

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+ Vector to the geocenter

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Satellite positions w.r.t. the center of mass of the Earth

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$\times$ Earth rotation parameters

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All orbit modelling. . .

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Satellite positions w.r.t. the center of mass of the Earth

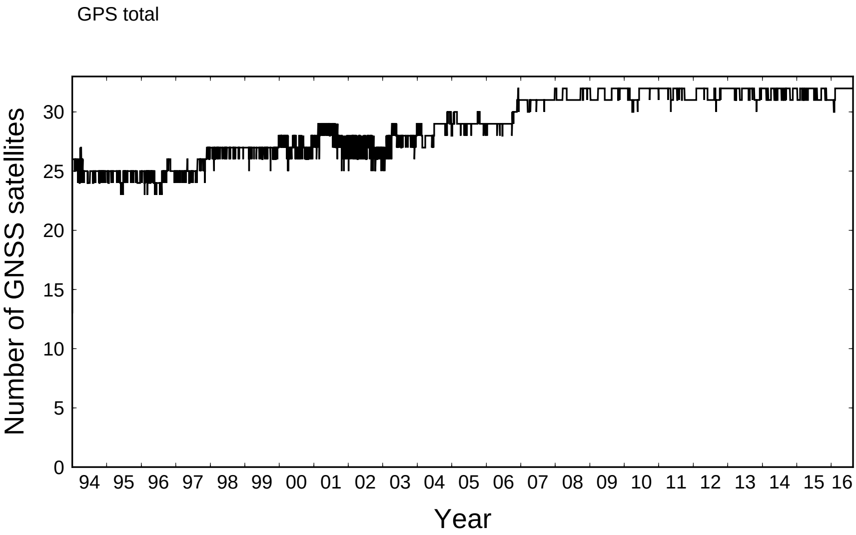
− Vector to the geocenter

Satellite positions in the terrestrial system

⋮

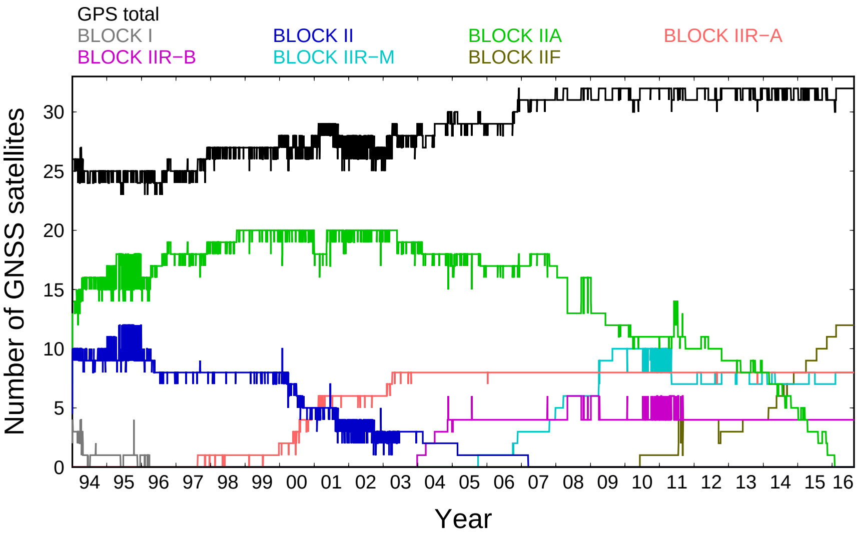
Satellite positions may be related to the station coordinates

GNSS Satellites in CODE Solution



Development of the GPS satellite constellation

GNSS Satellites in CODE Solution



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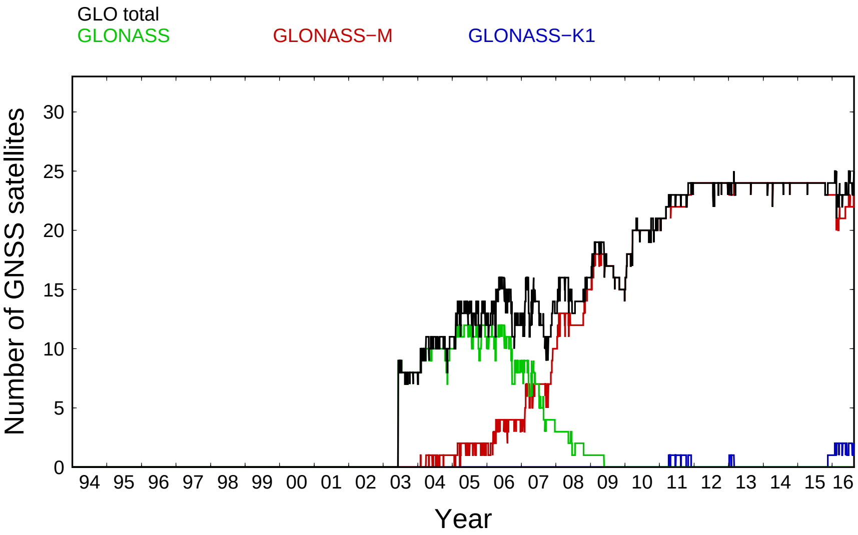
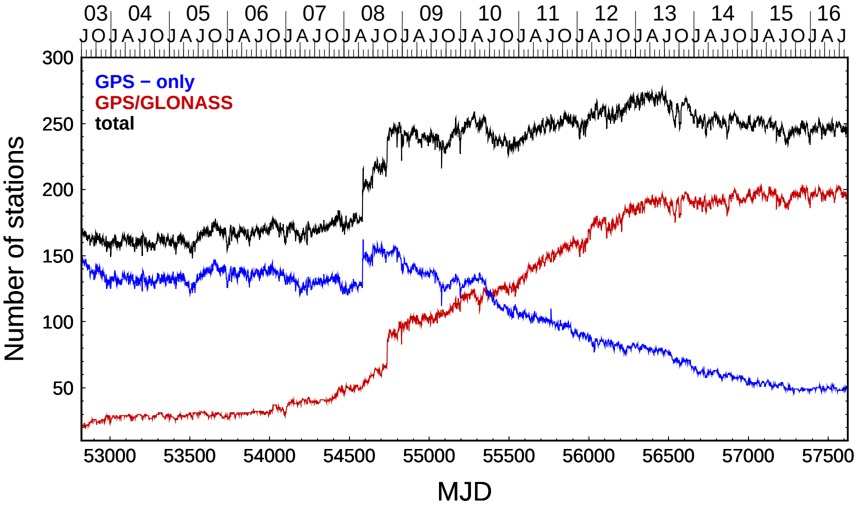
GNSS Satellites in CODE Solution



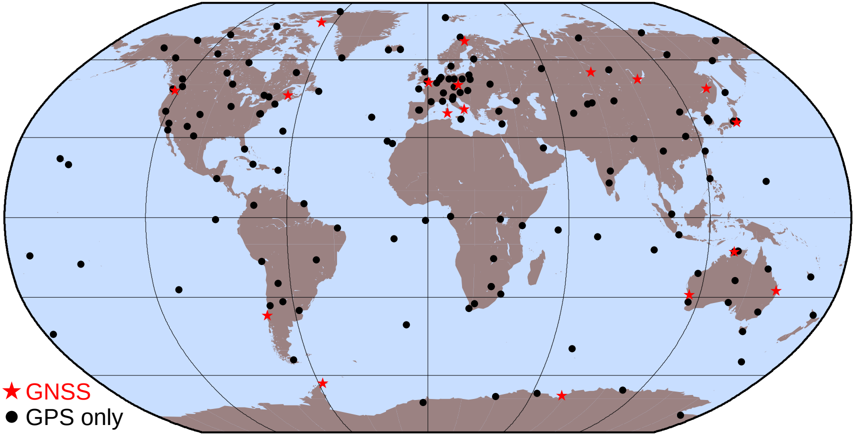
Development of the GLONASS satellite constellation

GNSS Stations in CODE Solution



Development of the number of GLONASS tracking stations

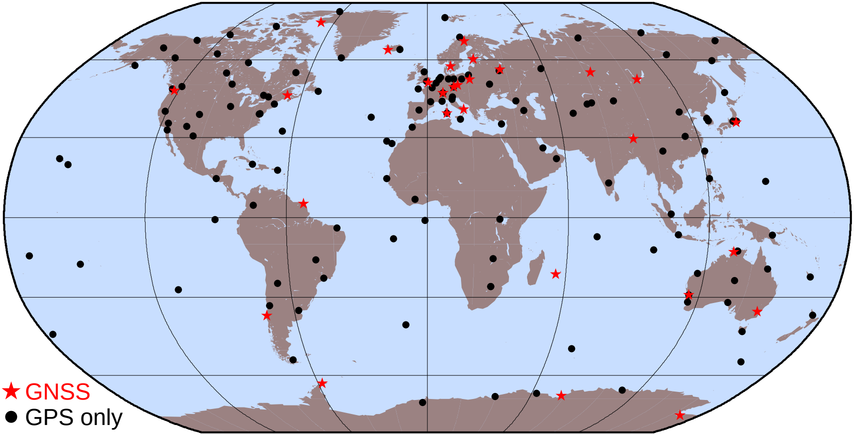
GNSS Stations in CODE Solution



Network used for the GNSS processing at CODE.

Status: June 2003

GNSS Stations in CODE Solution



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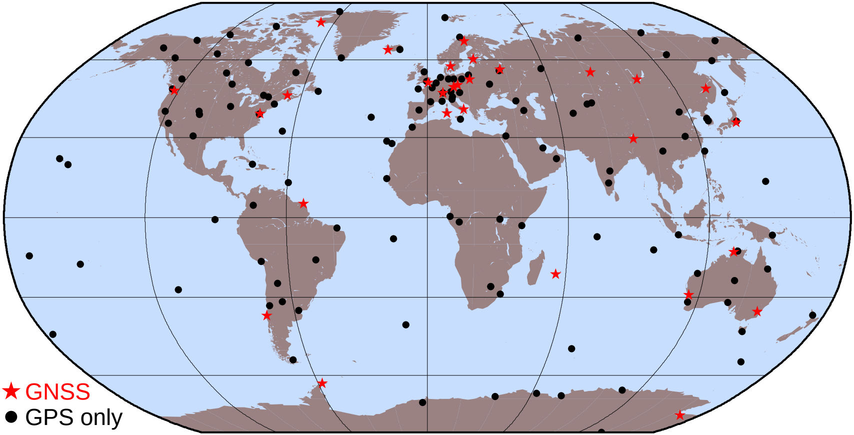
Status: June 2004

GNSS Stations in CODE Solution



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Status: June 2005

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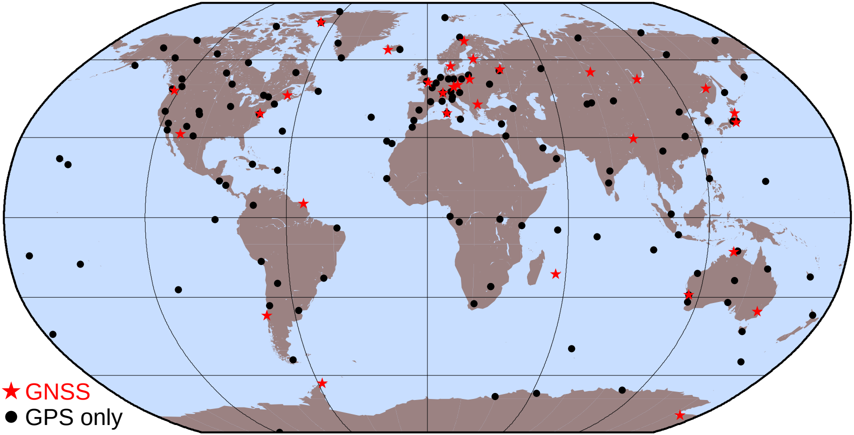

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Status: June 2006

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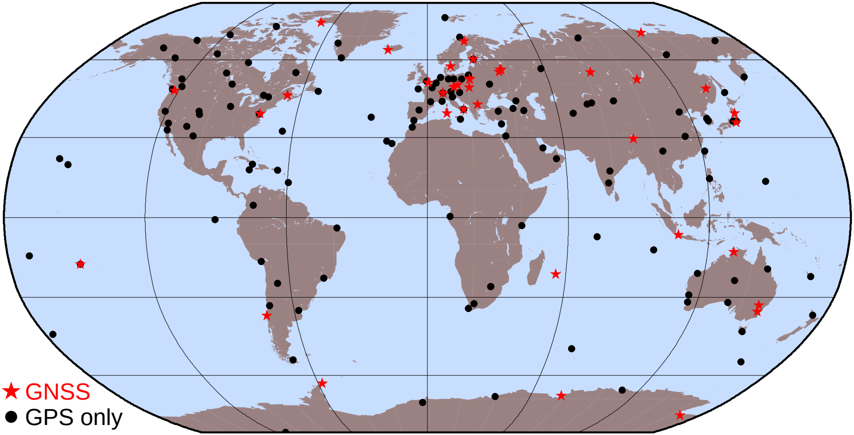
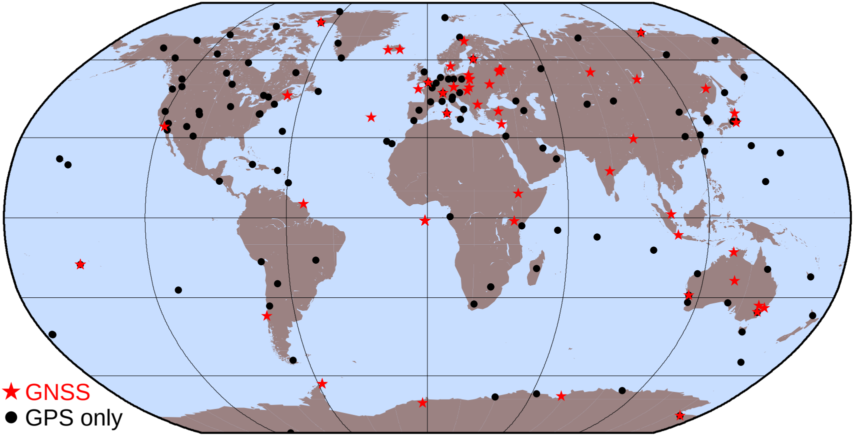


Network used for the GNSS processing at CODE.

Status: June 2007

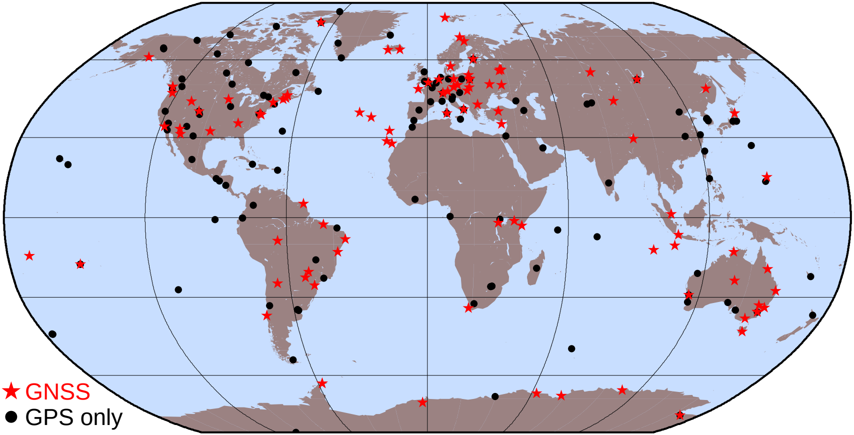
GNSS Stations in CODE Solution



Network used for the GNSS processing at CODE.

Status: June 2008

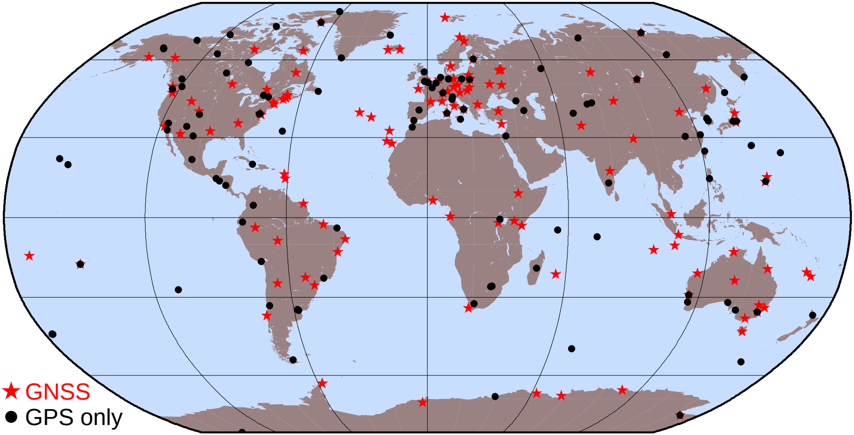
GNSS Stations in CODE Solution



Network used for the GNSS processing at CODE.

Status: June 2009

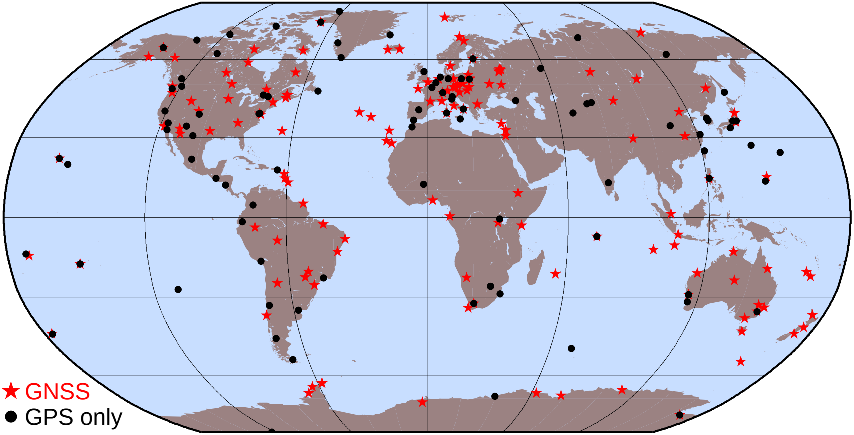
GNSS Stations in CODE Solution



Network used for the GNSS processing at CODE.

Status: June 2010

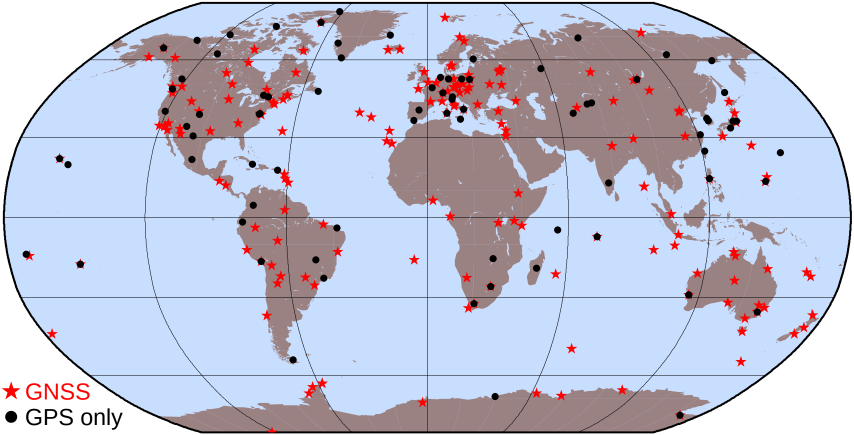
GNSS Stations in CODE Solution



Network used for the GNSS processing at CODE.

Status: June 2011

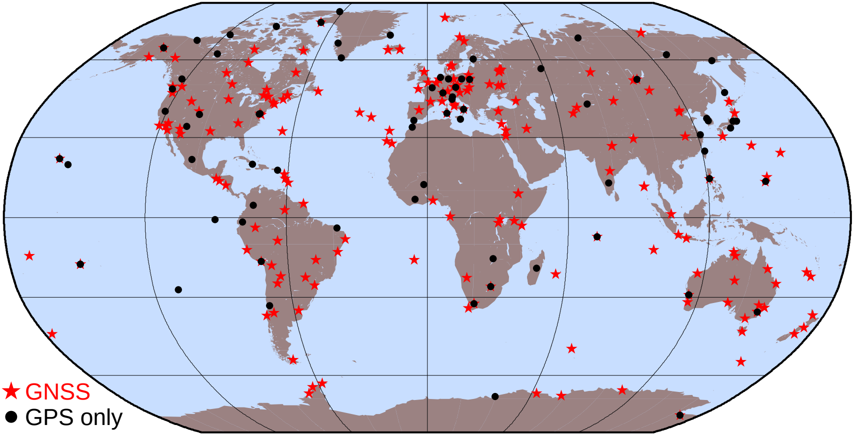
GNSS Stations in CODE Solution



Network used for the GNSS processing at CODE.

Status: June 2012

GNSS Stations in CODE Solution



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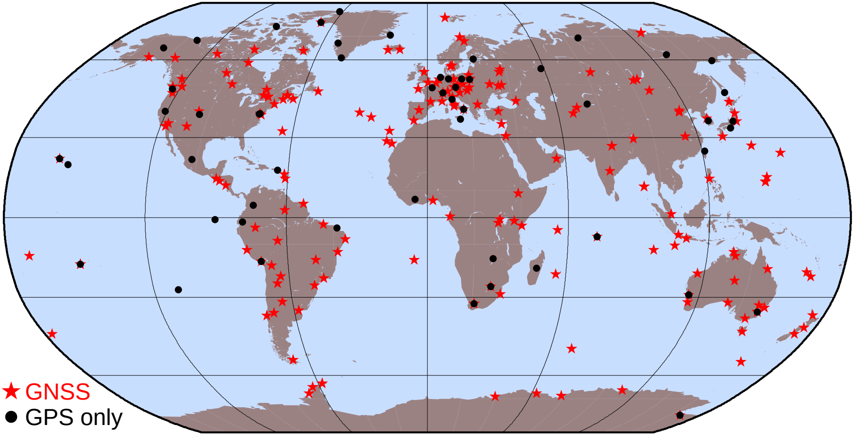
Status: June 2013

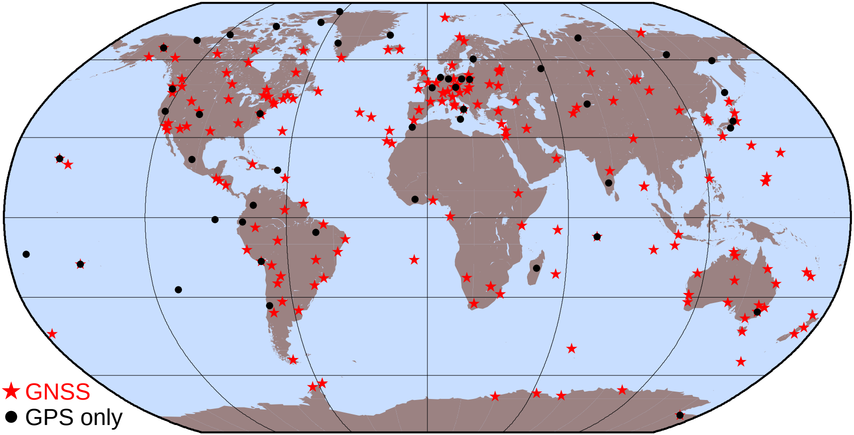
GNSS Stations in CODE Solution



Network used for the GNSS processing at CODE.

Status: June 2014

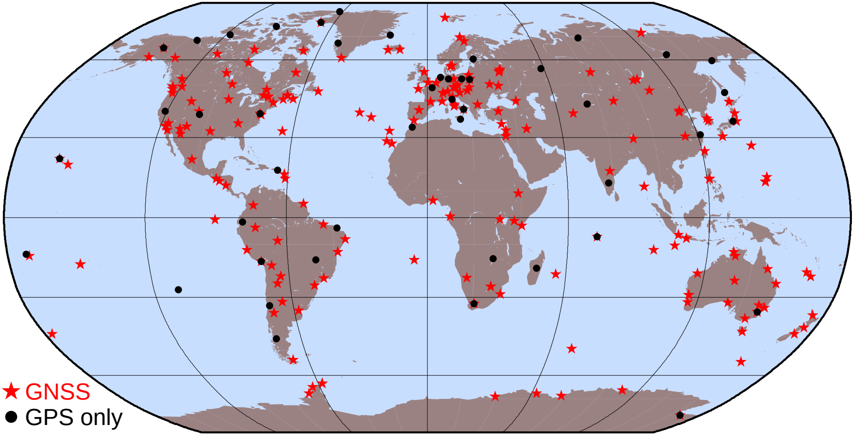
GNSS Stations in CODE Solution



Network used for the GNSS processing at CODE.

Status: June 2015

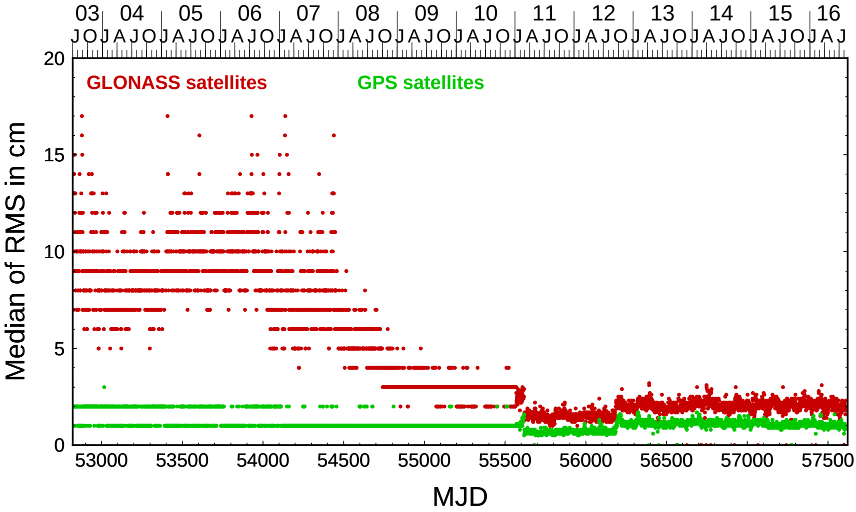
GNSS Stations in CODE Solution



Network used for the GNSS processing at CODE.

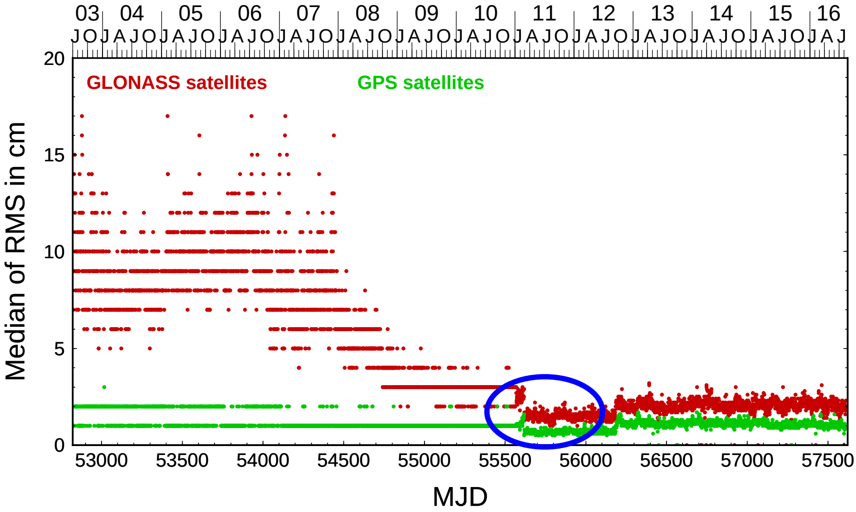
Status: June 2016

CODE GNSS Satellite Orbits



Development of the GLONASS orbit accuracy in the CODE final processing.

CODE GNSS Satellite Orbits



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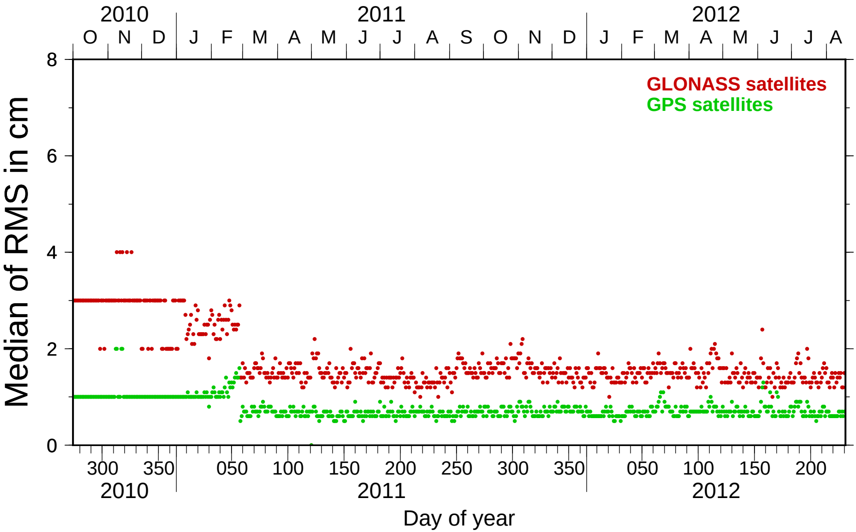

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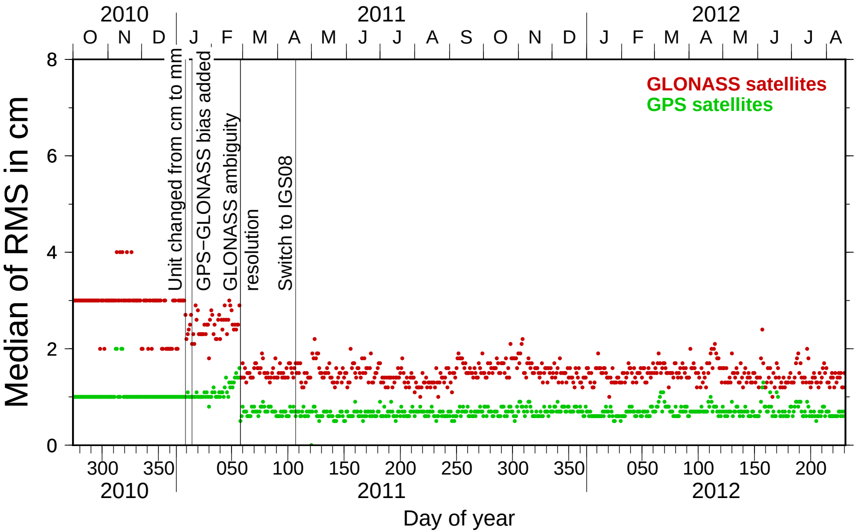

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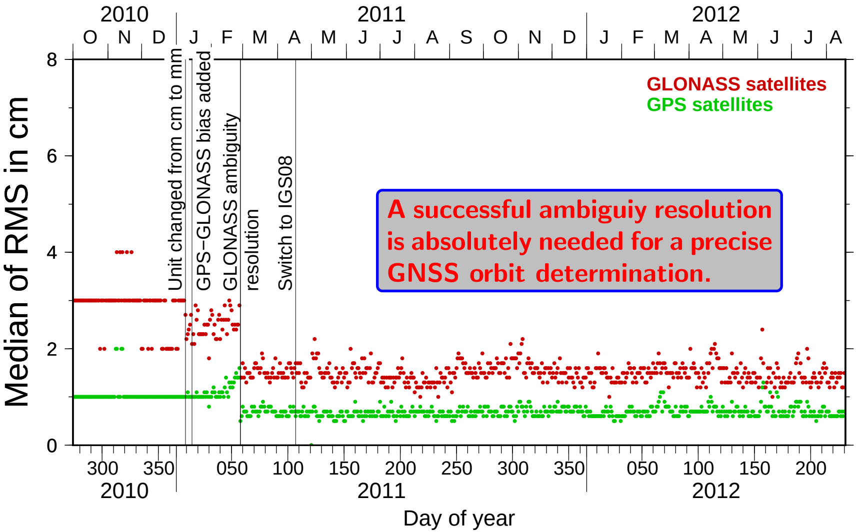
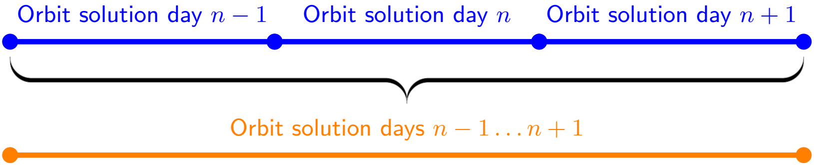


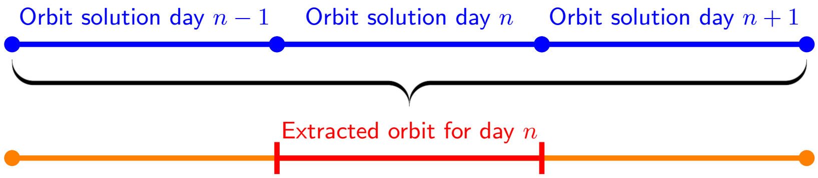
Multi-Day Solutions

Multi-Day Solutions

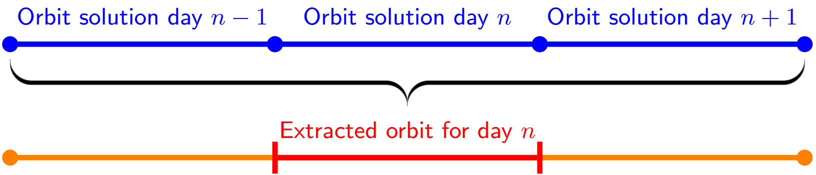
Multi-Day Solutions



Multi-Day Solutions



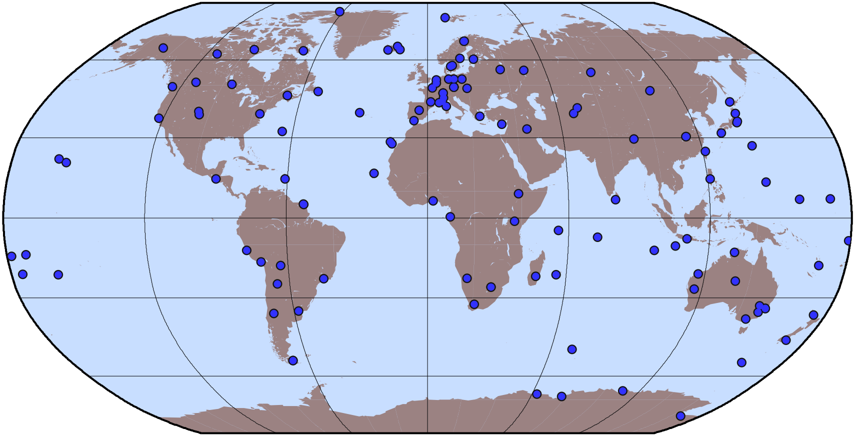
Multi-Day Solutions



Advantage of the "Extracted orbit for day n" with respect to the direct "Orbit solution day n":

- better decorrelation between orbit and Earth rotation parameters.
- no (or at least less) degradation of the orbit at the end of the boundary.
- smoothed day boundary discontinuities (in particular if the satellite was only weakly observed).

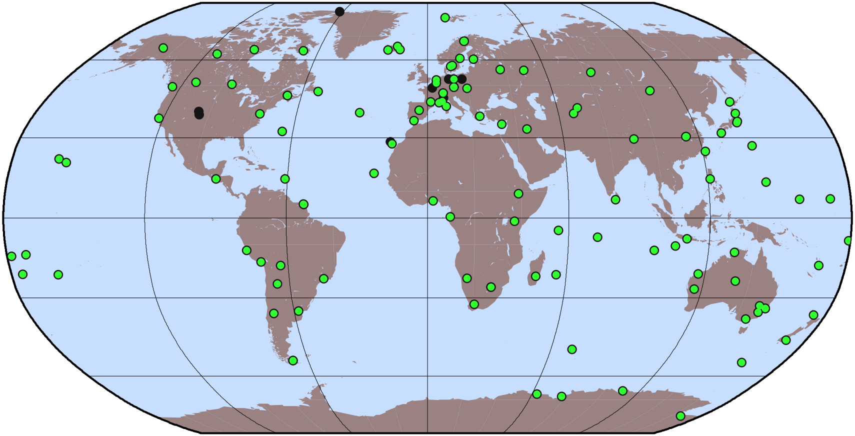
Tracking Situation in the MGEX Network



Network used for the CODE MGEX solution: stations tracking GPS

Status: July 2016.

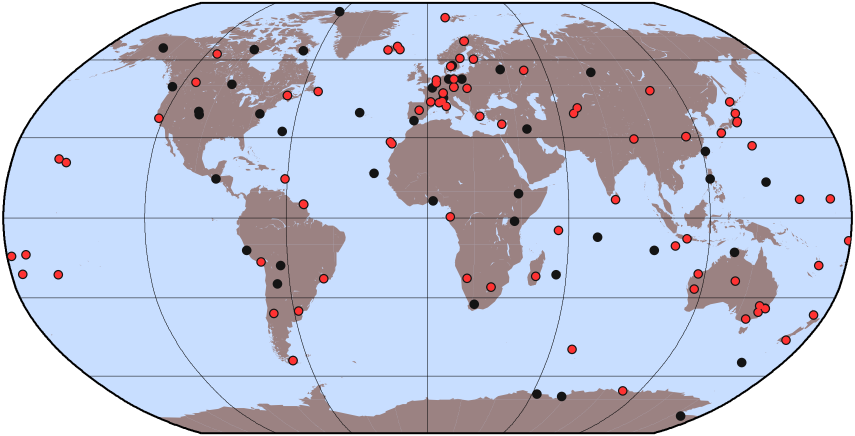
Tracking Situation in the MGEX Network



Network used for the CODE MGEX solution: stations tracking GLONASS

Status: July 2016.

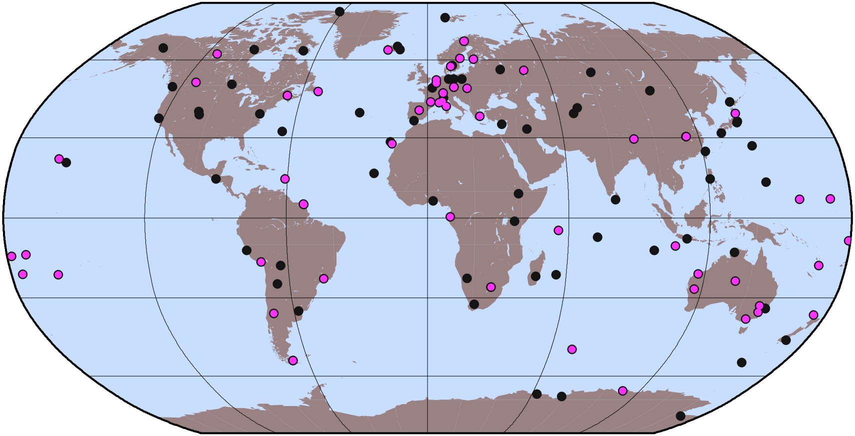
Tracking Situation in the MGEX Network



Network used for the CODE MGEX solution: stations tracking Galileo

Status: July 2016.

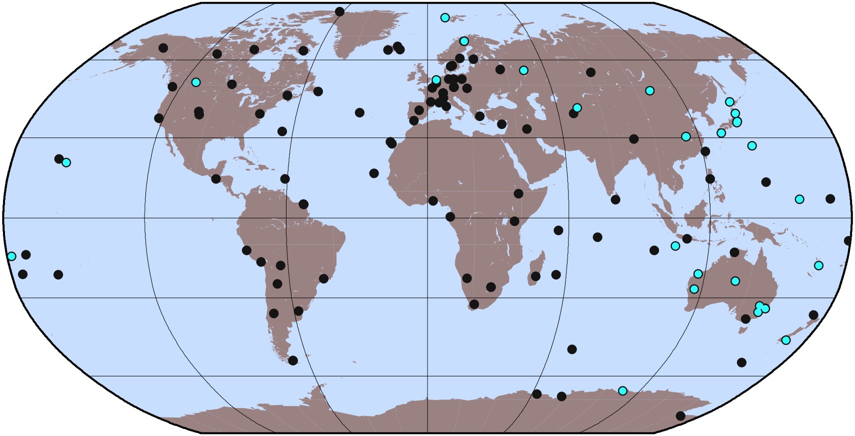
Tracking Situation in the MGEX Network



Network used for the CODE MGEX solution: stations tracking BeiDou

Status: July 2016.

Tracking Situation in the MGEX Network



Network used for the CODE MGEX solution: stations tracking QZSS

Status: July 2016.

Validation by Fitting Long Arcs

1. Fitting long arcs

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Validation by Fitting Long Arcs

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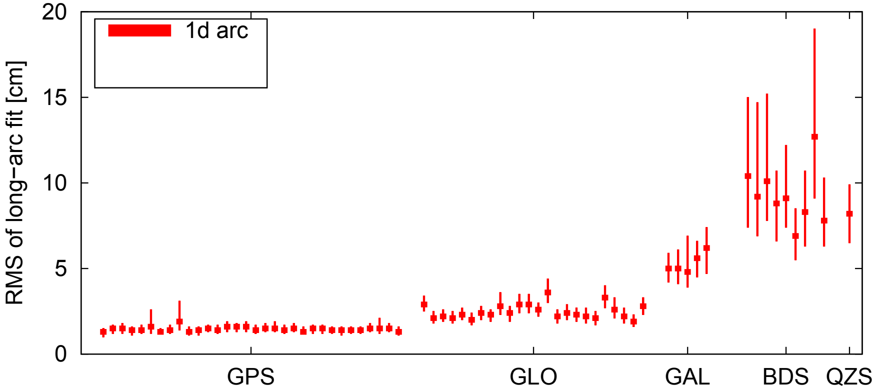
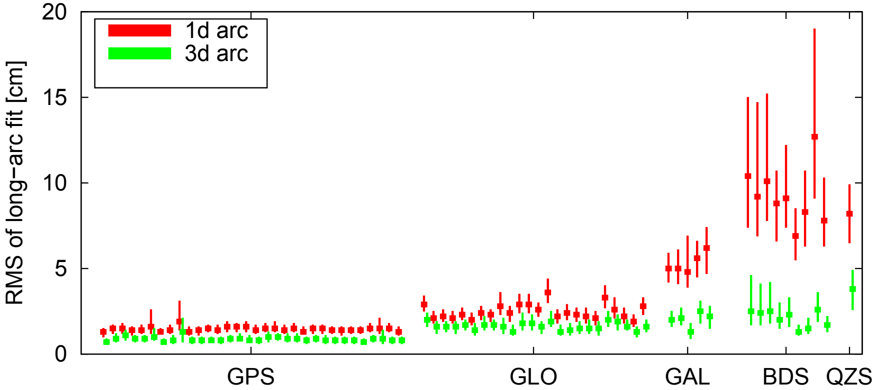
CODE MGEX solution for the year 2015



Median per satellite with associated quantiles

Validation by Fitting Long Arcs

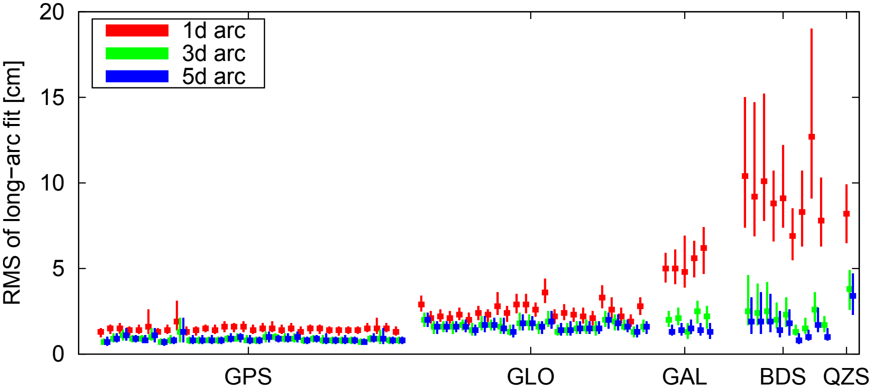
CODE MGEX solution for the year 2015



Median per satellite with associated quantiles

Validation by Fitting Long Arcs

CODE MGEX solution for the year 2015



Median per satellite with associated quantiles

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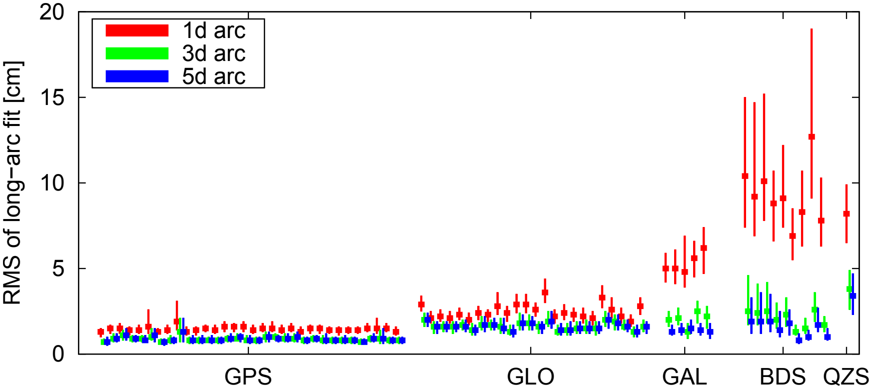
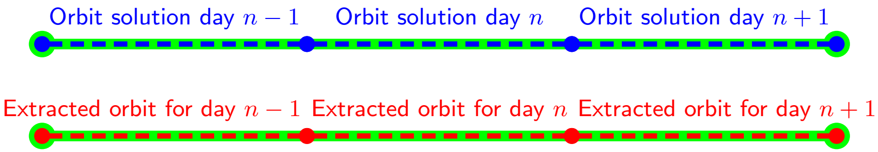
CODE MGEX solution for the year 2015



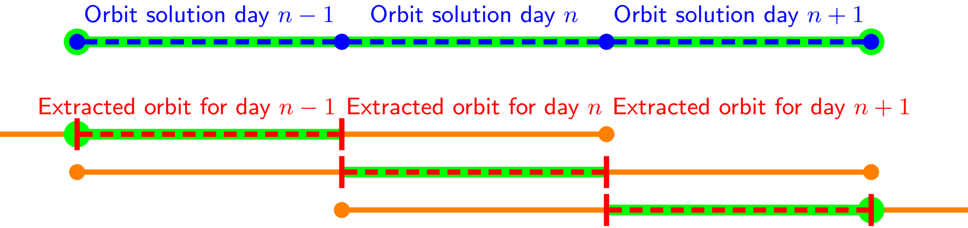
The multi-day long-arc solutions perform better than the one-day solutions for all satellites.

Validation by Fitting Long Arcs

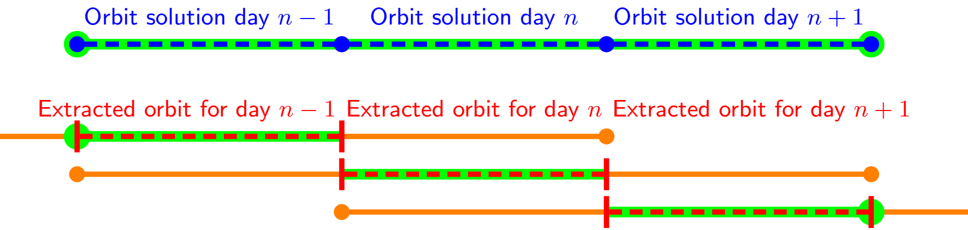
Validation by Fitting Long Arcs



Validation by Fitting Long Arcs



Validation by Fitting Long Arcs



Disadvantage of the "Extracted orbit for day n" with respect to the direct "Orbit solution day n":

- The orbits extracted from the three-day arc are not independent anymore.
- An orbit fit over several days cannot be used as a real quality indicator anymore.

Validation by Orbit Overlaps

1. Fitting long arcs

2. Orbit overlaps

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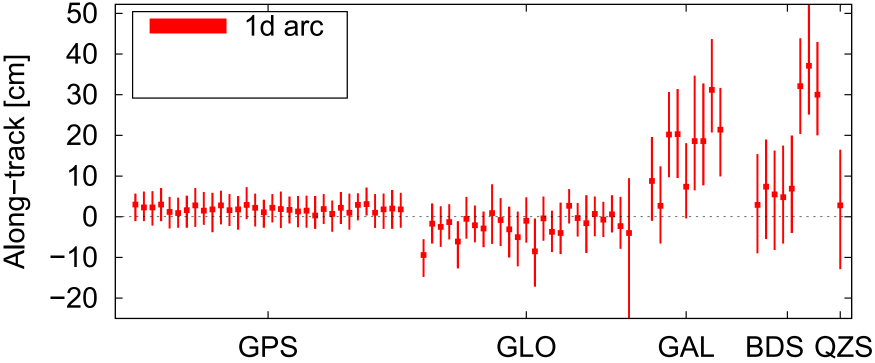
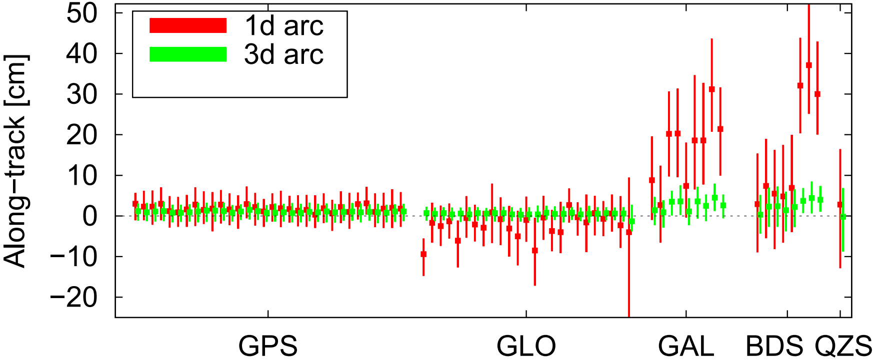
CODE MGEX solution for the year 2015



Median per satellite with associated quantiles

Validation by Orbit Overlaps

CODE MGEX solution for the year 2015



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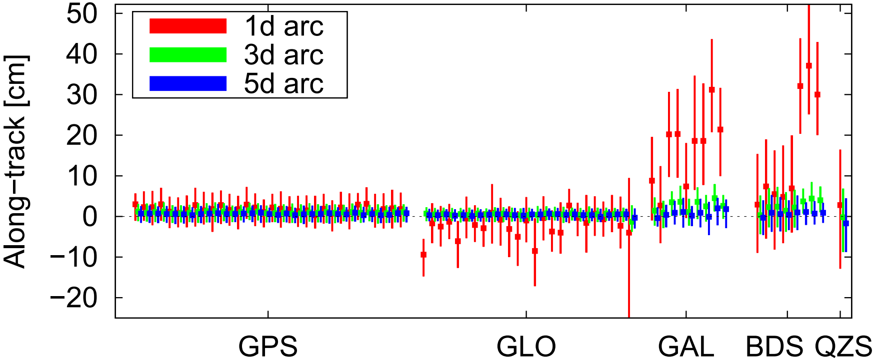

Median per satellite with associated quantiles

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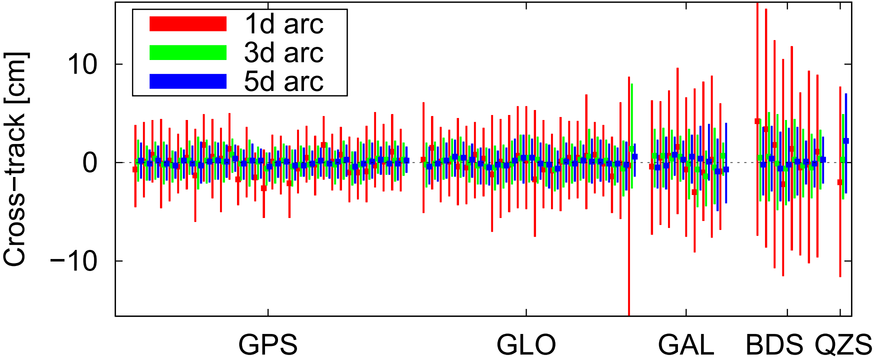

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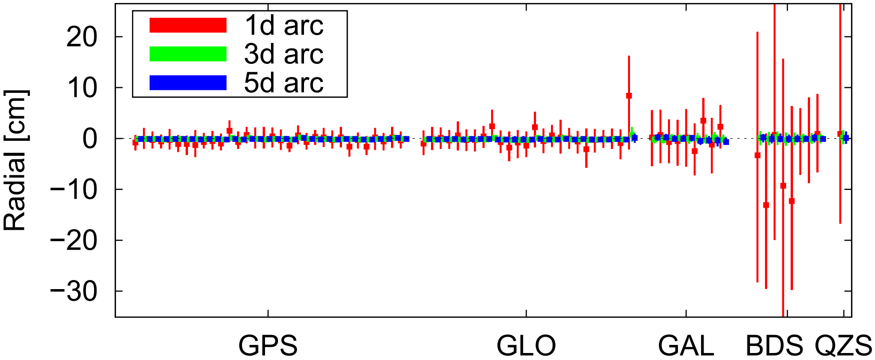

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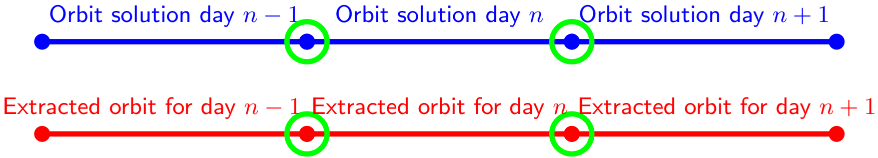

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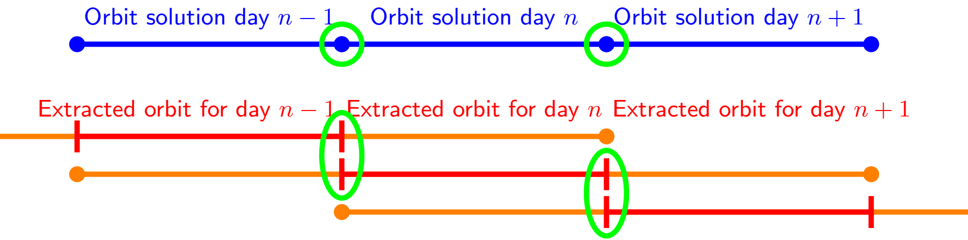

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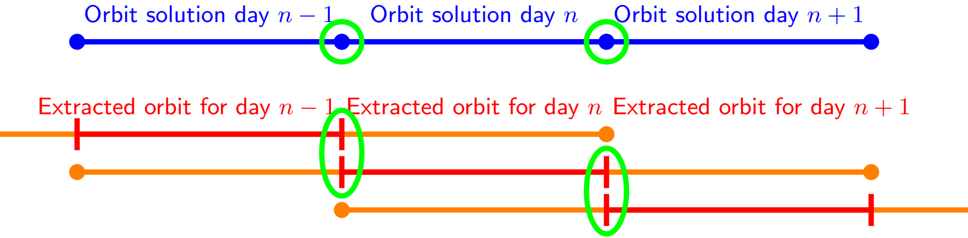


Disadvantage of the "Extracted orbit for day n" with respect to the direct "Orbit solution day n":

- The orbits extracted from the three-day arc are not independent anymore.
- Day boundary discontinuities cannot be used as a real quality indicator anymore.

Validation by SLR Measurements

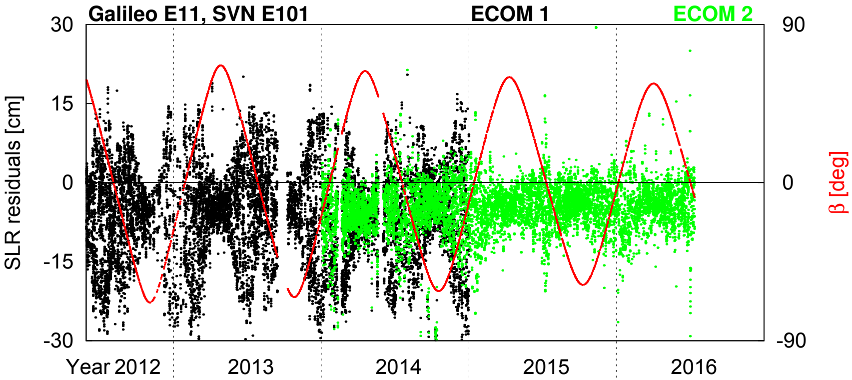
1. Fitting long arcs

2. Orbit overlaps

3. Comparison with independent measurements (e.g., SLR)
   - Consistency of the station coordinates between GNSS and SLR is required.
   - Biases of both techniques need to be known.
   - In case of problems an identification must be implemented to define which technique has caused the problem.

Validation by SLR Measurements

CODE MGEX solution (3d arc)



Validation by SLR Measurements

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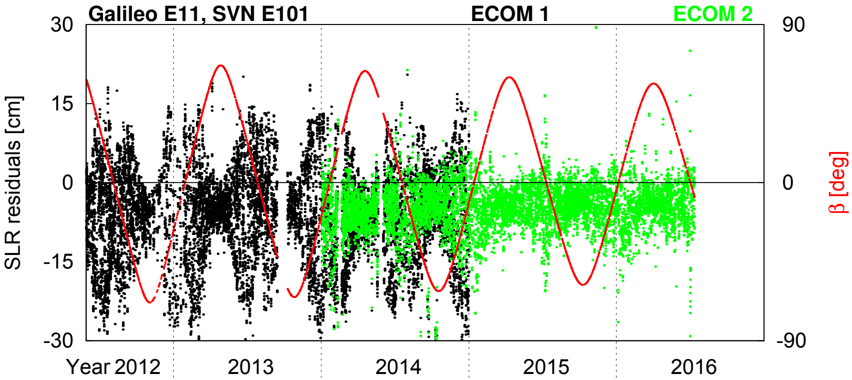
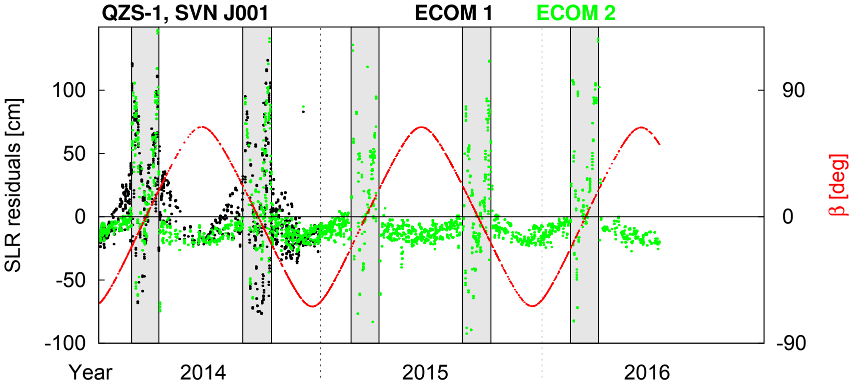


The new ECOM2 shows a clear improvement with respect to the old ECOM1 because of the stretched bodies of the Galileo satellites.

Validation by SLR Measurements

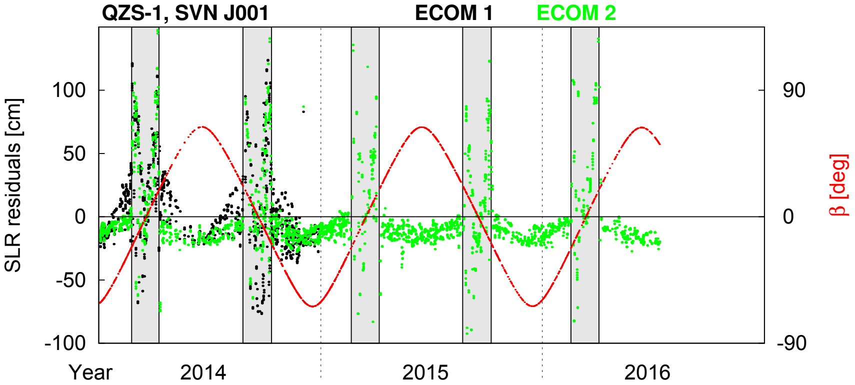
CODE MGEX solution (3d arc)



The new ECOM2 shows a clear improvement with respect to the old ECOM1 because of the stretched bodies of the QZSS satellites.

Validation by SLR Measurements

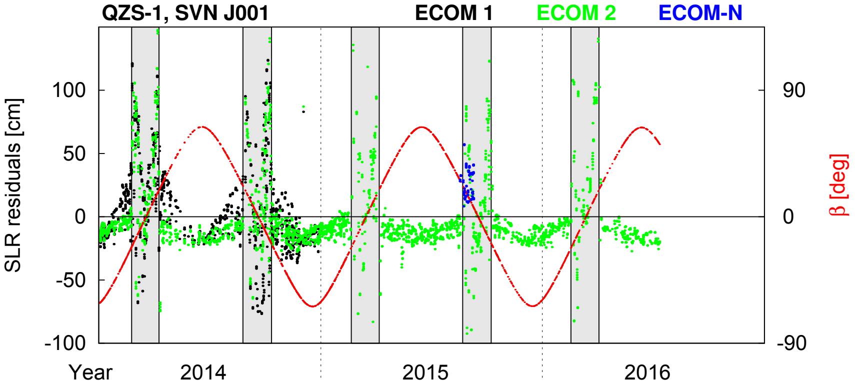
CODE MGEX solution (3d arc)



The ECOM2 decomposition is designed for the yaw-steering mode but not for the orbit normal mode.

Validation by SLR Measurements

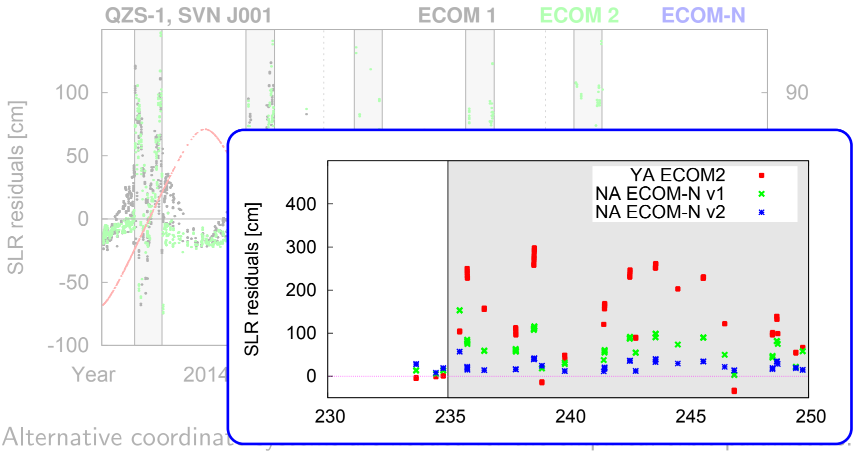
CODE MGEX solution (3d arc)



Alternative coordinate systems are needed for the empirical orbit parameters.

Validation by SLR Measurements

CODE MGEX solution (3d arc)



Alternative coordinat

Validation by Checking the Clock Performance

1. Fitting long arcs

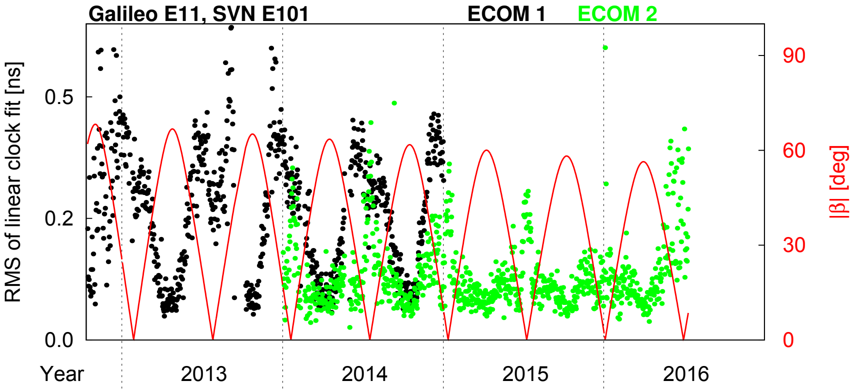
2. Orbit overlaps

3. Comparison with independent measurements (e.g., SLR)

4. Checking the performance of the GNSS satellite clock
   - Some of the GNSS satellites (Galileo, QZSS, GPS Block IIF) carry excellent clocks where a linear behaviour can be expected.
   - Orbit modelling problems (mainly in the radial component) may map into estimated satellite clock values.

Validation by Checking the Clock Performance

CODE MGEX solution (3d arc)



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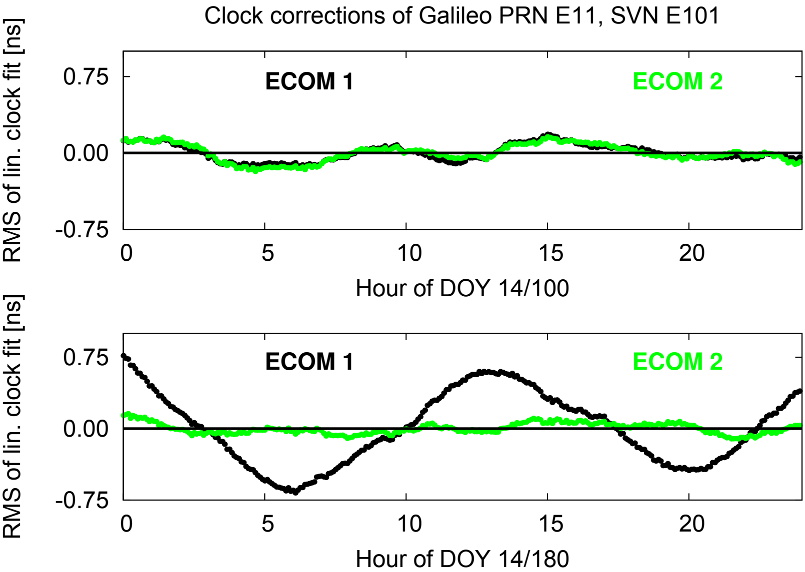

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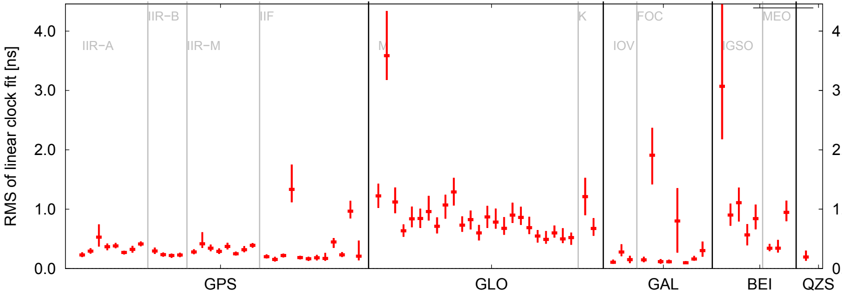

Median per satellite with associated quantiles

Validation by Checking the Clock Performance

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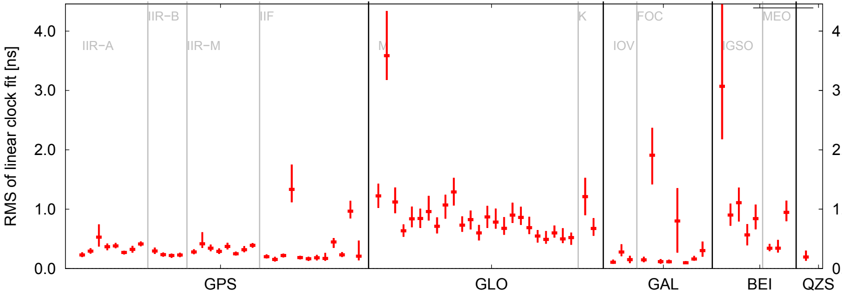

Median per satellite with associated quantiles

Not all GNSS satellite clocks perform well enough to serve for orbit validation purposes.

Handling of Repositioning Events

- Constellation keeping: GPS, GEO and IGSO satellites
- GPS Block IIF satellites during the injection procedure

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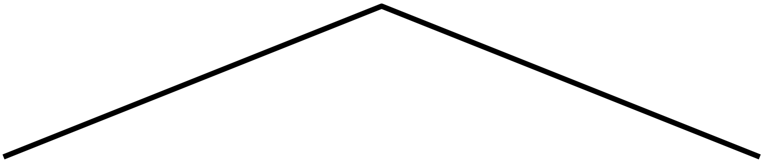

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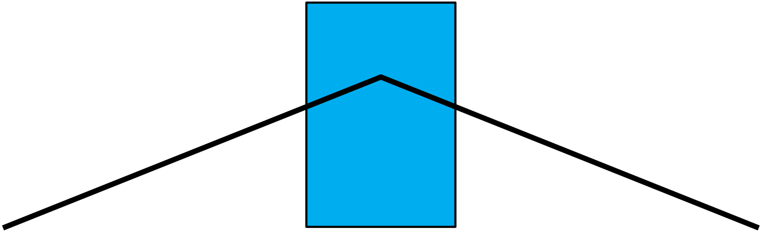

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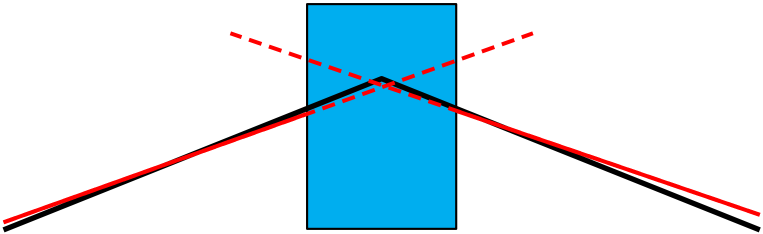

- Two independent satellite arcs are assumed (before and after the event)

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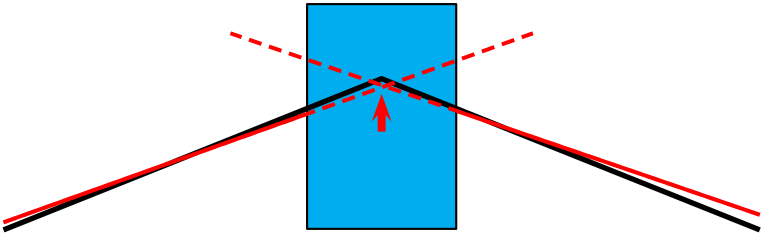
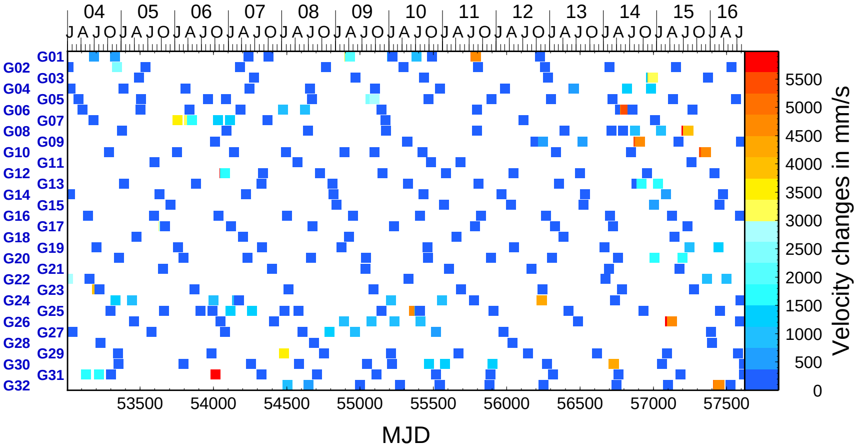
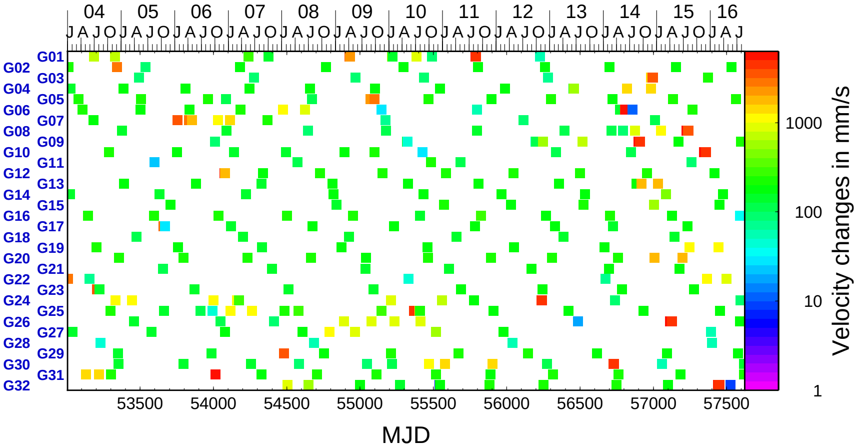


- Two independent satellite arcs are assumed (before and after the event)
- The smallest distance between both arcs gives the epoch and magnitude of the event.

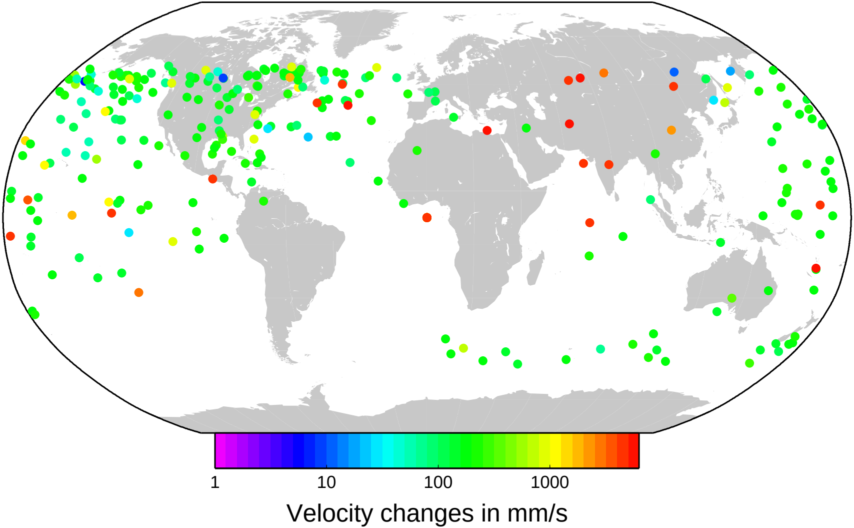
GPS Repositioning Events Estimated by CODE



GPS Repositioning Events Estimated by CODE



GPS Repositioning Events Estimated by CODE



GNSS Orbit Determination wihin the IGS

Introduction and Motivation

Overview on the GNSS Constellations

Effects Acting on Satellites and Related Models

Precise Orbit Determination for GNSS Satellites

GNSS Orbit Determination wihin the IGS

The IGS – a Service of the IAG

Precise GNSS satellite orbit determination is a challenging task requiring a global solution based on a well distributed network of stations.

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By 01. January 1994 the **IGS** was launched as an official service of the International Association of Geodesy (IAG).

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By 01. January 1994 the **IGS** was launched as an official service of the International Association of Geodesy (IAG).

IGS means:

- International GPS Service for Geodesy and Geodynamics
  January 1994
- International GPS Service
  May 1998
- International GNSS Service
  March 2005

The IGS and its Operational Orbit Products

Final series - ORB, ERP, CLK (300/30 sec. sampling), CRD

- available about two weeks after the end of the week
- GPS and GLONASS in compatible but independent series

Rapid series - ORB, ERP, CLK

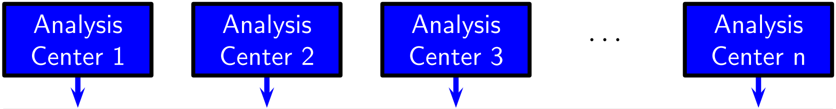
- available at the day after the measurements, 17:00 UTC
- quality very close to the final products

Ultra-rapid series - ORB, ERP, (CLK, 300 sec. sampling)

- four updates per day, latency 3 hours
- contains 24 hours estimated and 24 hours predicted orbits
- GLONASS series on an experimental stage

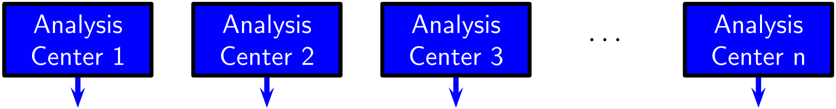
Combined IGS Products

Combined IGS Products



1. An unweighted mean orbit between the Analysis Centers is computed.

Combined IGS Products



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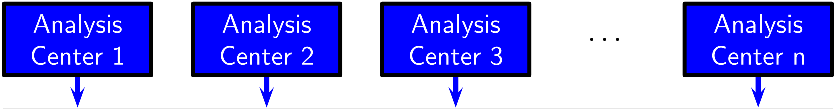
2. The standard deviation of each contribution to this mean orbit is computed to assign a weight to each Analysis Center.

Combined IGS Products



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Combined IGS Products

Analysis Center 1 | Analysis Center 2 | Analysis Center 3 | … | Analysis Center n

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Combined IGS orbit

R. Dach: GNSS Satellite Orbit Modelling
NGK Summer School, 29.Aug.–01. Sep. 2016, Båstad

Combined IGS Products

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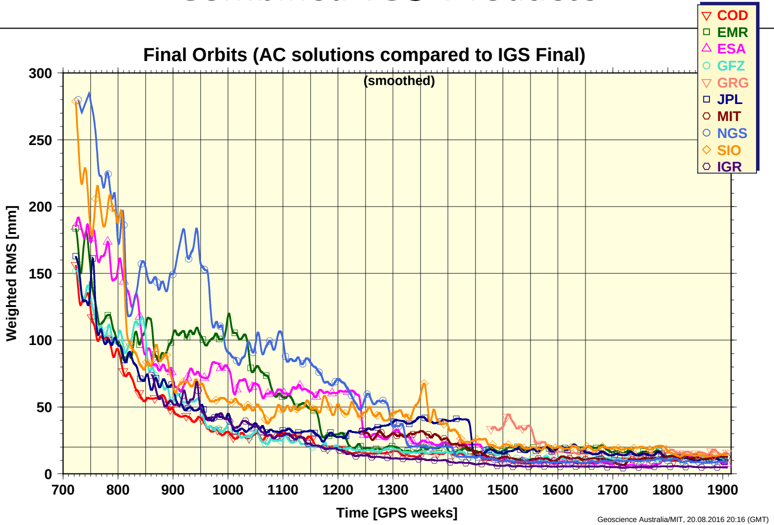
4. The mean errors and the transformation parameters of the individual solutions with respect to the IGS orbit are made available every week for each day of the (preceding) week.

↓

Combined IGS orbit

Combined IGS Products



Final Orbit Quality from November 1993 – August 2016 as computed by the IGS Analysis Center Coordinator (smoothed weekly RMS values).

Development of IGS Products

The consistency of the GNSS modelling between the individual Analysis Centers has significantly been increased during the last years.

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The biggest differences are currently in the GNSS satellite orbit modelling:

- All groups follow an empirical or semi-empirical approach where in most cases the parameters according to eqn. (4) are estimated.
- Significant differences exist in the a priori models that are introduced, e.g., for solar radiation pressure modelling.

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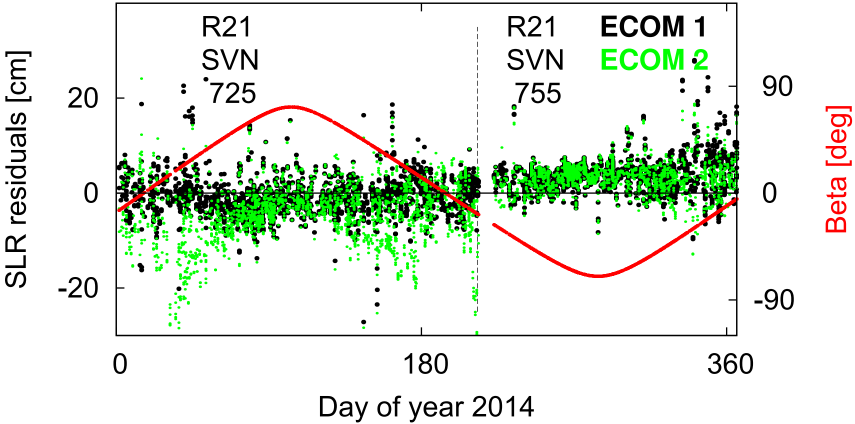
Many Analysis Centers focus currently on the development of their multi-GNSS processing capability.

The IGS needs also a multi-GNSS capable combination procedure.

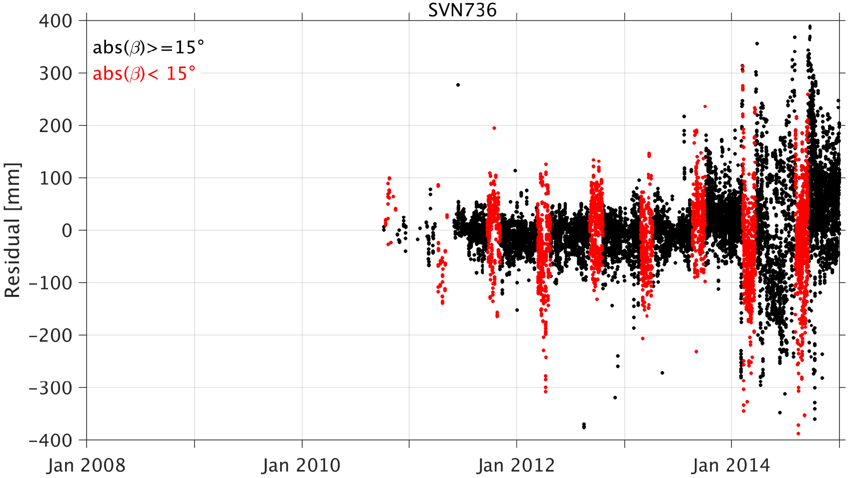
Other Challenges

The GLONASS miracle:



In the first part of the year the old ECOM1 outperforms the new ECOM2. This changes when a new satellite occupies the same slot in the constellation.

The GLONASS miracle:



Other Challenges

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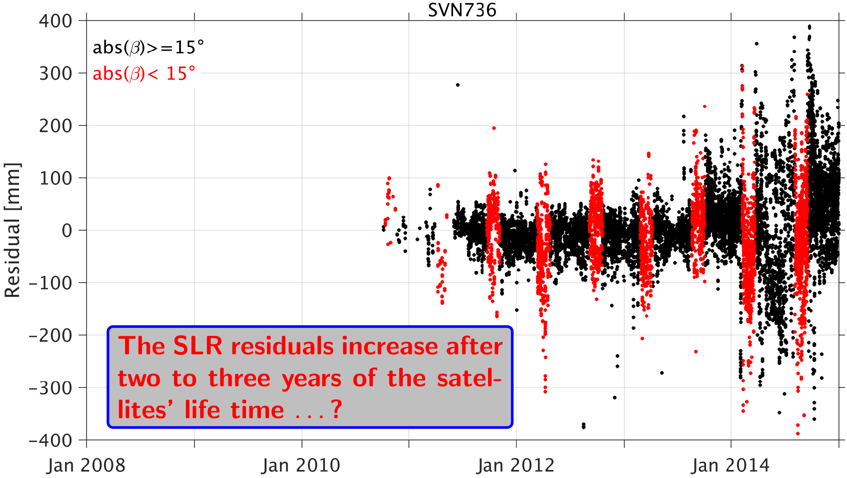

The SLR residuals increase after two to three years of the satellites' life time . . . ?

THANK YOU
for your attention

Publications of the satellite geodesy research group:

`http://www.bernese.unibe.ch/publist`