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## Earth gravity field recovery using GPS, GLONASS, and SLR satellites

Krzysztof Sośnica (1), Adrian Jäggi (1), Daniela Thaller (2), Ulrich Meyer (1),  
Christian Baumann (1), Rolf Dach (1), Gerhard Beutler (1)

(1) Astronomical Institute, University of Bern, Bern, Switzerland.

(2) Bundesamt für Kartographie und Geodäsie, Frankfurt am Main, Germany.  
sosnica@aiub.unibe.ch

**Abstract.** *The time variable Earth's gravity field provides the information about mass transport within the system Earth, i.e., the relationship of mass transport between atmosphere, oceans, and land hydrology. We recover the low-degree parameters of the time variable gravity field using microwave observations from GPS and GLONASS satellites and from SLR data to five geodetic satellites, namely LAGEOS-1/2, Starlette, Stella, and AJISAI.*

*GPS satellites are particularly sensitive to specific coefficients of the Earth's gravity field, because of the deep 2:1 orbital resonance with Earth rotation (two revolutions of the GPS satellites per sidereal day). The resonant coefficients cause, among other, a "secular" drift (actually periodic variations of very long periods) of the semi-major axes of up to 5.3 m/day of GPS satellites.*

*We processed 10 years of GPS and GLONASS data using the standard orbit models from the Center of Orbit Determination in Europe (CODE) with a simultaneous estimation of the Earth gravity field coefficients and other parameters, e.g., satellite orbit parameters, station coordinates, Earth rotation parameters, troposphere delays, etc. The weekly GNSS gravity solutions up to degree and order 4/4 are compared to the weekly SLR gravity field solutions. The SLR-derived geopotential coefficients are compared to monthly GRACE and CHAMP results.*

### Introduction

The main 'three pillars' of satellite geodesy include: (1) precise determination of geometrical three-dimensional positions and velocities (geometry), (2) modeling and observing of geodynamical phenomena including the rotation and orientation of the Earth (rotation), (3) determination of the Earth's gravity field and its temporal variations (gravity).

Even though all three pillars describe geodetic and geodynamic phenomena within the system Earth, the gravity has typically been treated separately from the geometry and rotation. E.g., the official products of the ILRS comprise SLR station coordinates, pole coordinates and the Length-of-Day excess (LoD) from the 7-day combined LAGEOS-Etalon solutions, whereas the gravity field parameters are not provided. In the solutions of the International GNSS Service (IGS), the gravity field parameters are not estimated as well. On the other hand, when estimating gravity field parameters from SLR data, the parameters related to geometry and rotation have typically been fixed and not simultaneously estimated (e.g., Devoti et al., 2001, Matsuo et al., 2013).

We present a simultaneous estimation of the gravity field, Earth rotation parameters, and station coordinates using microwave GNSS data and SLR observations to spherical geodetic satellites, namely LAGEOS-1/2, Starlette, Stella and AJISAI. We address benefits emerging from such an approach and discuss particular aspects and limitations of the gravity field recovery using GNSS or SLR data.

## Time-variable Earth's gravity field

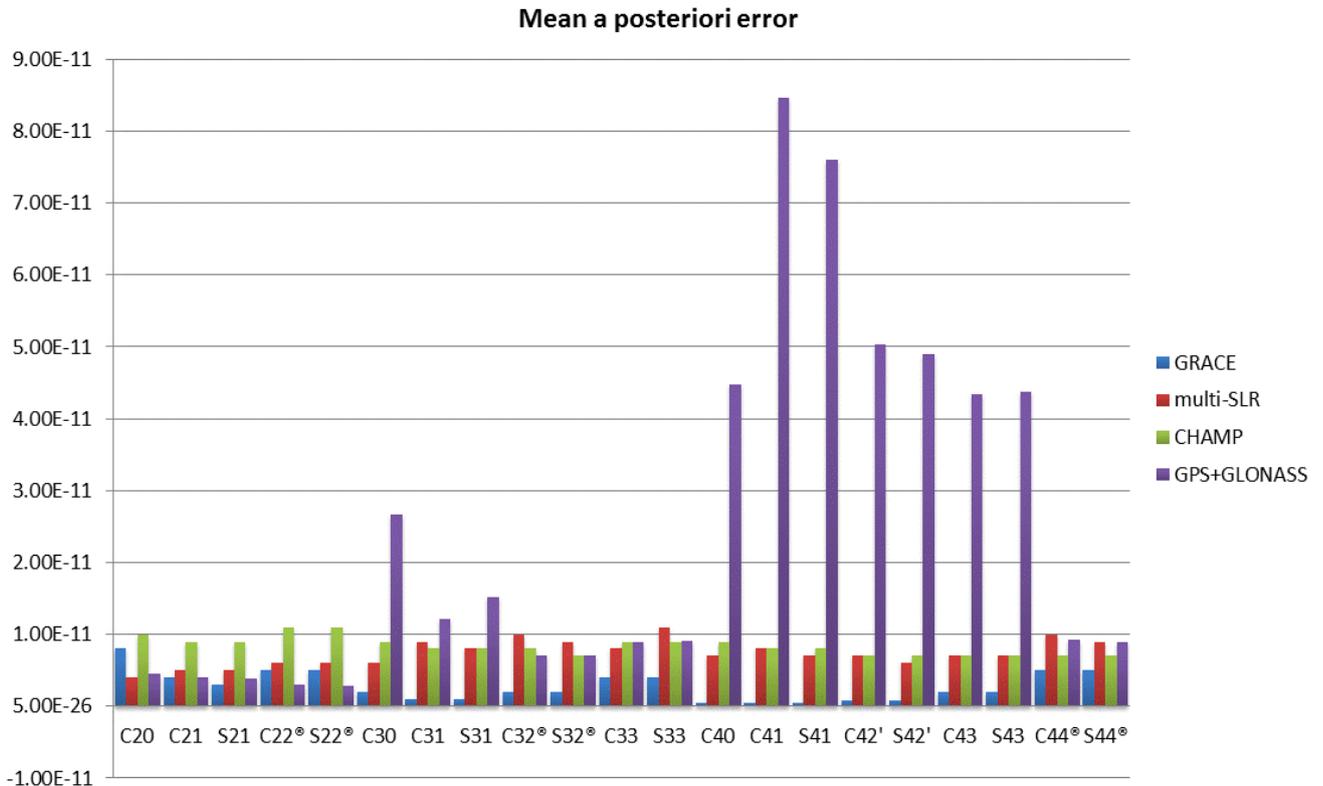
Time-variable Earth's gravity field can be determined from:

- K-Band GRACE observations,
- GPS-derived positions of LEO satellites (e.g., CHAMP),
- dynamic orbit perturbations:
  - using SLR to geodetic satellites,
  - using GPS + GLONASS (GNSS) microwave observations.

In this paper we determine the low-degree time-variable Earth's gravity field coefficients using the dynamic orbit perturbations of SLR and GNSS satellites and we compare the geopotential coefficients to the GRACE (Meyer et al., 2012) and CHAMP-derived results (Weigelt et al., 2013). The sensitivity of high-orbiting GNSS satellites to medium and high degree gravity field coefficients is low, due to their orbital altitude. GPS satellites are, however, particularly sensitive to specific gravity field coefficients (see Figure 1), because of the deep 2:1 orbital resonance with Earth rotation. The resonant coefficients cause, among other, a “secular” drift of the semi-major axes of up to 5.3 m/day of GPS satellites (Ineichen et al., 2003, Beutler 2005).

We processed 10 years of GPS and GLONASS data using the standard orbit modeling as from the IGS Analysis Center: Center of Orbit Determination in Europe (CODE, Dach et al., 2009) with two major exceptions:

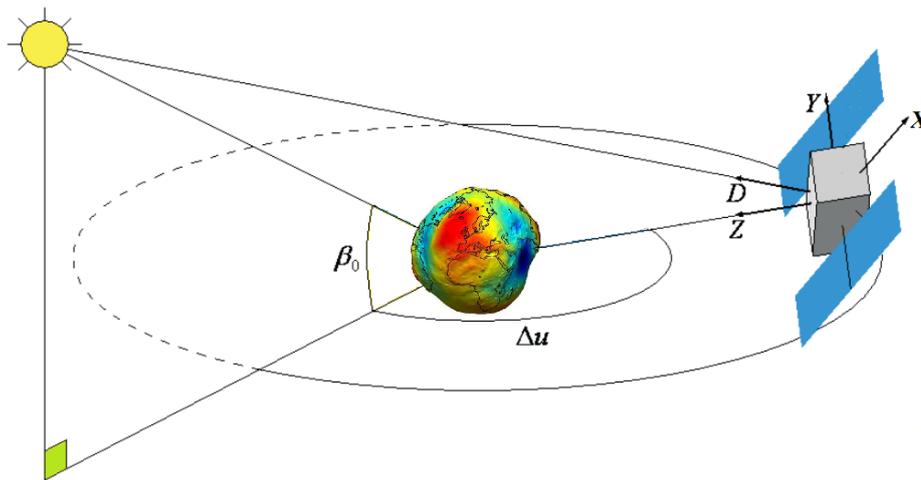
- 7-day solutions are generated instead of the standard IGS 1-day solutions,
- the Earth gravity field coefficients up to degree/order 4/4 are simultaneously estimated along with other parameters, e.g., satellite orbit parameters, station coordinates, Earth rotation parameters, etc. (see Table 1).



**Figure 1.** Mean a posteriori errors from gravity field determination using different techniques: 7-day SLR and GNSS solutions and 30-day CHAMP and GRACE solutions. ® and ' denote resonant coefficients which cause a secular drift of the semi-major axis of GPS satellites, and resonant coefficients which do not cause a drift, respectively.

**Table 1.** List of estimated parameters in the 7-day GNSS and 7-day SLR gravity field solutions. The modeling standards follow the IERS 2010 Conventions in both solutions. 7-day GNSS solutions are generated by stacking seven 3-day NEQs with overlapping orbits (by stacking all parameters and pre-eliminating the 3-day arc orbits). SLR solutions follow the method described in Sośnica et al., (2013a) and Sośnica et al., (2014).

Estimated parameters		GPS+GLONASS (GNSS) solutions	SLR solutions
		up to 32 GPS and 24 GLONASS satellites	LAGEOS-1/2, Starlette, Stella, Ajisai
Orbits	Osculating elements	a, e, i, $\Omega$ , $\omega$ , $u_0$ (1 set per 3 days)	a, e, i, $\Omega$ , $\omega$ , $u_0$ (1 set per 7 days)
	Dynamical parameters	$D_0$ , $Y_0$ , $X_0$ , $X_S$ , $X_C$ (1 set per 3 days), see Fig. 2	LAGEOS-1/2: $S_0$ , $S_C$ , $S_S$ (1 set per 7 days) Sta/Ste/Aji: $C_D$ , $S_C$ , $S_S$ , $W_C$ , $W_S$ (1 set per day)
	Pseudo-stochastic pulses	R, S, W (once per revolution)	LAGEOS-1/2: no pulses Sta/Ste/Aji: S (once per revolution)
Earth rotation parameters		$X_P$ , $Y_P$ , UT1-UTC (1 set per day)	$X_P$ , $Y_P$ , UT1-UTC (1 set per day)
Geocenter coordinates		1 set per 7 days	1 set per 7 days
Earth gravity field		Estimated up to d/o 4/4 (1 set per 7 days)	Estimated up to d/o 4/4 (1 set per 7 days)
Station coordinates		1 set per 7 days	1 set per 7 days
Other parameters		Troposphere ZD (2h), gradients (24h), GNSS-specific translations and ZTD biases	Range biases for selected stations



**Figure 2.** Sun fixed reference frame for dynamical orbit parameters of GNSS satellites (Beutler et al., 1994). GNSS dynamic orbit parameters estimated in standard CODE solutions read as follows:

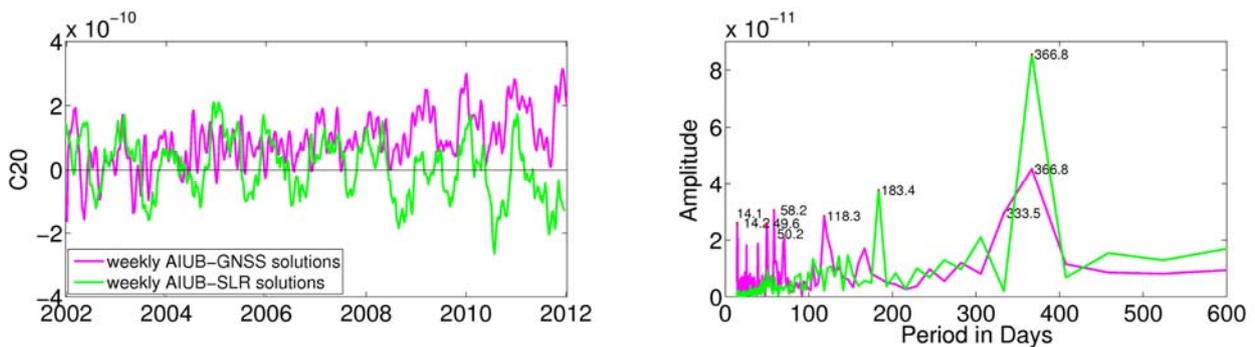
$$D = D_0; \quad Y = Y_0; \quad X = X_0 + X_S \sin \Delta u + X_C \cos \Delta u.$$

## Gravity field from GPS and GLONASS

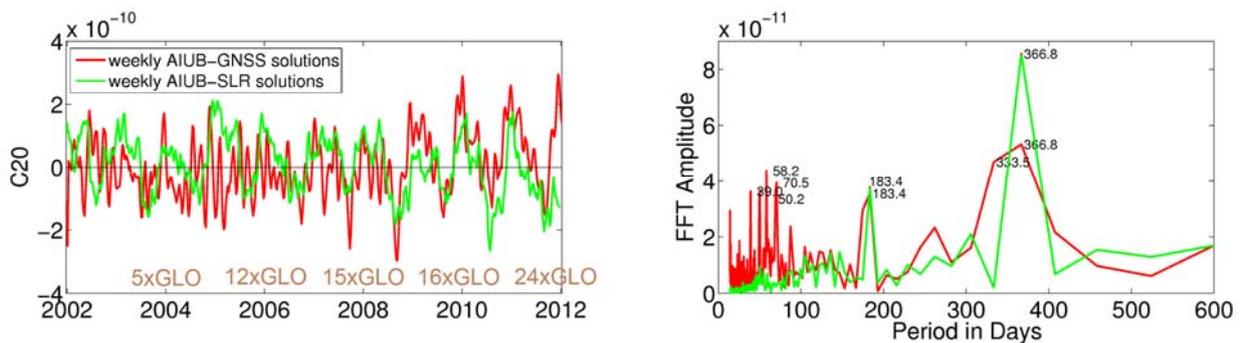
Figure 3 shows that the variations of  $C_{20}$  are not fully recovered from GNSS applying the standard CODE orbit parameterization, which is reflected in substantially smaller amplitudes of annual and semiannual signals as compared to the SLR solutions. Figure 4 shows that  $C_{20}$  can be much better determined from the GNSS solutions if the constant and once-per-revolution parameters in the X direction are not estimated: the semi-annual signal is well reproduced, the 3rd harmonic of 118 days disappears, and the correlation coefficient between the SLR and GNSS series increases from 0.02 to 0.28. A very good agreement between SLR and GNSS solutions is observed in particular for the period after 2008 when the contribution of GLONASS satellites becomes prominent and the GLONASS-observing network becomes more global (Sośnica et al., 2013b).

It is important to avoid the estimation of both, constant X and once-per-revolution orbit parameters in the X direction, because both parameters are correlated with  $C_{20}$  and all solutions with estimating one or both of these parameters result in inappropriate  $C_{20}$  estimates (as in Fig. 3).

The spectral analysis shows the 2<sup>nd</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, 6<sup>th</sup>, and 7<sup>th</sup> harmonics of the draconitic year in most of the GNSS-derived coefficients (Fig. 3-4). The amplitudes of these harmonics can be reduced for some parameters when not estimating  $X_0$ ,  $X_S$ ,  $X_C$  (see Fig. 8). The quality of other estimated parameters, e.g., ERPs and station coordinates, are, however, slightly degraded when  $X_0$ ,  $X_S$ ,  $X_C$  are not estimated.



**Figure 3.** Variations of  $C_{20}$  w.r.t. EGM2008 from the GNSS solutions with standard CODE modeling, compared to the SLR results for the time span 2002.0-2012.0.



**Figure 4.** Variations of  $C_{20}$  w.r.t. EGM2008 from the GNSS solutions without estimating constant and once-per-rev dynamical orbit parameters in the X direction, compared to the SLR results. The number of active GLONASS satellites is indicated in brown.

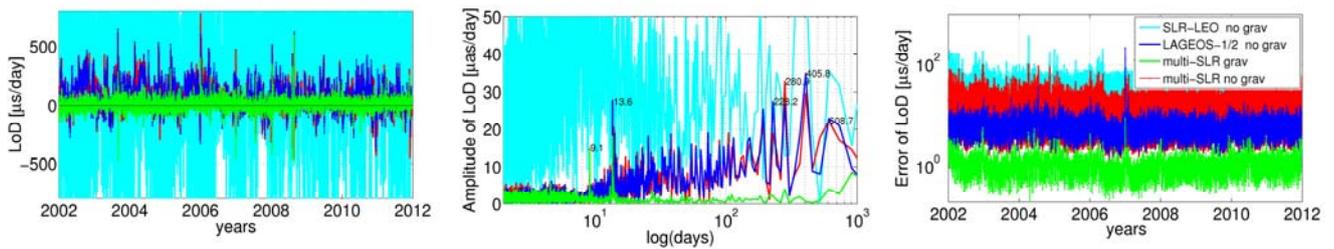
## Simultaneous estimation of gravity field along with other geodetic parameters

Before analyzing the SLR-derived Earth's gravity field, a critical question has to be answered: What is the benefit of a simultaneous determination of geopotential coefficients for other parameters derived from the SLR solutions? We study the particular case of LoD for the LAGEOS-1/2 solutions and the multi-SLR solutions incorporating LAGEOS-1/2, Starlette, Stella, and AJISAI.

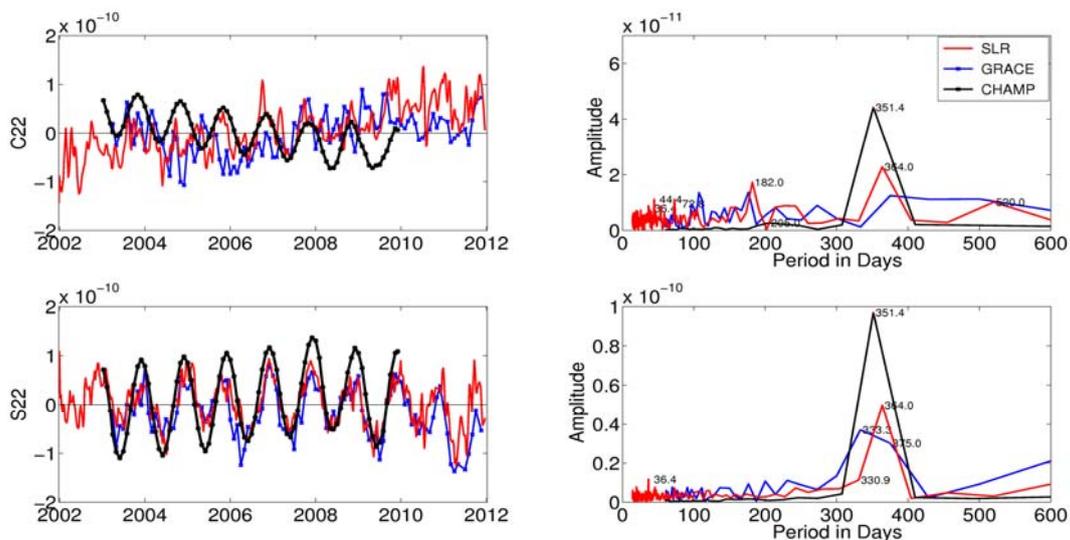
Figure 5 shows that for LoD the simultaneous estimation of the gravity field parameters:

- reduces the offset of LoD estimates w.r.t. IERS-08-C04 series (Figure 5, left), which is mostly due to absorption of the  $C_{20}$  variations by the LoD estimates (Thaller et al., 2013),
- reduces peaks in the spectrum analysis (Figure 5, middle), which correspond, e.g., to orbit modeling deficiencies (peaks of 222 days, i.e., a draconitic year of LAGEOS-2, 280 days, i.e., an eclipsing period of LAGEOS-1 or  $S_2$  alias period with LAGEOS-1 orbits),
- substantially reduces the a posteriori error of estimated LoD (Figure 5 right, notice a logarithmic scale for the y axis).

The a posteriori error of LoD in the multi-SLR solutions ( $16.9 \mu\text{s/day}$ ) is more than a factor of two higher than in the LAGEOS-1/2 solutions ( $7.1 \mu\text{s/day}$ ) when the gravity field parameters are not estimated. This quality degradation implies that the estimation of the gravity field parameters is essential for high-quality LoD estimates when using SLR data to low-orbiting geodetic satellites. When estimating the gravity field parameters the a posteriori error of LoD is remarkably reduced to  $1.3 \mu\text{s/day}$  in the multi-SLR solutions (i.e., a factor of 13 less than in the solution without gravity).



**Figure 5.** Differences of the LoD estimates w.r.t. IERS-08-C04 series (left), spectral analysis of the differences (middle), a posteriori errors of LoD estimates (right).



**Figure 6.**  $C_{22}$  and  $S_{22}$  gravity field coefficients from SLR, GRACE (Meyer et al., 2012), and CHAMP solutions (Weigelt et al., 2013).

## Gravity field from SLR

Most of the gravity field coefficients agree well between SLR and GRACE solutions (see, e.g.,  $C_{22}$  and  $S_{22}$  in Figure 6). The spectral analysis reveals, however, small aliasing problems in the SLR solutions. The periodogram shows peaks around 36 and 44 days corresponding to the  $S_2$  aliasing period with the orbits of Starlette and AJISAI, respectively. Thus, the deficiencies in the  $S_2$  tide modeling imply not only problems in the recovery of some gravity field coefficients from GRACE, but also disrupt the SLR solutions to a small extent. The CHAMP solutions overestimate the annual signal in both, the  $C_{22}$  and  $S_{22}$  coefficients, and they show an opposite drift in  $C_{22}$  as compared to the SLR and GRACE solutions.

## Summary

The gravity field determination using GPS+GLONASS data is very promising, but requires further investigations concerning orbit parameterization and the correlations between empirical orbit and gravity field parameters (e.g.,  $C_{20}$  &  $X_0$ ,  $X_S$ ,  $X_C$ ). Simultaneous estimation of gravity field with other SLR-derived parameters is in particular beneficial for the LoD estimates. When co-estimating the gravity field with other parameters the a posteriori error of LoD is one order of magnitude smaller in the multi-SLR solutions, as compared to the solution in which gravity field parameters are not estimated. Moreover, there is no offset of LoD estimates w.r.t. the IERS-08-C04 series and the spectrum analysis is free from the peaks that correspond, e.g., to orbit modeling deficiencies when the gravity field parameters are co-estimated.

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