

# Combined LARES-LAGEOS Solutions

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## Laser Relativity Satellite (LARES)

LARES is a new spherical geodetic satellite designed for SLR observations. It is made of solid tungsten alloy covered with 92 corner cubes (Fig.1). Due to a very small area-to-mass ratio (Tab.1), the sensitivity of LARES orbits to non-gravitational forces is greatly minimized. We process 82 weeks (Feb12-Aug13) of LARES observations from a global SLR network and we analyze the contribution of LARES data to the current SLR products (global scale, geocenter coordinates, station coordinates, Earth rotation and gravity field parameters). The quality of the combined LARES+LAGEOS-1/2 solutions is also addressed.

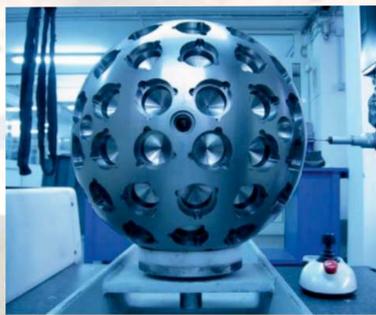


Fig. 1: LARES before embedding the corner cube retro-reflectors (courtesy of ASI)

	LAGEOS-1	LAGEOS-2	AJISAI	LARES	Starlette	Stella
Diameter [m]	0.60	0.60	2.15	0.36	0.24	0.24
Mass [kg]	407	405	685	386.8	47	48
Area-to-mass [m <sup>2</sup> /kg]	6.90E-04	7.00E-04	5.80E-03	2.70E-04	9.60E-04	9.40E-04
Radiation coeff. C <sub>R</sub>	1.13	1.12	1.03	1.07	1.134	1.131
Semi-major axis [km]	12274	12158	7866	7820	7334	7176
Orbit altitude [km]	5860	5620	1495	1450	812-1113	805
Eccentricity	0.0039	0.0137	0.0016	0.0007	0.0205	0.0010
Inclination [deg]	109.90	52.67	50.04	69.50	49.84	98.57
Draconitic year [days]	560	222	89	133	73	182
Drift of node [days]	1050	570	117	210	91	364
Drift of perigee [days]	1694	821	141	376	109	122
Decay of semi-major axis [m/y]	-0.203	-0.239	-12.000	-0.775	-14.000	-30.000

Tab. 1: Characteristics of geodetic SLR satellites. Draconitic years, nodal and perigee's drifts are estimated on the basis of first order orbit perturbations. Secular decays of semi-major axes are estimated on the basis of linear fit to time series of mean semi-major axes.

## LARES orbits

Table 2 shows the perturbing accelerations of gravitational, non-gravitational, and general relativistic origin, acting on geodetic satellites. Comparing LARES and AJISAI (two satellites of similar altitudes), the impact of gravitational accelerations is nearly the same, whereas the impact of non-gravitational accelerations is about 22 times smaller for LARES than for AJISAI. Thus, LARES orbits are remarkably well suited for the recovery of the Earth's gravity field or for the verification of the Lense-Thirring effect.

Figures 2-4 show the evolution of LARES mean orbital elements (eccentricity, inclination, semi-major axis, respectively). The secular drift of the LARES semi-major axis is mostly caused by the atmospheric drag, as opposed to the Yarkovsky effect for LAGEOS satellites. Nevertheless, the LARES' drift is about 16 times smaller than the AJISAI's drift. The residual LARES' along-track acceleration is just  $-3E-12$ , i.e., 47 times less than the impact of the Lense-Thirring effect.

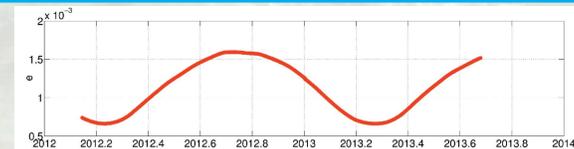


Fig. 2: Mean orbital eccentricity of LARES (the period of variations corresponds to the period of the perigee drift: 376 days).

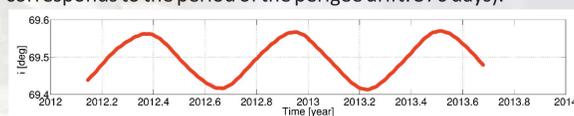
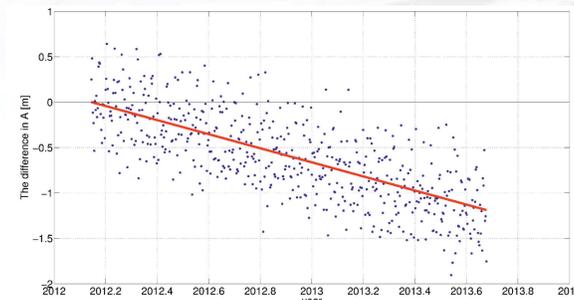


Fig. 3: Mean inclination of LARES (the period of variations corresponds to the period of the nodal drift: 210 days).



	LAGEOS-1/2	AJISAI	LARES	Star/Stella
Gravitational perturbations:	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]
Earth's monopole	2.7	6.4	6.4	7.7
C <sub>20</sub>	1.0E-03	6.2E-03	6.3E-03	8.8E-03
C <sub>22</sub>	6.0E-06	3.6E-05	3.7E-05	5.1E-05
C <sub>66</sub>	8.6E-08	3.1E-06	3.1E-06	6.3E-06
C <sub>20 20</sub>	8.1E-13	1.5E-08	1.6E-08	1.1E-07
Attraction of Moon	2.1E-06	1.9E-06	1.9E-06	1.8E-06
Attraction of Sun	9.6E-07	9.6E-07	9.6E-07	9.6E-07
Attraction of Venus	1.3E-10	1.3E-10	1.3E-10	1.3E-10
General relativity:	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]
Schwarzschild effect	2.8E-09	1.1E-08	1.1E-08	1.4E-08
Lense-Thirring effect	2.7E-11	1.3E-10	1.4E-10	1.8E-10
Geodetic precession	3.4E-11	4.2E-11	4.2E-11	4.3E-11
Non-gravitational perturbations:	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]	[m/s <sup>2</sup> ]
Solar radiation pressure	3.2E-09	2.5E-08	1.1E-09	4.4E-09
Albedo-infrared radiation	4.4E-10	8.6E-09	3.9E-10	1.8E-09
Thermal reradiation	5.0E-11	4.1E-10	1.9E-11	6.9E-11
Light aberration	1.1E-13	1.1E-12	5.1E-14	2.0E-13
Atmospheric drag (min)	8.0E-15	3.0E-11	2.6E-12	5.0E-11
Atmospheric drag (max)	2.0E-13	5.9E-10	4.8E-11	5.0E-08

Tab. 2: Perturbing accelerations acting on geodetic satellites.

Fig. 4: ← Mean semi-major axis of LARES with a linear fit. The estimated secular decay is  $-0.775 \pm 0.143$  m/year.

## Correlations

Figure 5 shows the correlation matrices of the LAGEOS, LARES, and combined solutions. LAGEOS solutions are affected, in particular, by the correlations between:

- > station coordinates & orbits,
- > orbits & Earth rotation parameters (ERPs),
- > orbits & zonal spherical harmonics (C<sub>20</sub>, C<sub>30</sub>, C<sub>40</sub>),
- > Length-of-Day (LoD) & even zonal spherical harmonics of gravity field (C<sub>20</sub>, C<sub>40</sub>),
- > ERPs & ERPs,
- > LAGEOS-1 orbits & LAGEOS-2 orbits.

All these correlations are substantially reduced in the combined LARES+LAGEOS-1/2 solutions.

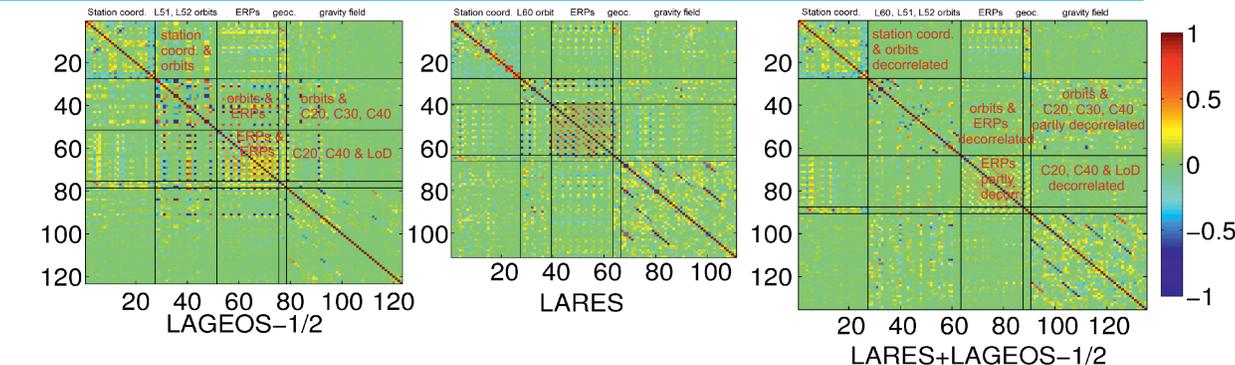


Fig. 5: Correlation matrices of LAGEOS-1/2, LARES-only, and LARES+LAGEOS-1/2 weekly solutions. The matrices contain the core station coordinates, satellites orbits, Earth rotation parameters, geocenter coordinates, and gravity field parameters up to degree/order 6/6. All remaining parameters were pre-eliminated (range biases, pseudo-stochastic pulses, non-core station coordinates).

## Combined multi-satellite solutions

The scale and origin (geocenter coordinates) of the international terrestrial reference frame (ITRF) are determined by the SLR observations. Thus, the highest quality of these parameters is crucial in the SLR solutions.

Figure 6 shows the scale from the LARES, LAGEOS, and combined LARES+LAGEOS solutions with a corresponding spectral analysis. The LARES-only scale is noisy and shows some deficiencies in orbit modeling, namely in modeling of the non-gravitational forces which are reflected in the draconitic year of the LARES satellite (the time interval between two consecutive passes of the Sun through the orbital plane). The scale defined by LAGEOS-1/2 is stable, but also show the variations related to the draconitic year of LAGEOS-2. The orbit modeling deficiencies are substantially reduced in the combined LARES+LAGEOS-1/2 solutions, resulting in the most stable scale estimates.

Figure 7 shows that in the combined solutions the offset w.r.t. SLRF2008 is smaller for the Y component, as compared to LAGEOS-1/2 solutions.

Moreover, the recovery of gravity field parameters can be greatly improved when including LARES data (Fig.8, compare the Star+Ste+Aji solution with the LARES+Star+Ste+Aji), because of improved observation geometry and large LARES sensitivity to the gravity field.

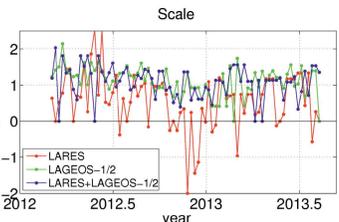


Fig. 6: Scale of reference frame derived from the Helmert transformation of SLR core stations.

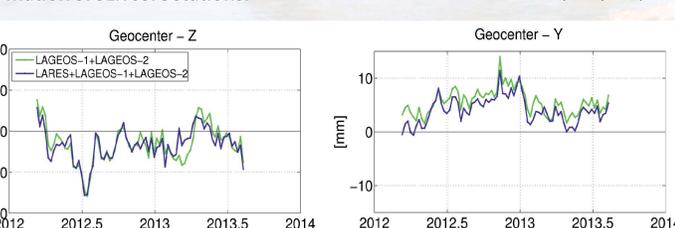


Fig. 7: Geocenter coordinates (Z and Y component) from the LAGEOS-1/2 and LARES+LAGEOS-1/2 solutions with respect to the SLRF2008.

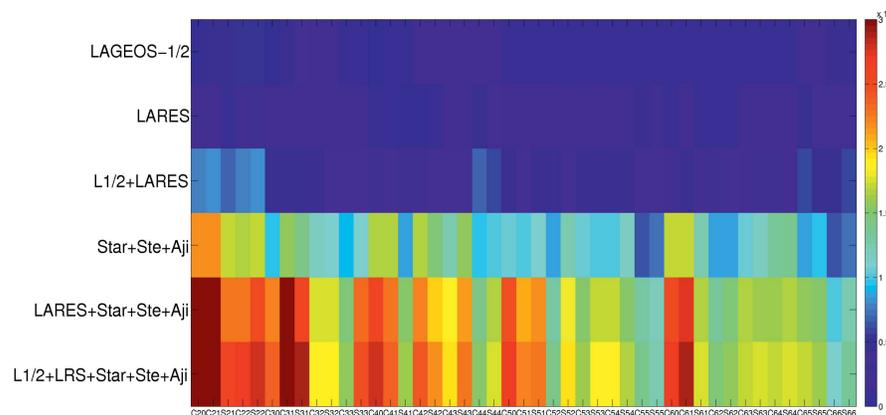


Fig. 8: Sensitivity of the SLR solutions to low-degree Earth gravity field parameters (C<sub>20</sub>-S<sub>66</sub>) as square roots of diagonal elements of normal equation systems. Compare the Star+Ste+Aji solution (line 4) with the LARES+Star+Ste+Aji (line 5) to see a substantial LARES contribution to determination of zonal terms: C<sub>20</sub>, C<sub>30</sub>, C<sub>40</sub>, C<sub>50</sub>, C<sub>60</sub>, and some of tesseral terms, e.g., C<sub>21</sub>, C<sub>31</sub>, S<sub>21</sub>, C<sub>32</sub>, C<sub>61</sub>.

## Conclusions

- > In the combined LARES+LAGEOS solutions, the correlations between parameters are reduced and the global scale is less affected by the deficiencies in LAGEOS orbit modeling.
- > A very small impact of non-gravitational forces and high orbit stability of LARES will allow the recovery of the Earth's gravity field and the validation of the effects of general relativity.

## References

- Sośnica K, Jäggi A, Thaller D, Dach R, Beutler G (2013) Contribution of Starlette, Stella, and AJISAI to the SLR-derived global reference frame. Submitted to J Geod
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