Non–Gravitational Forces Acting on Geodetic Satellites

Krzysztof Sośnica

Astronomical Institute, University of Bern, Switzerland

ETH Zurich, 14.11.2012
# Table of content

- Geodetic satellites
- Thermal effects
  - Yarkovsky effect
  - Yarkovsky–Schach effect
- Air drag
- Albedo
  - Reflectivity
  - Emissivity
- Solution combining many geodetic satellites
# Geodetic satellites

<table>
<thead>
<tr>
<th></th>
<th>AJISAI</th>
<th>Starlette</th>
<th>Stella</th>
<th>LAGEOS-1</th>
<th>LAGEOS-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter [m]</td>
<td>2.15</td>
<td>0.24</td>
<td>0.24</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Mass [kg]</td>
<td>685</td>
<td>47</td>
<td>48</td>
<td>407</td>
<td>405</td>
</tr>
<tr>
<td>Area-to-mass [m²/kg]</td>
<td>58.0e-4</td>
<td>9.6e-4</td>
<td>9.4e-4</td>
<td>6.9e-4</td>
<td>7.0e-4</td>
</tr>
<tr>
<td>Radiation coeff. $C_R$</td>
<td>1.03</td>
<td>1.134</td>
<td>1.131</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td>Semi-major axis [m]</td>
<td>7’866’500</td>
<td>7’334’700</td>
<td>7’176’100</td>
<td>12’274’000</td>
<td>12’158’000</td>
</tr>
<tr>
<td>Orbit altitude [m]</td>
<td>1’500’000</td>
<td>800’000 - 110’000</td>
<td>830’000</td>
<td>5’860’000</td>
<td>5’620’000</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.0016</td>
<td>0.0205</td>
<td>0.0010</td>
<td>0.0039</td>
<td>0.0137</td>
</tr>
<tr>
<td>Inclination [deg]</td>
<td>50.04</td>
<td>49.84</td>
<td>98.57</td>
<td>109.90</td>
<td>52.67</td>
</tr>
<tr>
<td>Draconitic year [days]</td>
<td>89</td>
<td>73</td>
<td>57</td>
<td>560</td>
<td>222</td>
</tr>
</tbody>
</table>

+ Etalon–1/2

---

Sosnički K.: Non-Gravitational Forces Acting on Geodetic Satellites
Seminar Presentation, ETH Zürich, 14.11.2012

Astronomical Institute University of Bern
New geodetic satellites

Blits – launched in September 2009

Blits–M to be launched in 2013

LARES – launched in February 2012

LARES–2 to be launched in 2014 (?)
Network of SLR stations

Regular observations: Yarragadee, Herstmonceux, Greenbelt, McDonald, Monument Peak, Graz, Zimmerwald, Wettzell, Changchung, Potsdam, Grasse, Matera, Mt. Stromlo, Haleakala, Arequipa, Riyadh, Hartebeesthoek, Borowiec, Simosato, San Fernando
Barycenter of SLR network

![Map showing the Barycenter of SLR network](image)

Number of stations

![Diagram showing the number of stations over time](image)

SLR Barycenter-Geocenter offset

![Graph showing the SLR Barycenter-Geocenter offset over time](image)
### Semi-major axis of LAGEOS-1

<table>
<thead>
<tr>
<th>Estimated parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ Station coordinates</td>
</tr>
<tr>
<td>➢ Osculating orbital elements</td>
</tr>
<tr>
<td>➢ Dynamic orbit parameters</td>
</tr>
<tr>
<td>(WC/WS, S0, SC/SS)</td>
</tr>
<tr>
<td>➢ Earth rotation parameters</td>
</tr>
<tr>
<td>(X-Pole, Y-Pole, LOD)</td>
</tr>
<tr>
<td>➢ Range biases (for the selected stations)</td>
</tr>
</tbody>
</table>

**Bernese Software, v.5.1**

**~110 parameters per week**

**~3000 obs. to LAGEOS-1/-2 per week**
Gravitational forces – Arg. of Perigee of LAGEOS–2

\[ \frac{d\omega}{dt} = \sqrt{\frac{p}{GM e^2}} \left( \cos v R' + \left(1 + \frac{r}{p}\right) \sin v S' \right) - \frac{d\Omega}{dt} \cos i \]

Only due to \( C_{20} \):

\[ \frac{d\omega}{dt} = \frac{3}{4} \frac{\sqrt{GM R_E^2}}{a^2 (1 - e^2)^2} C_{20} (1 - 5 \cos^2 i) \]

LAGEOS 1: 99.20% of drift explained by \( C_{20} \)
LAGEOS 2: 99.71% of drift explained by \( C_{20} \)
Rest can be explained by other zonal harmonics and relativistic effects

Gravitational forces dominate within forces acting on geodetic satellites
The secular decay of semi-major axis cannot be explained by gravitational forces

\[ \frac{da}{dt} = \sqrt{\frac{p}{GM}} \frac{2a}{1-e^2} \left( e \sin \nu R' + \frac{p}{r} S' \right) \]

**LAGEOS 1:**
\[ \frac{da}{dt} = -0.203 \text{ m/year} \]

**LAGEOS 2:**
\[ \frac{da}{dt} = -0.255 \text{ m/year} \]
Non–gravitational forces

1. Atmospheric drag
   - drag due to the electrically neutral atmosphere
   - drag due to charged particles
2. Radiation pressure
   - direct solar radiation pressure
     direct radiation
     umbra and penumbra radiations
     Light aberration
     - Earth radiation pressure
   - albedo infrared emissivity
   - albedo reflectivity
3. Thermal satellite re–radiation forces
   - Yarkovsky effect
   - Yarkovsky–Schach effect
4. Forces due to the satellite asymmetricity
5. Effects induced by the Earth’s magnetic field
6. Other non–gravitational forces
Thermal forces
Almost no net acceleration if there is no thermal inertia (because of cancellation when averaging over one orbit).

But the satellite does possess thermal inertia (delay in reradiation: 30-50 min.), which causes an acceleration in along-track (like a drag force).
Yarkovsky–Schach effect

No net acceleration if there is no satellite thermal inertia and no eclipses. Only once-per-rev components are affected.

During the eclipses the net acceleration in along-track occurs. The thrust decreases as the satellite cools down during the eclipse. The along-track acceleration can be positive or negative.
Empirical orbital forces – along-track

Red line – eclipsing day

\[ \frac{da}{dt} \approx \sqrt{\frac{p}{GM}} \]
Empirical orbital forces – along-track

<table>
<thead>
<tr>
<th></th>
<th>LAGEOS 1</th>
<th>LAGEOS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weeks</strong></td>
<td><strong>39%</strong></td>
<td><strong>40%</strong></td>
</tr>
<tr>
<td><strong>Outside the eclipses</strong></td>
<td><strong>61%</strong></td>
<td><strong>60%</strong></td>
</tr>
<tr>
<td><strong>Overall</strong></td>
<td><strong>100%</strong></td>
<td><strong>100%</strong></td>
</tr>
<tr>
<td><strong>S0 [10e-12 ms^-2]</strong></td>
<td><strong>-0.999</strong></td>
<td><strong>-1.366</strong></td>
</tr>
<tr>
<td></td>
<td><strong>-2.251</strong></td>
<td><strong>-2.276</strong></td>
</tr>
<tr>
<td></td>
<td><strong>-1.757</strong></td>
<td><strong>-1.913</strong></td>
</tr>
</tbody>
</table>
Empirical orbital forces – along-track

**Slide 16**

Sosnica K.: Non-Gravitational Forces Acting on Geodetic Satellites

Seminar Presentation, ETH Zurich, 14.11.2012

Astronomical Institute University of Bern

Period (days): 111 – eclipses, 222 – draconitic year, 365 – annual signal
Spin rate of LAGEOS–1/2

Values from Kucharski et al., “16 years of LAGEOS-2 Spin Data: From launch to present”
Proceedings of the conference held 12-17 October, 2008 in Poznan, Poland. Published online at http://cddis.gsfc.nasa.gov/lw16, p.61"
Semi-major axis of LAGEOS-1

The decay of Semi-major axis of LAGEOS-1:

'80: -0.40 m/y
'90: -0.34 m/y
'00: -0.14 m/y
Air drag
Air drag

\[ a_{drag} = -\frac{C_D}{2} \rho \frac{A}{m} v_{rel}^2 \frac{v_{rel}}{|v_{rel}|} \]

Air density \((\rho)\) of the high atmosphere depends on:

- Altitude of a satellite
- Time of the day, day of the year
- Geographic latitude and longitude,
- Solar activity
- Geomagnetic activity
Air drag

Time series of:
- daily F10.7cm solar flux index,
- APP Geomagnetic index

The relationship between atmospheric density at AJISAI’s and Stella’s altitudes and solar flux 10.7cm index
Total acceleration due to the air drag

![Graph showing the altitude of satellites over time](image)

- **AJISAI**
- **Starlette**
- **Stella**

X \(10^{-10} \text{ ms}^{-2}\)
Acceleration due to the air drag

- Acceleration due to air drag in out-of-plane is 20x smaller than in along-track.
- Acceleration in along-track is always negative.
- Acceleration in radial direction is negligible.
Decay of the semi-major axes

AJISAI: 12 m/year (2002-2012)

Stella: 30 m/year (2002-2012)
Decay of the semi-major axis and eccentricity

Starlette: 14 m/year
Semi-major axis
(2002-2012)

Starlette: $1.3 \times 10^{-6}$/year
orbital eccentricity
(2002-2012)

Secular drift
Albedo
Albedo

- albedo reflectivity (within visible spectrum)
- albedo emissivity (within infrared spectrum)

The Earth emissivity renders about 60% of total albedo effect and introduces a rather constant force in the radial direction.

Source of data: monthly global maps from the CERES mission, grids 2.5x2.5 (Rodriguez–Solano et al., 2011)
Coordinate system $\beta - \Delta u$

$\beta$ ... Elevation of the Sun above the orbital plane

$\Delta u$ ... Argument of latitude (satellite – Sun)
Acceleration due to albedo in radial direction

LAGEOS–2

Reflectivity only

Reflectivity and emissivity
Acceleration due to albedo

LAGEOS–2

Reflectivity and emissivity: along-track

Reflectivity and emissivity: out-of-plane
Impact of albedo on LAGEOS–2 orbit

Differences of empirical accelerations in along-track w.r.t. Solution 1

### Emissivity

<table>
<thead>
<tr>
<th>Solution</th>
<th>Emissivity</th>
<th>Reflectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution 1</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 2</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 3</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Solution 4</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>
Impact of albedo on LAGEOS–2 orbit

\[ n_c = n + \frac{1-e^2}{n a e} \left[ \left( \cos \nu - 2e \frac{r}{p} \right) R' - \left( 1 + \frac{r}{p} \right) \sin \nu S' \right] \]

\[ n_c = n - 2 \frac{r}{n a^2} R' = n - 2 \frac{2}{n a} R' \]

\[ n^2 a^3 = GM \]

Albedo introduces a force in an opposite direction to the Earth’s gravitational attraction

<table>
<thead>
<tr>
<th>Solution</th>
<th>Emissivity</th>
<th>Reflectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution 1</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 2</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 3</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Solution 4</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>
Impact of albedo on LAGEOS–2 orbit

In total LAGEOS orbits are diminished due to albedo by about 1.5 mm

Differences of semi–major axis w.r.t. Solution 1

<table>
<thead>
<tr>
<th>Solution</th>
<th>Emissivity</th>
<th>Reflectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution 1</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 2</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 3</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Solution 4</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>
Impact of albedo on station coordinates

A shift in up component of \(-0.2\) mm due to emissivity and \(-0.4\) mm due to albedo reflectivity

Differences of station coordinates w.r.t. Solution 1

<table>
<thead>
<tr>
<th>Solution</th>
<th>Emissivity</th>
<th>Reflectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution 1</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 2</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 3</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Solution 4</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>
Impact of albedo on station coordinates

Differences of station coordinates w.r.t. Solution 1

Wettzell

No range bias

Range bias estimated

A range bias absorbs the impact of albedo

<table>
<thead>
<tr>
<th>Solution</th>
<th>Emissivity</th>
<th>Reflectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution 1</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 2</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Solution 3</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Solution 4</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>
Impact of albedo on scale

Scale differences w.r.t. ITRF2008

Scale from 7 days LAGEOS-1 and -2 solution - All stations

<table>
<thead>
<tr>
<th>Mean bias</th>
<th>Emissivity</th>
<th>Reflectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale [ppb]</td>
<td>Std dev [ppb]</td>
<td>Solution 1</td>
</tr>
<tr>
<td>Solution 1</td>
<td>0.16</td>
<td>0.54</td>
</tr>
<tr>
<td>Solution 2</td>
<td>0.21</td>
<td>0.54</td>
</tr>
<tr>
<td>Solution 3</td>
<td>0.18</td>
<td>0.54</td>
</tr>
<tr>
<td>Solution 4</td>
<td>0.23</td>
<td>0.54</td>
</tr>
</tbody>
</table>
Multi-satellite solution
Multi-satellite solution

Current International Laser Ranging System’s (ILRS) products:

- LAGEOS–1/2 & Etalon–1/2 solutions only,
- On average ~3000 normal points to LAGEOS–1/2 and ~300 normal points to Etalon–1/2 per week,
- The impact of Etalon–1/2 on the solution is virtually negligible
Multi-satellite solution

- Some of the SLR stations do not observe LAGEOS, e.g., Helwan, Egypt (7831), Mendeleeevo, Russia (1870)

<table>
<thead>
<tr>
<th></th>
<th>LAGEOS-1</th>
<th>LAGEOS-2</th>
<th>AJISAI</th>
<th>Starlette</th>
<th>Stella</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of normal points per week</td>
<td>1500</td>
<td>1500</td>
<td>3000</td>
<td>1600</td>
<td>800</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>LAGEOS-1/2</th>
<th>LEO</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of stations per week</td>
<td>19.8</td>
<td>21.1</td>
<td>22.4</td>
</tr>
</tbody>
</table>
Why LEO satellites are neglected?

It is more difficult to model Starlette, Stella, AJISAI (LEO) orbits, because of:

- higher sensitivity to the Earth gravity field and to the temporal variations of the gravity field,
- uncertainties in air drag models and variations of air density in the high atmosphere,
- deficiencies of SLR station–specific Center–of–Mass corrections (CoM), due to different laser systems used in SLR stations.
Solution set-up

In the Starlette, Stella, and AJISAI (LEO) 7–day solutions we apply the air drag NRLMSISE–00 model with fixed scaling factors and we estimate:

- **Orbits:**
  - six osculation elements (daily)
  - constant (S0) and OPR sine and cosine terms (SS/SC) in along-track (daily),
  - OPR sine and cosine terms (WS/WC) in out-of-plane (daily),
  - *pseudo-stochastic pulses in along-track* (every revolution period) \(\rightarrow\) similar to CHAMP/GRACE/GOCE orbit solutions

- station coordinates (one set per 7–day arc),
- *range biases* (for all satellites and all stations, one set per 7–day arc),
- *gravity field coefficients* up to degree/order 4/4 (one set per 7–day arc),
- Piece Wise Linear (PWL) pole coordinates and Length–of–Day (one set at the daily boundary).
Results: Station coordinates

Time series of Up component of 7090

Time series of North component of 7090

Time series of East component of 7090
Results: Station coordinates

Station coordinate repeatability in LAGEOS–1/2 and the combined solutions

The station repeatability is improved in the combined solutions for East and North components of non-core SLR station.

Stations ordered by increasing number of weekly solutions.
Results: Earth Rotation Parameters (w.r.t. IERS C04)

<table>
<thead>
<tr>
<th></th>
<th>Starlette, Stella, Ajisai</th>
<th>Lageos -1/2</th>
<th>Combined solution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean bias</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X pole</td>
<td>57.7 μas</td>
<td>4.1 μas</td>
<td>6.4 μas</td>
</tr>
<tr>
<td>Y pole</td>
<td>-8.7 μas</td>
<td>-8.0 μas</td>
<td>-8.5 μas</td>
</tr>
<tr>
<td>LOD</td>
<td>-3.6 μs</td>
<td>6.1 μs</td>
<td>6.3 μs</td>
</tr>
<tr>
<td><strong>Weighted RMS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X pole</td>
<td>269.8 μas</td>
<td>160.0 μas</td>
<td>148.9 μas</td>
</tr>
<tr>
<td>Y pole</td>
<td>218.1 μas</td>
<td>155.2 μas</td>
<td>140.3 μas</td>
</tr>
<tr>
<td>LOD</td>
<td>106.5 μs</td>
<td>57.0 μs</td>
<td>56.3 μs</td>
</tr>
</tbody>
</table>

7% for X pole
10% for Y pole
Results: Geocenter

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LEO</td>
<td>LAGEOS</td>
<td>LEO+ LAGEOS</td>
</tr>
<tr>
<td>RMS</td>
<td>6.67</td>
<td>4.02</td>
<td>4.23</td>
</tr>
<tr>
<td></td>
<td>9.06</td>
<td>3.67</td>
<td>4.56</td>
</tr>
<tr>
<td></td>
<td>7.97</td>
<td>5.99</td>
<td>5.33</td>
</tr>
</tbody>
</table>

| Amplitude of annual signal | 2.97 | 2.99 | 3.40 | 5.28 | 2.49 | 2.94 | 4.68 | 3.64 | 4.13 |

![Graphs showing Geocenter X, Y, and Z movements over time in various satellite configurations, including LEO, LAGEOS, LEO+ LAGEOS, and a combined solution.](image-url)
Results: Gravity field coefficients

Comparison between $C_{20}$ obtained from SLR (weekly sol.) and GRACE (monthly sol.)

![Graph comparing $C_{20}$ from SLR and GRACE]
SLR observations of geodetic satellites are still a very efficient tool for the determination of low gravity field harmonics.
Summary

- Special handling of non-gravitational forces is important in order to achieve orbits of the highest precision,
- Thermal effects and atmospheric drag cause a decay of satellites' semi-major axes,
- Earth Albedo has an impact on SLR-derived scale,
- LAGEOS-only solutions can be improved by combining with lower geodetic satellites,
- Low spherical can be well established using SLR observations of geodetic satellites.
Thank you for your attention

Astronomical Institute, University of Bern, Switzerland

Special acknowledgments to:

- AIUB: Daniela Thaller, Adrian Jäggi, Rolf Dach, Christian Baumann, Gerhard Beutler, et al.,
- Swiss National Science Foundation for the financial support within the SNF Project 200021E-131228

ETH Zurich, 14.11.2012