

# Combining GNSS and SLR Measurements: Lessons Learned for GNSS Processing

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## INTRODUCTION

The combination of Global Navigation Satellite System (GNSS) and Satellite Laser Ranging (SLR) data is done commonly by using the connection at co-located GNSS-SLR sites (station coordinates together with local ties) and by common Earth rotation parameters (ERP). Co-location at the satellites is, however, generally ignored, although they offer an alternative - or additional - connection between the observing techniques. Such an alternative is of special interest because the problems related to the local ties and their discrepancies with the coordinate differences at co-located sites are well known. Fig. 1 visualizes the principle of the co-location of the GNSS and SLR technique at GNSS satellites. Using satellite co-location implies that one common set of orbit parameters is estimated based on GNSS microwave and SLR observations together. As the orbit parameters refer to the CoM, the vectors between the CoM and the phase center of the GNSS antenna and the LRA are needed.

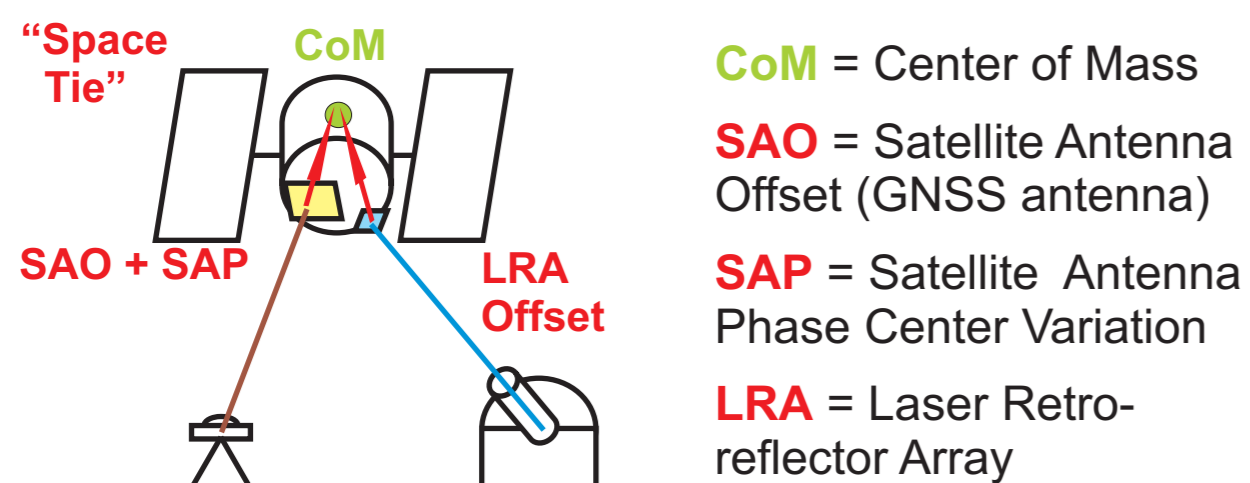


Fig. 1: Co-location of GNSS and SLR at GNSS satellites.

From SLR solutions using LAGEOS data we know that the strengths are the terrestrial reference frame parameters, i.e., geocenter and scale. Both parameters are problematic for GNSS data analysis due to correlations with un-modelled effects in the solar radiation pressure and with the SAO (see Tab. 1). The latter parameters should therefore benefit from a combined GNSS+SLR analysis. We performed a combined analysis of GNSS and SLR data employing the satellite co-locations using the *Bernese GPS Software*.

Correlation of parameters	GNSS @GNSS	SLR @GNSS	SLR @Lageos
Radiation pressure GEOCENTER	Problems in RPR modelling	Problems in RPR modelling	RPR well modelled
GNSS satellite antenna phase center SCALE	Problems in phase center modelling	independent	-
Range biases SCALE	-	Decorrelated if different elevation angles	For a few sites only

Tab. 1: Correlation between estimated parameters and the reference frame parameters for different observation types.

## SLR TRACKING OF GNSS SATELLITES

Two GPS satellites carry an LRA and are tracked by the SLR sites (G05, G06). All GLONASS satellites carry an LRA, but normally only three of them are tracked in parallel by the SLR sites. Unfortunately, the amount of SLR normal point (NP) observations to the GNSS satellites is small: 10 - 30 NP per day as sum over all stations (see Fig. 2). Many days are even without any SLR data.

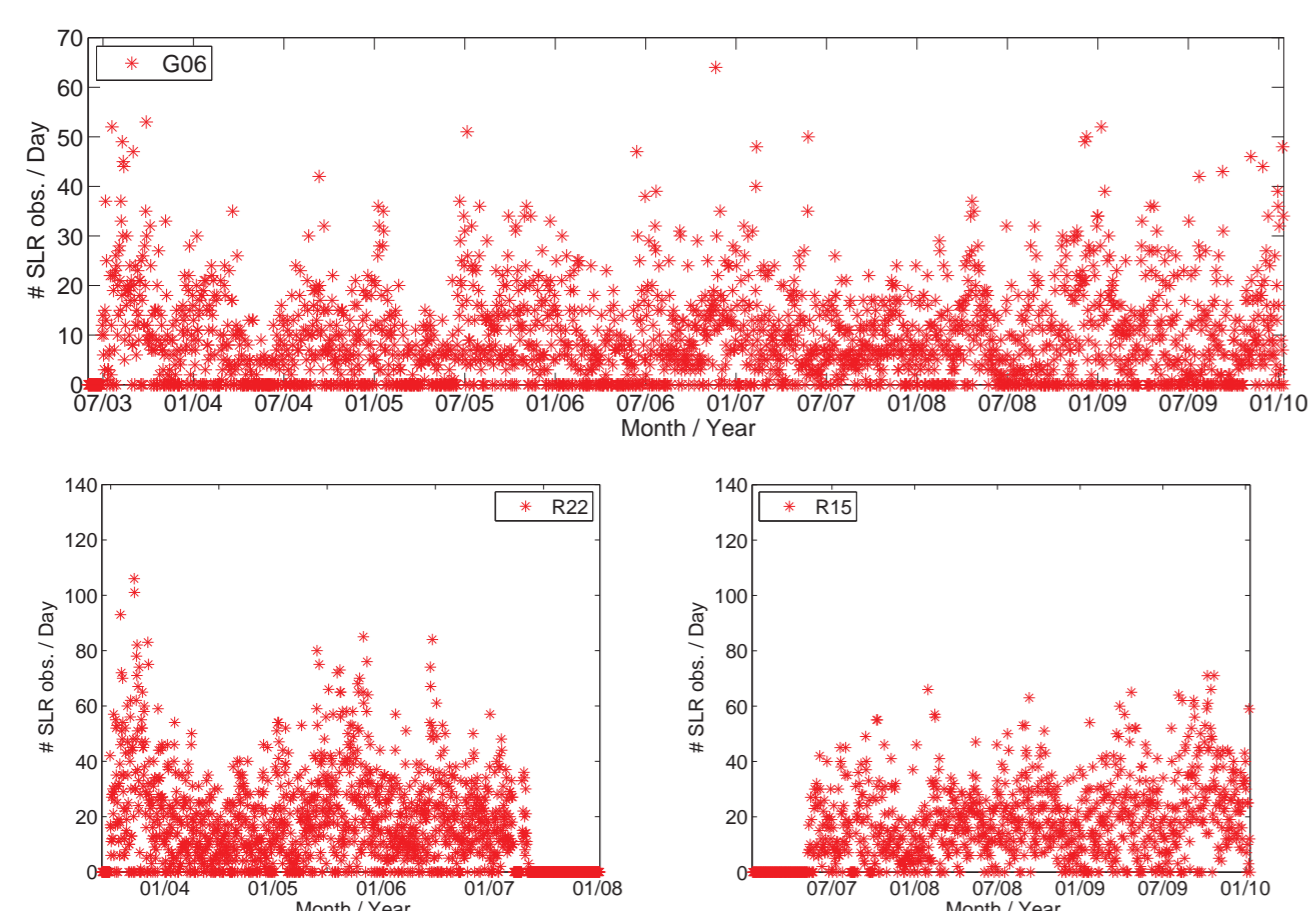


Fig. 2: Number of SLR NP per day to GNSS satellites.

## SLR RESIDUAL ANALYSIS

The SLR range residuals are computed with fixed  
 ✓ GNSS orbits based on microwave data only (taken from the CODE reprocessing) with GNSS SAO and SAP fixed on igs05.atx  
 ✓ SLR station coordinates: SLRF2005

Fig. 3 shows the mean biases per station for the two GPS satellites over the time span 2003-2009. All stations show similar biases of a few cm. Possible reasons for the biases might be:

- ✓ Errors in SLR station coordinates,
- ✓ Errors in GNSS SAO (and SAP),
- ✓ Errors in LRA offsets,
- ✓ SLR range biases not taken into account.

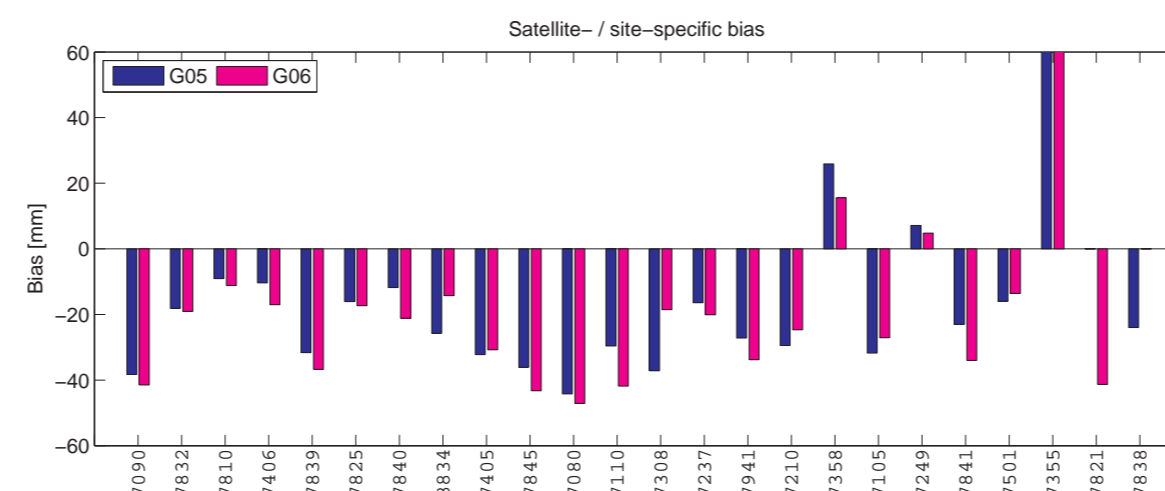


Fig. 3: Mean biases of SLR range residuals per station and satellite for 2003 - 2009. The mean bias over all stations is **-24.0 mm** and **-26.4 mm** for G05 and G06, respectively.

## COMBINED GNSS+SLR ANALYSIS

Daily combined normal equations (NEQ) are generated using microwave data and SLR data to GPS and GLONASS satellites. In order to find the reason for the discrepancies seen in Fig. 3, we estimated parameters that can account for the above mentioned error sources. The set-up of the relevant parameters can be seen in Tab. 2. The combination is done using the satellite co-locations only instead of the station co-locations, i.e., the local ties are not introduced into the solution.

	GNSS data	SLR data	Remark
Station coord.	estimated	estimated	not combined
GNSS sat. orbits	estimated	estimated	combined
ERP	estimated	estimated	combined
GNSS SAO	a) fixed b) estimated		scale defined scale from SLR
LRA offsets		fixed	Errors will be absorbed by the range biases
SLR range biases		estimated	

Tab. 2: Set-up of parameters in combined analysis. Two types of solutions are generated w.r.t. the SAO parameters.

Due to the small amount of data (see Fig. 2) the SLR-specific parameters (coordinates, range biases) cannot be estimated reasonably from daily solutions. Therefore, we accumulated the daily NEQs in order to get a long-term solution. Range biases have the same size for all directions of the observations, whereas the impact of the GNSS SAO in the observation modelling depend on azimuth and nadir angle in the satellite-fixed system. As a consequence, GNSS SAO and range biases are de-correlated in a combined analysis provided that the SLR observations are distributed well w.r.t. the nadir angle. Fig. 4 shows that this pre-requisite is fulfilled, thus, allowing for the common estimation of GNSS SAO and SLR range biases without relying on the scale of any a priori reference frame.

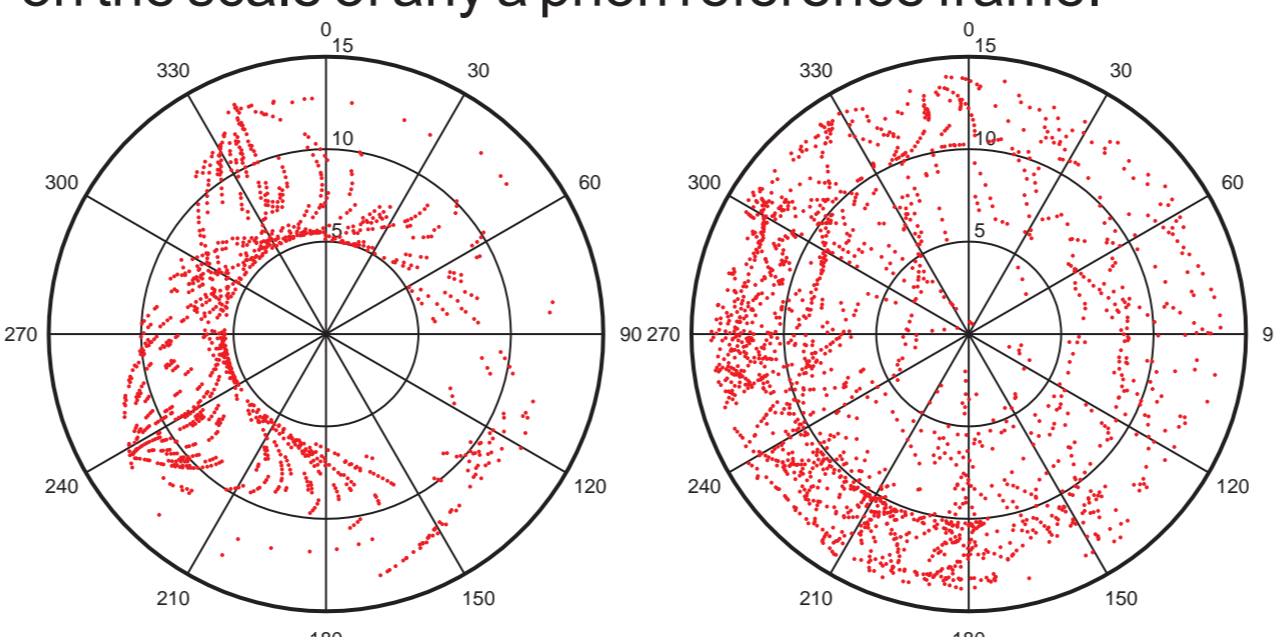


Fig. 4: Distribution of SLR observations of the station Yarragadee seen in the satellite-fixed system (azimuth and nadir angle) for the satellites G05 (left) and R15 (right).

## RESULTS

The estimated GNSS SAO are shown in Fig. 5. The corrections w.r.t. igs05.atx are at the cm-level. The corrections for GPS and GLONASS are quite different, demonstrating that there is an inconsistency in the igs05.atx.

The scale difference between the solution with fixed SAO and that with estimated SAO is:

- ✓ **0.59 ppb** for the GNSS sub-network
- ✓ **0.00 ppb** for the SLR sub-network

This demonstrates that the SLR scale is not influenced by estimating range biases so that the estimation of GNSS SAO becomes possible.

The scale difference for the GNSS network shows additionally that the SAO values of igs05.atx are not consistent with the SLR scale.

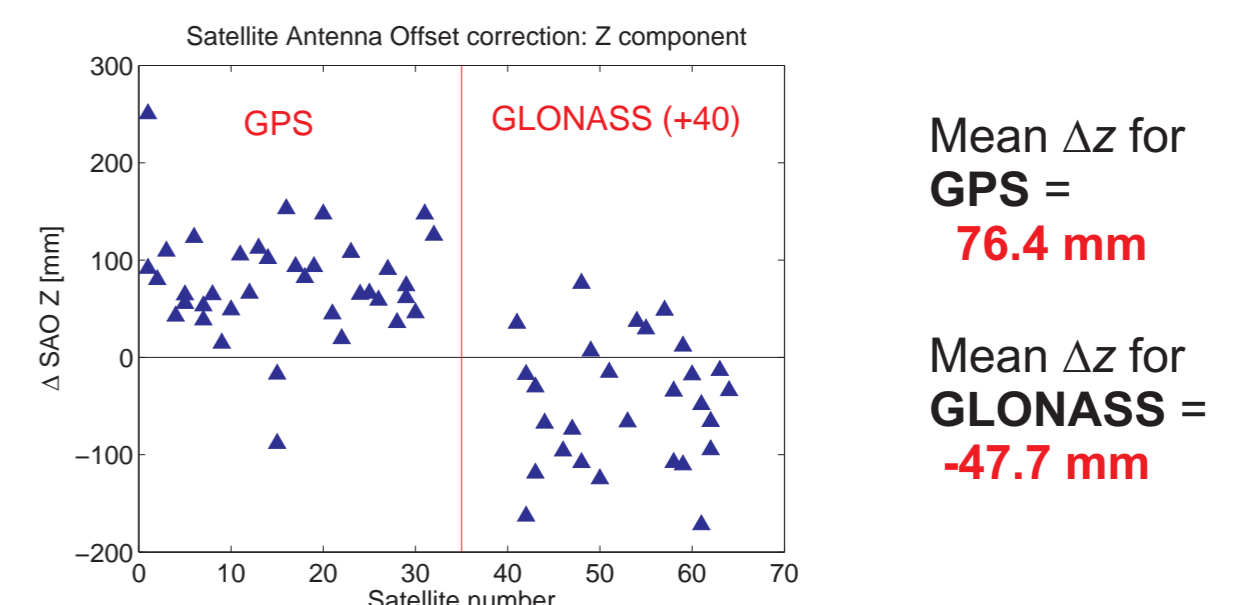


Fig. 5: GNSS SAO corrections w.r.t. igs05.atx (z-direction).

The satellite-specific SLR range biases are shown in Fig. 6 for two types of solutions: GNSS SAO estimated and fixed on igs05.atx values. The differences between both solution types are small (<5mm), what proves that the common estimation of GNSS SAO and SLR range biases is possible.

Fig. 7 shows a validation of the estimated station coordinates at co-located GNSS-SLR sites with the local ties. The agreement is better for most sites if the GNSS SAO are estimated and the scale is defined solely by SLR.

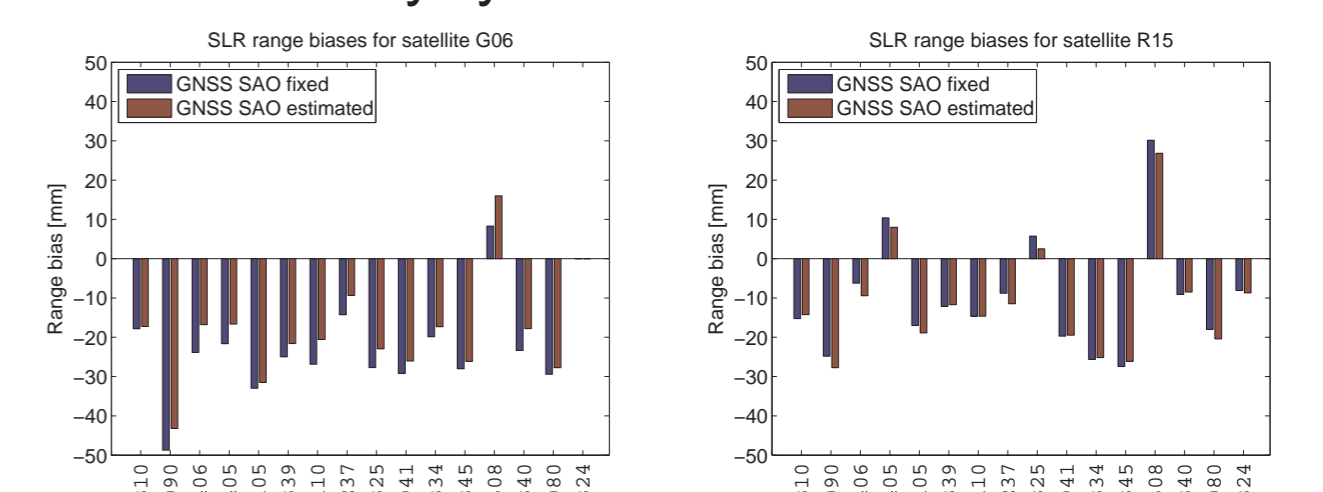


Fig. 6: SLR range biases to GNSS satellites estimated from combined solutions.

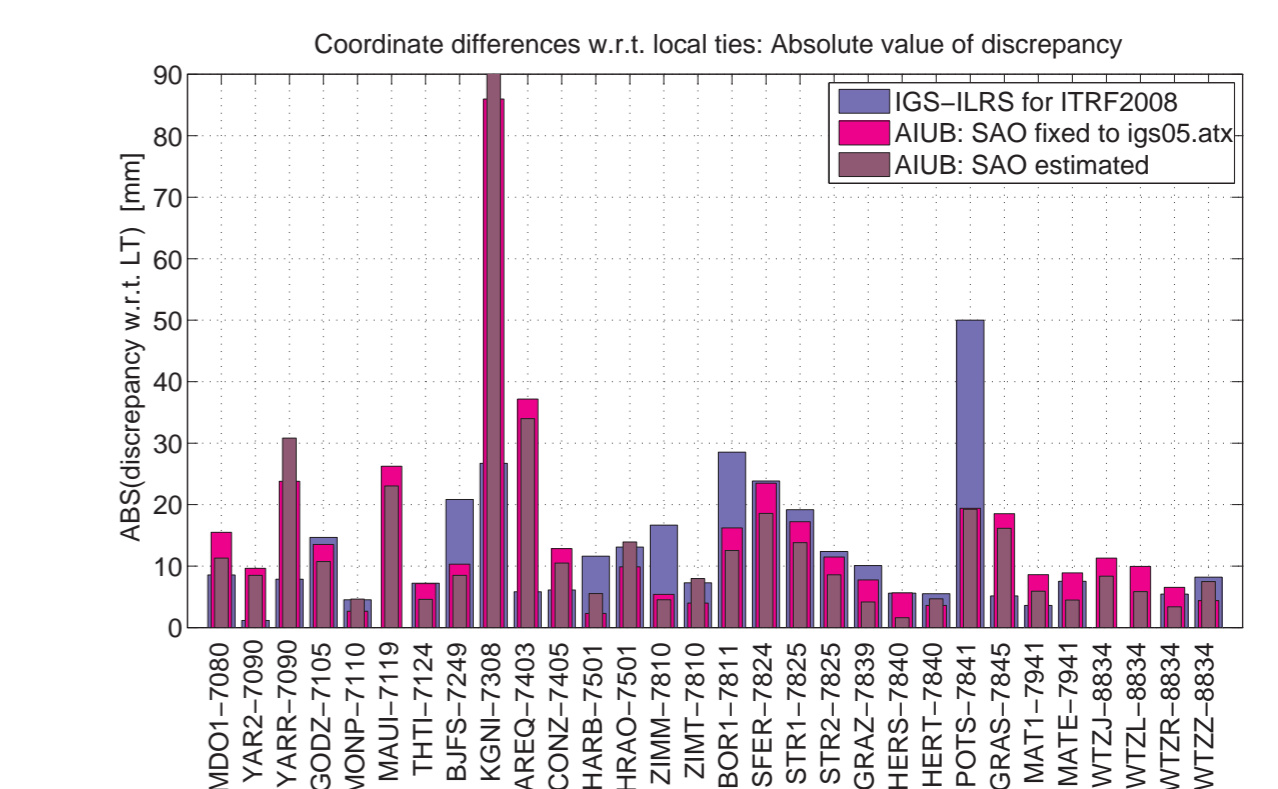


Fig. 7: Comparison of local ties with the coordinate differences between co-located GNSS-SLR sites.

## CONCLUSIONS

The estimation of common orbit parameters allows it to transfer the scale directly from SLR to GNSS. We demonstrated that the SLR-derived scale does not suffer from estimating range biases. Therefore, a combined GNSS-SLR analysis using satellite co-locations at GNSS satellites allows the estimation of the GNSS SAO without fixing the scale of the a priori reference frame. The resulting GNSS SAO are consistent with the SLR scale, what is actually not the case for the official IGS values provided in igs05.atx. Additionally, the SAO values for the GLONASS satellites need to be updated in order to be consistent with those for the GPS satellites.