PRECISE ORBIT DETERMINATION OF LOW EARTH SATELLITES AT AIUB

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ABSTRACT

Many low Earth orbiting (LEO) satellites are nowadays equipped with on-board receivers to collect the observations from Global Navigation Satellite Systems (GNSS), such as the Global Positioning System (GPS), or with retro-reflectors for Satellite Laser Ranging (SLR). At the Astronomical Institute of the University of Bern (AIUB) LEO precise orbit determination (POD) using either GPS or SLR data is performed for satellites at very different altitudes. The classical numerical integration techniques used for dynamic orbit determination of LEO satellites at high altitudes are extended by pseudo-stochastic orbit modeling techniques for satellites at low altitudes to efficiently cope with force model deficiencies. Accuracies of a few centimeters are achieved by pseudo-stochastic orbit modeling, e.g., for the Gravity field and steady-state Ocean Circulation Explorer (GOCE).

Key words: Low Earth orbiting (LEO) satellites; Precise orbit determination (POD); Pseudo-stochastic orbit modeling.

1. INTRODUCTION

The Astronomical Institute of the University of Bern (AIUB) has a well-documented record concerning the scientific analysis of Global Navigation Satellite System (GNSS) data with the Bernese GPS Software [4]. The Center for Orbit Determination in Europe (CODE), a global analysis center of the International GNSS Service (IGS), generates the full IGS product line, in particular GNSS orbits and high-rate satellite clock corrections, which are used as input for spaceborne applications relying on GNSS data. Spaceborne measurements of the Global Positioning System (GPS) are used at AIUB to determine precise kinematic and reduced-dynamic orbits for a variety of low Earth orbiting (LEO) satellites. The classical dynamic orbit determination of LEO satellites at high altitudes is extended by so-called pseudo-stochastic orbit modeling techniques for satellites at low orbital altitudes to efficiently cope with force model deficiencies. Currently the procedures are used by AIUB to derive the precise science orbits (PSOs) for the GOCE mission in the frame of the High-level Processing Facility (HPF) [7].

2. ORBIT DETERMINATION

The equation of motion of an Earth orbiting satellite including all perturbations reads in the inertial frame as

$$\ddot{\boldsymbol{r}} = -GM\frac{\boldsymbol{r}}{r^3} + \boldsymbol{f}_1(t, \boldsymbol{r}, \dot{\boldsymbol{r}}, q_1, ..., q_d) \doteq \boldsymbol{f} \ , \qquad (1)$$

where GM denotes the gravity parameter of the Earth, $\boldsymbol{r}$ and $\dot{\boldsymbol{r}}$ represent the satellite position and velocity, and $\boldsymbol{f}_1$ denotes the perturbing acceleration. The initial conditions $\boldsymbol{r}(t_0) = \boldsymbol{r}(a, e, i, \Omega, \omega, T_0; t_0)$ and $\dot{\boldsymbol{r}}(t_0) = \dot{\boldsymbol{r}}(a, e, i, \Omega, \omega, T_0; t_0)$ at epoch t_0 are defined by six Keplerian osculating elements, e.g., $a, e, i, \Omega, \omega, T_0$. The parameters $q_1, ..., q_d$ in Eq. 1 denote additional dynamical orbit parameters considered as unknowns.

Based on a numerically integrated a priori orbit $\boldsymbol{r}_0(t)$ solving Eq. 1, dynamic orbit determination may be formulated as an orbit improvement process. The actual orbit $\boldsymbol{r}(t)$ is expressed as a truncated Taylor series with respect to n unknown orbit parameters p_i about the a priori orbit, which is represented by the parameter values p_{i0}:

$$\boldsymbol{r}(t) = \boldsymbol{r}_0(t) + \sum_{i=1}^{n} \frac{\partial \boldsymbol{r}_0}{\partial p_i}(t) \cdot (p_i - p_{0,i}) \ . \qquad (2)$$

Provided that the partial derivatives of the a priori orbit with respect to the unknown parameters are known, corrections to the a priori orbit parameters p_{i0} may be computed in a standard least-squares adjustment of tracking data together with measurement-specific parameters, e.g., ambiguity parameters for GPS carrier phase data or range biases for SLR data. The improved orbit may be eventually obtained by either using Eq. 2 or by propagating the improved state vector by numerical integration and by taking into account the improved dynamical orbit parameters.

2.1. Variational equations

The initial value problem associated with the partial derivative $\boldsymbol{z}_{p_i} \doteq \partial \boldsymbol{r}_0 / \partial p_i$ is referred as the system of variational equations [1] and obtained by taking the partial derivative of Eq. 1. The variational equations for parameter p_i read as

$$\ddot{\boldsymbol{z}}_{p_i} = \boldsymbol{A}_0 \cdot \boldsymbol{z}_{p_i} + \boldsymbol{A}_1 \cdot \dot{\boldsymbol{z}}_{p_i} + \frac{\partial \boldsymbol{f}_1}{\partial p_i} \ , \qquad (3)$$

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where the 3×3 matrices $\boldsymbol{A}_0$ and $\boldsymbol{A}_1$ are defined by

$$A_{0[i;k]} \doteq \frac{\partial f_i}{\partial r_{0,k}} \qquad \text{and} \qquad A_{1[i;k]} \doteq \frac{\partial f_i}{\partial \dot{r}_{0,k}}\,, \qquad (4)$$

where f_i denotes the i-th component of the total acceleration $\boldsymbol{f}$ in Eq. 1. For $p \in \{a, e, i, \Omega, \omega, T_0\}$ Eq. 3 is a linear, homogeneous, second-order differential equation system with initial values $\boldsymbol{z}_p(t_0) \neq \boldsymbol{0}$ and $\dot{\boldsymbol{z}}_p(t_0) \neq \boldsymbol{0}$, which may be solved by numerical integration techniques. For $p \in \{q_1, ..., q_d\}$ Eq. 3 is inhomogeneous with zero initial values. As the homogeneous part of Eq. 3 is the same as for the parameters p defining the initial values, the inhomogeneous system may be solved by the method of variation of constants, which reduces the problem to numerical quadrature [1].

2.2. Pseudo-stochastic orbit modeling

Purely dynamic LEO POD is a challenge for satellites at low orbital altitudes due to unavoidable deficiencies in the non-gravitational force models. If dense tracking data are available, however, use may be made of their geometric strength by adopting reduced-dynamic orbit determination techniques [16]. At AIUB, so-called pseudo-stochastic parameters, e.g., instantaneous velocity changes or piecewise constant accelerations with a user-specified spacing in the radial, along-track, and cross-track direction, are added to the deterministic equation of motion (1). They may be efficiently set up because the solution of Eq. 3 is obtained as a linear combination of a few independent variational equations [9].

Pseudo-stochastic parameters are primarily intended to compensate for force model deficiencies and are characterized by a priori variances which constrain them to zero. If dense tracking data are available, pseudo-stochastic parameters may be set up frequently and can be used to replace deterministic force models to some extent, e.g., atmospheric drag models.

3. GOCE ORBIT DETERMINATION

AIUB is responsible for the generation of the PSO product of the GOCE mission [2, 6]. The 5 s GPS clock corrections [3] and the GPS final orbits from CODE [5] are used to process the full amount of 1 s GPS data for an arclength of 30 hours. The parameters of the reduced-dynamic orbit of the PSO product are the six initial osculating elements, three constant empirical accelerations acting over the entire arc in the radial, along-track, and cross-track directions, and piecewise-constant accelerations over 6 min acting in the same directions. No use is made of the GOCE common-mode accelerometer data or non-gravitational force models, which implies that the piecewise constant accelerations mainly compensate the not explicitly modeled non-gravitational accelerations. Due to the low orbital altitude only weak constraints are imposed on the piecewise constant accelerations.

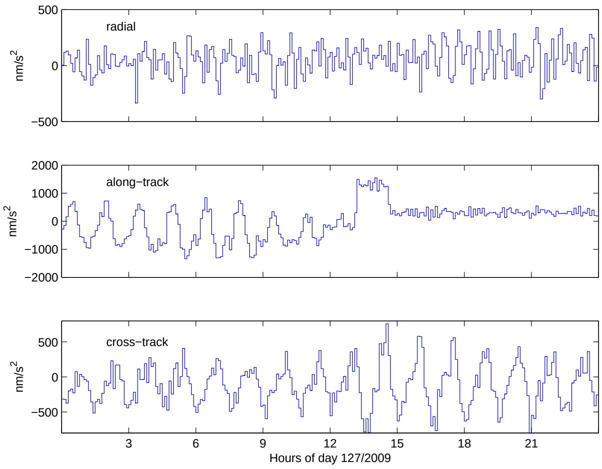

Figure 1. Piecewise constant accelerations estimated for GOCE on 7 May, 2009. Note the different scales for the three components and the transition into drag-free flight.

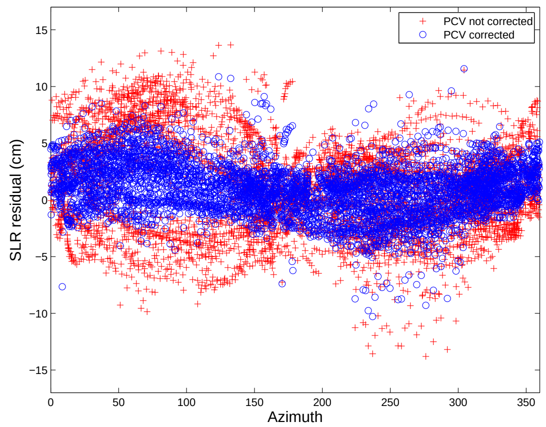

Figure 2. GOCE SLR residuals plotted as a function of the azimuth of the SLR stations for PSO solutions computed with PCV corrected or not corrected.

Fig. 1 shows the estimated piecewise constant accelerations on 7 May, 2009. On that day thrust biases of 4 mN at maximum and follow-up biases of about 2-2.5 mN brought GOCE into the first drag-free flight ever [12]. Fig. 1 illustrates that the along-track drag is compensated to a large extent during the drag-free flight and that the remaining variations are reduced to a magnitude similar to the radial direction. No adaption for POD had to be made on that day.

Independent SLR measurements may be used to compare the computed ranges between the GPS-based GOCE orbit solutions and the SLR ground stations with the observed ranges. Fig. 2 shows for a three-months period (August - October, 2009) that special care has to be taken to model the phase center variations (PCVs) of the GOCE GPS helix antenna. Adopting the methodologies described by Jäggi et al. [11], the GOCE orbit solutions may be improved from 4.4 cm to 2.5 cm SLR RMS.

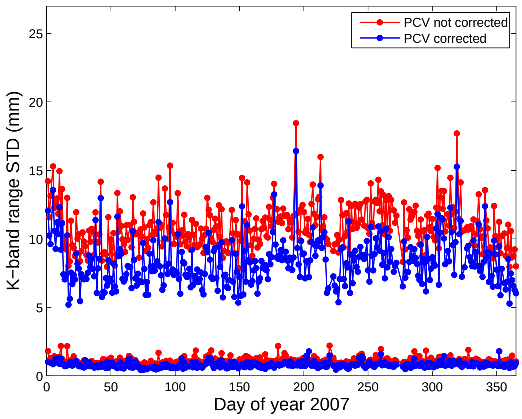

Figure 3. Daily K-band range STD for distances between reduced-dynamic GRACE-A and GRACE-B orbits using undifferenced (top) or doubly-differenced GPS data with resolved carrier phase ambiguities (bottom).

4. GRACE ORBIT DETERMINATION

Undifferenced or doubly differenced GRACE GPS data have been extensively used at AIUB for various studies on LEO POD, e.g., [10] and [11], and for gravity field recovery with inter-satellite K-band data [13]. The parametrization used for reduced-dynamic GRACE POD is the same as applied for GOCE, but more tight constraints are imposed on the piecewise constant accelerations due to the higher orbital altitude of the GRACE satellites.

Independent K-band measurements may be used to compare the GPS-derived distances between the reduced-dynamic orbits of GRACE-A and GRACE-B with the biased ranges which are directly observed by the K-band ranging system. The daily K-band range standard deviations (STDs) of the year 2007 (see Fig. 3) confirm that the GRACE GPS choke-ring antennas exhibit significant PCVs as well. Adopting the methodologies described by Jäggi et al. [11], the GRACE orbit solutions may be improved from 10.9 mm to 8.3 mm K-band STD when using undifferenced GPS data, and from 1.1 mm to 0.8 mm when using doubly differenced GPS data with resolved carrier phase ambiguities. The latter aspect is of importance for the TanDEM-X interferometry mission, where baseline vectors between TerraSAR-X and TanDEM-X have to be determined with an accuracy of 1 mm [14].

PCVs of LEO GPS receiver antennas are not only important for LEO POD, but also for orbit-based applications such as gravity field recovery. Fig. 4 shows the square-roots of difference degree variances of gravity field recoveries based on kinematic GRACE orbits of the year 2007. The differences to ITG-GRACE03S (based on K-band data, [8]) show that unmodeled PCVs propagate via kinematic orbits into the gravity field solutions and deteriorate the low-degree coefficients, especially for GRACE-A due to receiver internal cross-talk caused by the active GPS occultation antenna.

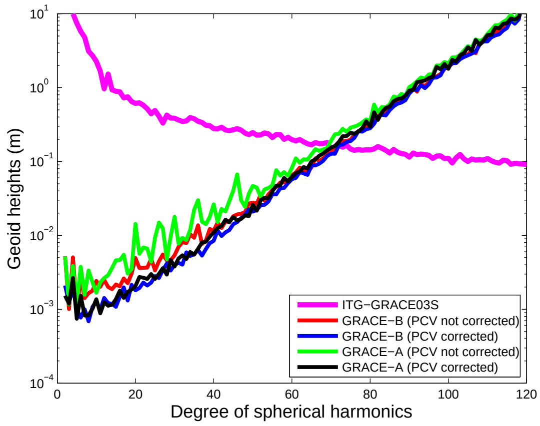

Figure 4. Square-roots of degree difference variances of gravity field recoveries based on one year of kinematic GRACE orbits.

5. JASON-2 ORBIT DETERMINATION

Undifferenced GPS data of JASON-2 have been processed at AIUB to gain experience on LEO POD at higher orbital altitudes, where no longer air-drag but solar radiation pressure is the dominating non-gravitational perturbation [1]. Therefore, and due to a worse GPS tracking performance compared to GRACE and GOCE, a more dynamic orbit representation is aimed at. Apart from the six initial osculating elements, constant and once-per-revolution periodic terms acting over the entire arc are set up in the direction sun-satellite, in the perpendicular direction pointing along the solar panel axis, and in the direction complementing the right-handed orthogonal orbital frame. In order to cope with remaining model deficiencies (no a priori radiation pressure model is taken into account due to the complicated shape of the satellite), pulses are set up every 15 min in the radial, along-track, and cross-track direction. As opposed to Sects. 3 and 4 spacecraft orientation data have to be taken into account to distinguish between sinusoidal or yaw-fixed attitude steering.

Independent SLR measurements may be used to validate the GPS-based JASON-2 orbit solutions. Fig. 5 shows that the accuracy is currently at a level of about 5 cm, which indicates that further investigations are required to find the optimal trade-off between pseudo-stochastic and dynamic orbit modeling.

6. LAGEOS ORBIT DETERMINATION

The Bernese GPS Software has been extended to become a full SLR analysis software [15] for processing SLR data

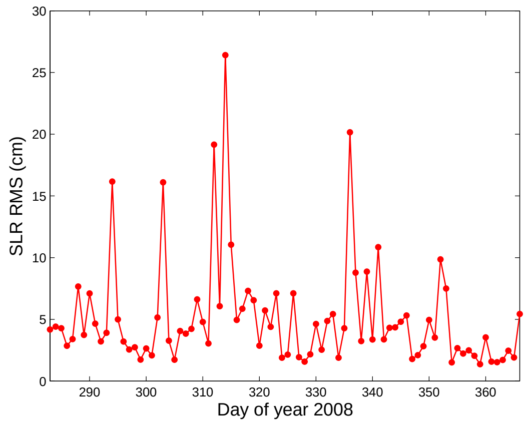

Figure 5. Daily SLR RMS for reduced-dynamic JASON-2 orbits using undifferenced GPS data.

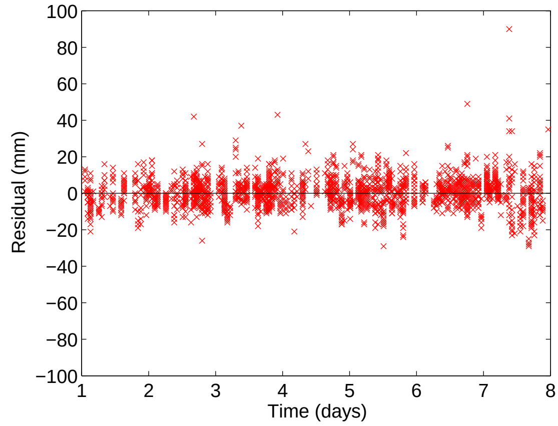

Figure 6. SLR residuals of one weekly solution based on LAGEOS-1 and -2 data.

to spherical satellites, e.g., to the LAGEOS and ETALON satellites.

As opposed to the LEO satellites mentioned in the previous sections, the LAGEOS satellites are orbiting the Earth at a considerably larger orbital altitude. Therefore, and due to the sparse SLR tracking coverage, a purely dynamic orbit representation is aimed at. The strongest non-gravitational perturbations are caused by solar radiation pressure, but may be well described for spherical satellites and are thus taken into account by a priori models. As a consequence of the simple modeling, 7-day arcs are generated with only a few additional parameters estimated during orbit determination. Apart from the six initial osculating elements, only one constant acceleration in the along-track direction and once-per-revolution accelerations in the along-track and cross-track directions acting over the entire arc are set up.

Weekly solutions for the LAGEOS satellite orbits are estimated at AIUB together with Earth rotation parameters and station coordinates following the standards of the analysis centers of the International Laser Ranging Service (ILRS). Figure 6 shows the SLR residuals of one weekly solution using LAGEOS-1 and -2 data. Only about 2000 observations are available for both satellites. The orbits are fitted with an RMS of 8.3 mm.

7. CONCLUSIONS

LEO POD using either GPS or SLR data is performed at AIUB for satellites at very different altitudes. Provided that dense GPS tracking data is available, accuracies of about 2 cm are achieved by pseudo-stochastic orbit modeling for LEO satellites at low orbital altitudes such as GOCE and GRACE. Systematic GPS carrier phase errors, e.g., LEO receiver antenna PCVs, are important to be carefully modeled for high-precision LEO POD and further applications such as gravity field recovery. Experience on GPS-based LEO POD at higher orbital altitudes has been gained at AIUB as well by processing data from JASON-2. The current accuracy level of 5 cm indicates that the optimal trade-off between pseudo-stochastic orbit modeling and a dynamic orbit representation has not yet been found for JASON-2 POD, which requires further investigations. Purely dynamic orbit fits of the LAGEOS satellites based on SLR data are at a satisfactory level of about 8 mm.

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