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Gravity Field Recovery with Highly Reduced-Dynamic Orbits compared with Kinematic Orbits

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Introduction

In this study, the so-called highly reduced-dynamic (HRD) orbit determination strategy, which is a compromise of the two extreme cases of dynamic and purely geometrically determined kinematic orbits, and its use for the determination of the Earth's gravitational field, are analyzed. For gravity field (GF) recovery the energy integral approach is applied, where the satellite velocity is the basic input. The potential of HRD orbits is studied in the frame of a test environment based on a realistic GOCE orbit configuration. The results are analyzed, assessed, and compared with the respective reference solutions based on a kinematic orbit scenario. The main advantage of HRD orbits is the fact that they contain orbit velocity information, thus avoiding numerical differentiation of the orbit positions. The error characteristics are usually much smoother, and the computation of gravity field solutions are more efficient, because less densely sampled orbit information is sufficient. Nevertheless, the main drawback of HRD orbits is that they contain external GF information, and thus yield the danger to obtain GF results which are biased towards this prior information.

Mathematical Description

HRD Orbit Determination

We use the so-called pseudo-stochastic orbit modeling technique for computing reduced-dynamic orbits, which are characterized by six initial osculating elements and a user-specified number of additional pseudo-stochastic orbit parameters. These parameters are either set up as unconstrained velocity changes (pulses) at predefined epochs or as unconstrained piecewise constant accelerations over predefined time intervals. The latter are presented in this poster. The partial derivatives of an a priori orbit with respect to pseudo-stochastic parameters, which are needed to set up the GPS observation equations for reduced-dynamic orbit determination, may be expressed as linear combinations of the partial derivatives with respect to six initial osculating elements $o_1,...,o_6$. The partial derivative $z_{a_{ij}}$ with respect to a constant acceleration a_{ij} between times t_{i-1} and t_i in direction j may be expressed by Eq. (1).

$$\mathbf{z}_{a_{ij}}(t)=\begin{cases}\mathbf{0} & ;t<t_{i-1}\\ \sum_{k=1}^{6}\alpha_{ij,k}(t)\,\mathbf{z}_{o_k}(t) & ;t\in[t_{i-1},t_i)\\ \sum_{k=1}^{6}\alpha_{ij,k}(t_i)\,\mathbf{z}_{o_k}(t) & ;t\geq t_i\,.\end{cases} \qquad (1)$$

GF Recovery using the energy integral approach

Since the gravitational potential V is basically calculated according to Eq. (2), the error of V is, besides smaller effects, directly related to the error of the velocities $|\dot{\mathbf{x}}|$. Hence, the error can be approximated by Eq. (3). Since the velocities are computed by applying numerical differentiation, it automatically amplifies high-frequency noise. Therefore, highly accurate velocities are the key for a high-quality GF solution within the energy integral approach. For this reason HRD orbits, where in contrast to kinematic orbit solutions also orbit velocities are part of the solution, are an interesting alternative.

$$V=\frac{1}{2}|\dot{\mathbf{x}}|^2 \qquad (2)$$

$$\sigma_V=|\dot{\mathbf{x}}|\cdot\sigma_{\dot{\mathbf{x}}} \qquad (3)$$

Detailed information can be found in:
Jäggi A., (2006). Pseudo-Stochastic Orbit Modeling of Low Earth Satellites Using the Global Positioning System. PhD Thesis, Astronomical Institute, University of Bern, Switzerland.
Jäggi, A., H. Bock, R. Pail, and H. Goiginger (2008). Highly reduced-dynamic orbits and their use for global gravity field recovery - a simulation study. Studia Geophysica et Geodaetica. In print.

Numerical case studies

Initial point of the numerical case studies is a set of test configurations. Different kinds of simulated GOCE orbits, which contains to a greater or lesser extent external gravity information, were generated by the University of Bern. Furthermore, as a reference, a kinematic orbit was produced. For comparison a kind of 'true' orbit is needed which is provided by a noise-free scenario. The orbits have a temporal coverage of one month, a sampling rate of 1 Hz, and are based on the GF model EIGEN-2 complete to degree/order (d/o) 90. Alternatively, a modified, i.e., an artificially degraded GF model is used as a priori model for the HRD orbit determination. The modified model is the EIGEN-2 model truncated at d/o 20, whereas the remaining low degree spherical harmonic coefficients are slightly modified by applying random errors corresponding to the standard deviations of the EIGEN-2 coefficients. Furtermore, a GPS phase noise of 1 mm is superposed on both the kinematic and the HRD orbit positions.

Kin. vs. HRD Orbits, 1 s and 30 s Sampling

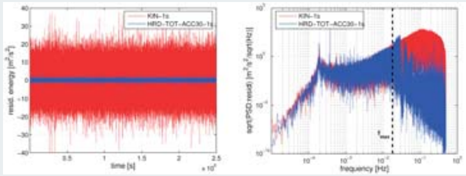

Fig. 1 Residual energy and corresponding PSD (right) based on 'KIN-1s' and 'HRD-TOT-ACC-1s'.

The more convenient error characteristics of the HRD positions can be well recognized. Although the magnitude of the residuals of the kinematic orbit is about four times larger than of the HRD orbit, the final GF solutions are very similar (cf. Fig. 2). The minor differences in the low degrees can be attributed to the incorrect GF information introduced in the 'HRD-TOT-ACC30-1s' orbit solution. The vertical line in Fig. 1 (right) shows the maximum frequency that can be attributed to a GF signal complete to d/o 90. Consequently, the filtering effect of the HRD scenario acts almost exclusively in the spectral region which is not relevant for a GF solution up to d/o 90. This explains the fact that there are no signicant differences between these two solutions.

This is not true in case of 30 s sampled positions. While the 30 s HRD solution is quite similar to the corresponding 1 s kinematic and HRD solutions, the 'KIN-30s' solution is markedly degraded. This underlines that compared to the 30 s sampled HRD orbit, the additional observations of the 1 s HRD orbit do not contain additional GF information, i.e., the GF is fully represented by the coarser 30 s sampling. This, however, is not true for the kinematic orbit. The redundancy due to the 1 s sampling is absolutely necessary to achieve a similar performance as in the case of the HRD orbits, because every data point contains additional GF information.

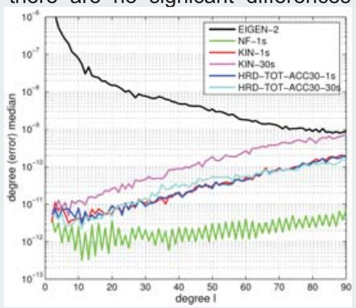

Fig. 2 Deviations from the true GF model in terms of the degree error median: Kinematic vs. HRD orbits, 1 s vs. 30 s sampling interval.

The Effect of a priori GF Information

One of the key parameters of the HRD orbit determination is the time period for the piecewise constant accelerations. A longer period results in a stronger a priori GF signal introduced into the orbit solution and vice versa.

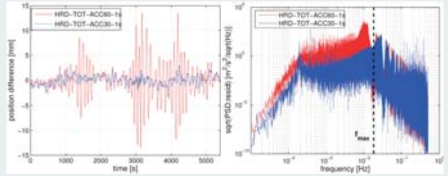

Fig. 3 Absolute pos. differences of the HRD orbit based on 30 s and 60 s piecewise constant accelerations w.r.t. to the true orbit for a period of one revolution (left) and PSD of corresponding residual energies (right).

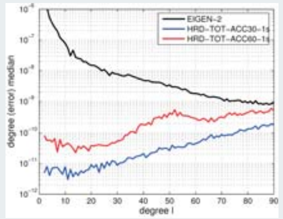

Fig. 4 DEM of the GF solutions based on HRD orbits with 30 s and 60 s accelerations.

Obviously, compared to the 'ACC30-1s' orbit the position differences of the 'ACC60-1s' are significantly larger (cf. Fig. 3 (left)) and show a systematic error component, which is introduced by the longer period of (artificially degraded) piecewise constant accelerations. This effect is also evident in the PSD of the residuals (Fig. 3 (right)). Firstly, the amplitude of the residual energy of 'ACC60-1s' is in general higher than the amplitude of 'ACC30-1s', and secondly, the high-frequency signal component is cut off, leading to a substantially degraded GF solution particularly for degrees 35 and higher.

Conclusion

The final GF solution is unavoidably more or less biased towards the prior information as shown by the studies. If the goal is to produce a GF solution, which is GOCE-only to the highest possible extent, i.e., ideally without using external GF information, the kinematic orbit type has to be favored. The price to pay when using kinematic orbits are less convenient error characteristics and higher noise amplitudes. Since onboard GPS receivers demonstrated their acceptable performance in determining the position of LEOs, the orbit determination process based on geometry without the involvement of any a priori information should be the favored procedure for GF mapping.